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Does a CBDC Reinforce Inefficiencies?

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Does a CBDC Reinforce Inefficiencies?

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Abstract: This paper examines whether a central bank digital currency (CBDC) reinforces inefficiencies in transactions with cash. In this case, the gap between the traded quantity and the welfare-maximizing one, which arises due to discounting or a suboptimal amount of money, increases further. To get some answers, the monetary search model of Trejos and Wright (1995) is extended by a CBDC. We show that an interest-bearing CBDC reinforces inefficiencies in transactions with cash since opportunity costs for cash holders and money supply increase. Nevertheless, a CBDC is able to increase welfare as long as the share of CBDC holders is limited.

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1 Introduction

Monetary search models show that the first best allocation, the welfare-maximizing quantity of goods where marginal utility equals marginal cost, is missed due to inefficiencies, which either occur owing to discounting or a suboptimal amount of money. In the case of discounting, sellers produce less while costs of production incurred today, the utility of consumption is tomorrow. In addition, if money supply is too high, sellers also produce less since they know that they are in the minority and about their good position. If a central bank issues a central bank digital currency (CBDC) to tilt the playing field for rising private digital payment systems, money supply and thus inefficiencies in transactions with cash could increase further. This effect would be even stronger if a CBDC is interest-bearing.

To answer whether an interest-bearing CBDC reinforces inefficiencies in transactions with cash, the monetary search model of Trejos and Wright (1995, henceforth TW) is extended by a CBDC. We show that a CBDC reinforces inefficiencies in transactions with cash since opportunity costs for cash holders and money supply increase. Nevertheless, a CBDC increases welfare as long as the share of CBDC holders is limited. Otherwise, money supply and thus the price level is too high.

As generation two of monetary search models, the TW framework is the only one which enables an examination of the effects of two payment systems on another. In generation one, i.e., Kiyotaki and Wright (1993), goods and money are indivisible. As a consequence, the value of a currency, that is the quantity of goods a buyer receives for one monetary unit, is always one. Generation one is mainly used to mark off partially from fully accepted payment systems. For instance, Fuchs and Michaelis (2021) show that a partially accepted currency, which circulates as a second payment system next to fully accepted cash, only increases welfare if the second payment system and cash are complements. If the second payment system is also fully accepted, on the other hand, welfare increases if both payment systems are close substitutes. In generation three, i.e. Lagos and Wright (2005), money demand is not affected by properties of other payment systems. Generation three is mainly used to model further groups next to sellers and buyers; see Fuchs (2022).

Of course, there are already studies with two payment systems building on TW, e.g,. Craig and Waller (2000) or Camera et al. (2004), but these studies make use of a take-it-or-leave-it offer. In this case, buyers purchase the quantity where costs of production are equal to the surplus of switching positions for sellers. As a consequence, the welfare-maximizing quantity of goods is always traded and questions about inefficiencies cannot be answered.

The structure of the paper is as follows: chapter 2 deals with the framework and, in particular, with the traded quantity in transactions with cash. Chapter 3 describes the dual currency regime and examines how a CBDC affects the traded quantity in transactions with cash. Chapter 4 compares welfare between a single and a dual currency regime. Finally, chapter 5 concludes.

2 Framework

As in TW, there is a [0,1]-continuum of agents which is divided into sellers $1-\mu$ and buyers $\mu \in (0,1)$. In the initial stage, sellers have no endowment while buyers have one monetary unit. Agents of type $i \in \{1,...,I\}$ with $I \geq 3$, prefer only goods of type i but produce goods of type $i+1$ (modulo I); see also Matsuyama et al. (1993). Thus, nobody consumes own production and pure barter does not take place.

As a consequence, money is necessary for trading. As soon as buyers meet sellers who are able to produce the preferred good of buyers, a Nash bargaining process decides about the traded quantity for one monetary unit. Afterwards, sellers start to produce the preferred good and receive one monetary unit. In the next period, sellers act as buyers, while buyers act as sellers.

2.1 Bellman Equations

Now, buyers are looking for sellers who produce their preferred good, while sellers are looking for buyers who demand their production. Meetings are pairwise and occur according to a Poisson process with constant arrival rate β with $\beta/I = 1$. With $r > 0$ as discount rate, the Bellman equations are

$$rV_s = \mu[V_c - V_s - c(q)] \tag{1}$$

$$rV_c = (1 - \mu)[V_s - V_c + u(q)] + \gamma_c, \tag{2}$$

where V_s and V_c denote the expected return for sellers and buyers (cash holders). The subscript s (c) denotes sellers (cash holders). Equation (1) displays the Bellman equation of a seller. With a probability of μ a trade with a buyer takes place. A seller has a surplus of switching position, $V_c - V_s$, minus costs $c(q)$ of producing quantity q with $c(0) = 0$, $c'(q) > 0$, $c''(q) \geq 0$ and $c'(0) = 0$. On the other side, a buyer trades with a probability of $1 - \mu$, has a loss of switching position, $V_s - V_c$, and utility $u(q)$ of consumption quantity q with $u(0) = 0$, $u'(q) > 0$, $u''(q) < 0$ and $u'(0) > 0$, see equation (2). Thus, q is the quantity a buyer receives for one monetary unit. The reciprocal is the price, $p = 1/q$. As long as $\gamma_c > 0$, buyers have a monetary benefit for holding cash. If $\gamma_c < 0$, there are storage costs.[1]

[1]One can also implement a transaction fee for paying with cash, but this does not affect results. Even if utility decreases, the traded quantity is not affected, at least as long as the fee does not depend on the traded quantity.

2.2 Nash Bargaining Process

As mentioned above, if buyers find adequate sellers who are able to produce the preferred good of buyers, a Nash bargaining process takes place to determine q. The optimization problem reads:

$$\max_q (\underbrace{V_s - V_c + u}_{\text{buyer's surplus}})^{\theta} (\underbrace{V_c - V_s - c}_{\text{seller's surplus}})^{1-\theta}, \tag{3}$$

where V_c (V_s) denotes the threat point for buyers (sellers); see TW for further details. Here, $\theta \in (0, 1)$ is a buyer's bargaining power. For $\theta \to 1$, buyers make a take-it-or-leave-it offer and purchase the quantity where costs of production, $c(q)$, are equal to the surplus of switching position, $V_c - V_s$, for sellers. In this case, sellers make neither profits nor losses, they are indifferent between selling or not, from equation (1) we get $V_s = 0$. Assuming that they are producing, the buyer's surplus now coincides with the overall trade surplus, defined by $\Delta(q) \equiv u(q) - c(q) > 0$. They buy the the welfare-maximizing quantity q^* which satisfies $u'(q^*) = c'(q^*)$. The traded quantity will always be lower than $\overline{q} > 0$, where $\overline{q}$ is defined by $u(\overline{q}) = c(\overline{q})$.

A trade only takes place if both trade surpluses are non-negative. Buyers' utility of consumption has to be at least equal to the loss of switching position, while sellers' surplus of switching position has to be at least equal to costs of producing. This is true if

$$rc - (1 - \mu)\Delta < \gamma_c < ru + \mu\Delta. \tag{4}$$

If γ_c is too small, a seller's trade surplus, $\phi \equiv (1 - \mu)\Delta - rc + \gamma_c$, is negative. If γ_c is too large, a buyer's trade surplus, $\psi \equiv \mu\Delta + ru - \gamma_c$, is negative since opportunity costs are too big.

Let us have a closer look at the maximization problem (3). In the bargaining process, agents take $V_s(Q)$ and $V_c(Q)$ as given, where Q is the exogenous traded quantity on macroeconomic level. In equilibrium, $q = Q$ always holds; see also TW. The first order condition is

$$\rho(q) = \frac{(1-\theta)\psi}{\theta\phi} \quad \text{with} \quad \rho(q) \equiv \frac{u'(q)}{c'(q)}. \tag{5}$$

Equation(5) is the inverse demand function. There is a unique monetary equilibrium, $q \in (0, \overline{q})$, solving equation (5), see figure 1.

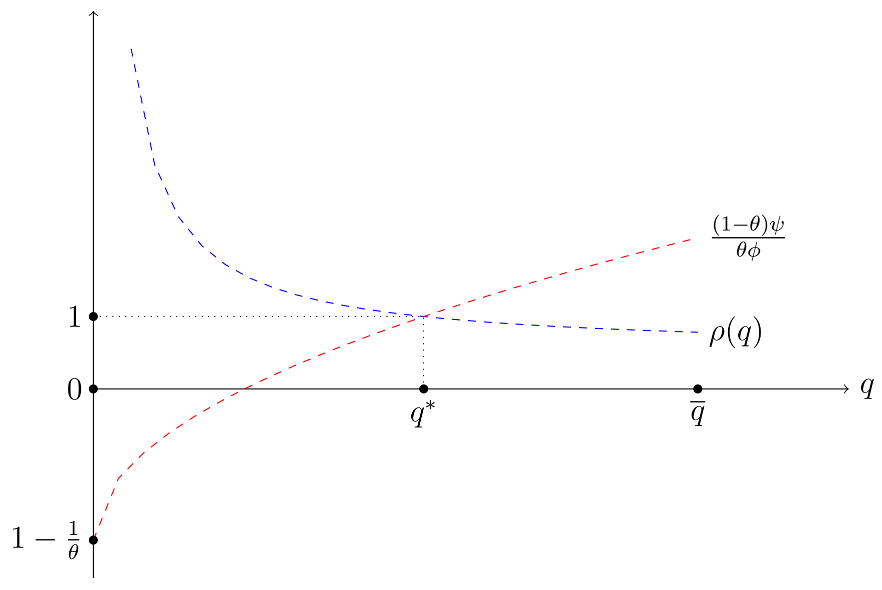

Figure 1: Monetary Equilibrium

Since $u'(0) > c'(0) = 0$, the lhs of equation (5) is large for small q and decreasing in q, see the blue line in figure 1. The rhs is small for small q and increasing in q, see the red line in figure 1. Note that $\frac{\psi(q)}{\phi(q)}$ only increase in q if $\gamma_c \in [rc, ru]$. Thus, condition (4) is no longer sufficient.

As mentioned above, the welfare-maximizing quantity of goods q^* is traded if marginal utility equals marginal costs so that $\rho = 1$. For instance, this is true if the bargaining power and trade surpluses of sellers and buyers are equal, $\theta = 1/2$ and $\phi = \psi$. Equation (5) implies several points:

- First, since $\rho(q)$ is strictly decreasing in q, it is strictly increasing in p. If q decreases, buyers receive a lower quantity of goods for one monetary unit. Thus, prices are higher.

- Second, equation (5) captures the fact that buyers accept higher prices if their surplus increases. In this case, ψ/ϕ increases and the red line in figure 1 turns to the left. Thus, the traded quantity is lower, $q < q^*$. On the other hand, if the trade surplus of sellers increases, the traded quantity is higher, $q > q^*$.

- Third, the notation describes the positive relation between the amount of money and the price level. If μ increases, a buyer's (seller's) surplus increases (decreases), the red line turns to the left and q decreases. Buyers know about the challenge of finding an adequate seller. If they find sellers who are able to produce the preferred good of buyers, they are willing to pay more. Assuming that the goods market is fully competitive for $\mu \to 0$ and monopolistic for $\mu \to 1$, one can also argue that the goods market is less competitive if μ increases since there are only a few sellers who are able to raise prices.

- And fourth, if the bargaining power of sellers or the discount rate increases, so do prices. Again, the red line turns to the left and q decreases. If the bargaining power of sellers increases, sellers are in a better position. In addition, if the discount rate increases, sellers' earnings are discounted higher. In both cases sellers react by raising prices. On the other hand, if the monetary benefit increases, prices decrease since sellers are compensated more highly for costs of production.

Proposition 1: *The welfare-maximizing quantity q^* is traded if marginal utility equals marginal costs, $\rho = 1$. For $\rho > 1$, the traded quantity is too small, $q < q^*$. In this case, an increasing discount rate, an increasing money supply, a decreasing monetary benefit and an increasing seller bargaining power reinforce inefficiencies, q goes down. For $\rho < 1$, all factors also reduce q but inefficiencies decrease.*

2.3 Welfare

If a trade takes place, q^* is the welfare-maximizing quantity. But welfare is also affected by the number of trades. In general, welfare is given by average utility. Wealth of sellers is weighted by $1 - \mu$, while wealth of buyers is weighted by μ. Rearranging yields welfare in a single currency regime (SR)

$$rW^{SR} = \mu(1 - \mu)\Delta + \mu\gamma_c. \tag{6}$$

Welfare is given by the trade probability times the overall trade surplus plus the share of cash holders times their monetary benefit. Now, $\rho \neq 1$ implies $\Delta(q) < \Delta(q^*)$. Thus, for a given μ and γ_c, welfare is below the case where q^* is traded.

In TW the bargaining power is equal, $\theta = 1/2$, while there is no monetary benefit, $\gamma_c = 0$. In this case, there is a trade-off between maximizing the number of trades and optimizing the traded quantity. TW distinguish between a liquidity and price level effect. As long as there is a liquidity shortage, $\mu < 1/2$, an increase in money supply increases the number of trades (liquidity effect) but also raises prices (price level effect). Now, the number of trades is maximized if $\mu = 1/2$ which implies a trade probability of $\mu(1-\mu) = 1/4$. But $\mu = 1/2$ also implies that the traded quantity is below the welfare-maximizing one. In this case, there are too many buyers and the goods market competitiveness is too small. As a consequence, sellers do not produce the welfare-maximizing quantity of goods. Indeed, the welfare-maximizing quantity of goods is only produced if $\mu = \frac{1}{2} - \frac{r(u+c)}{2}$. In this case, the goods market is more competitive so that sellers are willing to produce the welfare-maximizing quantity, with the disadvantage of a lower number of trades since $\mu(1-\mu) < 1/4$.

Proposition 2: *For $\theta = 1/2$ and $\gamma_c = 0$, there is a trade-off between maximizing the number of trades and optimizing the traded quantity. If $\mu = 1/2$ the number of trades is maximized, but the traded quantity is below the welfare-maximizing one. On the other hand, q^* requires $\mu = \frac{1}{2} - \frac{r(u+c)}{2\Delta}$, so that the number of trades is lower than optimal.*

Suppose that γ_c may serve as a policy parameter. In this case, the number of trades as well as the traded quantity can be optimized simultaneously. By choosing $\mu = 1/2$ first, the number of trades is maximized. In a second step, the monetary benefit is determined by $\gamma_c = \frac{r(u+c)}{2}$ so that q^* is traded. Compared to the previous situation with where q^* requires $\mu < 1/2$, the goods market is less competitive. But sellers are still willing to produce q^* since they are compensated for their losses due to accepting lower prices by receiving a subsidy.

3 Dual Currency Regime

Now, the dual currency regime (DR) is considered next. The continuum of agents remains but there are three types now. Next to sellers, μ_s, and cash traders, μ_c, there is a fraction of agents receiving digital money, i.e., a CBDC. It is assumed that $\lambda \in (0, 1)$ of the agents from an SR receive a CBDC. Thus, the fractions of agents are given by $\mu_s = (1-\lambda)(1-\mu)$, $\mu_c = (1-\lambda)\mu$ and $\mu_d = \lambda$. The subscripts s, c and d denote sellers, cash holders and digital money (CBDC) holders.

3.1 Bellman Equations

The Bellman equations of a seller, cash holder and CBDC holder are

$$rV_s = \mu_c(V_c - V_s - c) + \mu_d(V_d - V_s - c)$$
$$rV_c = \mu_s(V_s - V_c + u) + \gamma_c$$
$$rV_d = \mu_s(V_s - V_d + u) + \gamma_d.$$

Sellers have two options to sell their goods: with a probability of μ_c they meet a cash holder, with μ_d they meet a CBDC holder. Cash and CBDC holders search for an adequate seller and receive a monetary benefit. A trade between a cash and CBDC holder does not make both agents better off. Since we rule out side-payments, money traders continue with their own money; see also Aiyagari et al. (1996).

Compared to cash, a CBDC can be interest-bearing. Since the monetary benefit covers all properties of a payment system, e.g., an interest payment or storage costs, we assume that the difference in the monetary benefits between a CBDC and cash, $\delta \equiv \gamma_d - \gamma_c$, is positive, $\delta > 0$ holds. In this way, one can argue that a CBDC is a better payment system than cash. The gap in the monetary benefits, δ, also covers opportunity costs for cash holders. As long as they hold cash, they hold a payment system with a lower monetary benefit. Only after consuming and switching the status from a buyer to a seller, they are able to sell their goods for CBDC.

3.2 Monetary Equilibrium

Compared to previous studies which deal with TW and two currencies, e.g., Craig and Waller (2000) or Camera et al. (2004), there is no take-it-or-leave-it offer here. In this way, the model is less tractable but more realistic. Since sellers' interests also matter, they only produce the quantity which ensures that costs of production are below the surplus of switching position. As a consequence, q^* is not traded for sure and inefficiencies may arise.

To secure a trade now, $c < V_c - V_s < u$ as well as $c < V_d - V_s < u$ have to be fulfilled. The first condition secures a trade with cash, the second one with a CBDC. If the surplus of switching position for sellers does not exceed costs of production, sellers do not produce and sell. Conversely, if the utility of consumption does not exceed the loss of switching position, cash (CBDC) traders do not buy. A trade with cash takes place if the gap between the monetary benefits, δ, is within a certain range:

$$-\frac{(r+\mu_s)[(1-\mu_s)\Delta + ru - \gamma_c]}{\mu_d} \equiv \underline{\delta}_c < \delta < \overline{\delta}_c \equiv \frac{(r+\mu_s)(\mu_s\Delta - rc + \gamma_c)}{\mu_d}. \quad (7)$$

If $\mu_d \to 0$, condition (7) is equal to (4). Here, $\underline{\delta}_c < \delta$ ensures that cash holders trade, while $\delta < \overline{\delta}_c$ ensures that sellers accept cash. Otherwise, cash holders (sellers) do not trade since the monetary benefit of cash is too high (low). If the share of CBDC holders decreases, the upper bound $\overline{\delta}_c$ increases. Even if δ is high, sellers accept cash since they know that the probability of a trade with a CBDC is low. In addition, a trade with a CBDC takes place if

$$-\frac{(r+\mu_s)(\mu_s\Delta - rc + \gamma_d)}{\mu_c} \equiv \underline{\delta}_d < \delta < \overline{\delta}_d \equiv \frac{(r+\mu_s)[(1-\mu_s)\Delta + ru - \gamma_d]}{\mu_c}. \quad (8)$$

The argumentation is similar here. If the monetary benefit of a CBDC is too high (low), CBDC holders (sellers) do not trade since δ exceeds the upper (lower) bound of condition (8). Thus, the interval for a monetary equilibrium with two payment systems is

$$\max\{\underline{\delta}_c, \underline{\delta}_d\} < \delta < \min\{\overline{\delta}_c, \overline{\delta}_d\}. \quad (9)$$

For instance, if $\mu_c \gg \mu_d$, condition (9) simplifies to $\underline{\delta}_d < \delta < \overline{\delta}_d$. Even if δ can be large by considering condition (7) only, the interval for δ is limited. If δ is too large, $\delta < \overline{\delta}_d$ is no longer fulfilled. In this case, CBDC holders do not trade for two points: first, the monetary benefit for a CBDC is too large, $\delta \geq \overline{\delta}_d$, and second, there is a high probability that CBDC holders only receive cash instead of a CBDC after consuming and selling since $\mu_c \gg \mu_d$.

Proposition 3: *A dual currency regime requires that the gap between the monetary benefits fulfills condition* (9)*. If the gap is too small, sellers do not accept a CBDC, while cash holders do not buy. On the other hand, if the gap is too large, sellers do not accept cash, while CBDC holders do not buy.*

3.3 Price Level for Transactions with Cash

Using condition (7), the trade surplus for sellers in a DR in transactions with cash is $\phi_c \equiv \mu_s \Delta - rc + \gamma_c - \frac{\mu_d}{r+\mu_s}\delta$, while the trade surplus for cash holders is $\psi_c \equiv (1-\mu_s)\Delta + ru - \gamma_c + \frac{\mu_d}{r+\mu_s}\delta$. If the gap in the monetary benefits increases, the trade surplus of sellers decreases since they have opportunity costs by holding cash after selling. On the other hand, the trade surplus of cash holders increases since they avoid opportunity costs by continuing to hold cash. It can be also argued that the probability of receiving a CBDC in the next period decreases for sellers by accepting cash. By refusing to accept cash, sellers need one transaction to receive a CBDC, by accepting cash, they need two. Cash holders, on the other hand, need two transactions to receive a CBDC. After the transaction with a seller, they need just one transaction to receive a CBDC. Now, the inverse money demand function, ρ_c, and the traded quantity, q_c, is given by

$$\rho_c(q_c) = \frac{(1-\theta)\psi_c}{\theta\phi_c}.$$

Here, $\mu_d = 0$ implies $\rho = \rho_c$. Again, there is a unique monetary equilibrium. The lhs is large for small q_c and decreasing in q_c. On the other hand, the rhs is small for small q_c. In addition, we assume that the rhs increases in q_c, i.e., $-\frac{(r+\mu_s)(ru-\gamma_c)}{\mu_d} \leq \delta \leq \frac{(r+\mu_s)(\gamma_c - rc)}{\mu_d}$ holds.

If the money supply, $1-\mu_s$, or the gap in the monetary benefits, δ, increases, the price level in transactions with cash does too since sellers value cash less. On the other hand, if the buyer's bargaining power increases, the price level decreases since buyers have a higher power in the bargaining process, even if money supply or opportunity costs are high. Thus, for $\rho_c > 1$, an increasing money supply and rising gap in the monetary benefits reinforce inefficiencies, while an increasing buyer bargaining power mitigates inefficiencies. For $\rho_c < 1$, all factors have the same effect on q_c but inefficiencies decrease since $q_c > q^*$.

In addition, the positive relation between the price level in transactions with cash and the discount rate is not for sure anymore, $\partial\rho_c/\partial r < 0$ is possible. In this case, discounting mitigates inefficiencies. Now, $\partial\rho_c/\partial r < 0$ is true if

$$\delta > \tilde{\delta} \equiv \frac{(r+\mu_s)^2[\mu_s u + (1-\mu_s)c + \gamma_c]}{\mu_d(1+2r+\mu_s)}.$$

If the gap in the monetary benefits is zero, the discounted gap is also zero; for small as well as for large discount rates. On the other hand, if the gap is large, the discounted gap is still large for small discount rates, while it converges towards zero for large discount rates. Since a positive gap represents opportunity costs for cash holders, discounting mitigates inefficiencies in transactions with cash in a DR if the gap exceeds the threshold $\tilde{\delta}$.

But $\delta > \tilde{\delta}$ is only possible if $\tilde{\delta} < \min\left\{\frac{(r+\mu_s)(\gamma_c - rc)}{\mu_d}; \frac{(r+\mu_s)(ru-\gamma_d)}{\mu_c}\right\}$. The first upper bound ensures a unique equilibrium with cash, i.e., that the rhs of $\rho_c(q_c)$ is increasing in q_c. The second upper bound ensures a unique equilibrium with a CBDC, i.e., that the rhs of $\rho_d(q_d)$ is increasing in q_d. For instance, $\tilde{\delta}$ is below both upper bounds if there are numerous CBDC holders, $\mu_d \to 1$, while $\gamma_c > 2rc$ and $\gamma_d < ru$. On the other hand, if $\tilde{\delta}$ exceeds one upper bound, there is no δ which exceeds $\tilde{\delta}$ and satisfies the conditions for a monetary equilibrium.

Proposition 4: *For $\rho_c > 1$, discounting mitigates inefficiencies in transactions with cash in a dual currency regime if opportunity costs for cash holders are large, $\delta > \tilde{\delta}$.*

In general, a CBDC reinforces inefficiencies in transactions with cash in a DR if $\rho_c > \rho > 1$. To make a comparison possible, the shares of an SR must be linked with the shares of a DR. As mentioned above, it is assumed that $\lambda \in (0,1)$ of the agents from an SR receive a CBDC. Thus, $\mu_s = (1-\lambda)(1-\mu)$, $\mu_c = (1-\lambda)\mu$ and $\mu_d = \lambda$ are implemented in ρ_c. In this case, $\rho_c > \rho$ holds if

$$\frac{\psi + \Gamma}{\phi - \Gamma} > \frac{\psi}{\phi}, \quad \text{with} \quad \Gamma \equiv \lambda \left[(1-\mu)\Delta + \frac{\delta}{r + (1-\lambda)(1-\mu)}\right].$$

Since $\Gamma > 0$, a CBDC increases inefficiencies in transactions with cash in a DR, at least if $\rho > 1$. On the one hand, a CBDC increases money supply by λ so that prices increase. On the other hand, a CBDC causes opportunity costs for cash holders since a CBDC is interest-bearing, $\delta > 0$ holds. But a CBDC is also able to mitigate inefficiencies in transactions with cash if the goods market in an SR is too competitive, $\mu \to 0$ and hence $\rho < 1$. In this case, the liquidity effect reduces the share of sellers. Thus, sellers produce less and the traded quantity converges to the optimum, $q^* < q_c < q$ and hence $\rho < \rho_c < 1$. But there is a specific point where inefficiencies increase again. If the money supply increases further, the traded quantity q_c decreases even more until $q_c < q^* < q$. Thus, if the liquidity effect is too large, $q^* - q_c > q - q^*$ holds and a CBDC reinforces inefficiencies.

Proposition 5: *If $\rho > 1$, a CBDC reinforces inefficiencies in transactions with cash in a DR since a CBDC increases money supply and causes opportunity costs for cash holders. If the goods market in an SR is too competitive, on the other hand, $\rho < 1$, a CBDC mitigates inefficiencies.*

4 Welfare Analysis

As proved above, a CBDC can reinforce as well as mitigate inefficiencies in transactions with cash in a DR. To answer how a CBDC affects welfare, we compare welfare between an SR and DR. Again, welfare is given by average utility,

$$rW^{DR} = \mu_s(\mu_c\Delta_c + \mu_d\Delta_d) + \mu_c\gamma_c + \mu_d\gamma_d,$$

where $\Delta_c(q_c) \equiv u(q_c) - c(q_c)$ and $\Delta_d(q_d) \equiv u(q_d) - c(q_d)$ are the overall trade surplus in transactions with cash and a CBDC. Thus, q_c and q_d are the traded quantities for cash and CBDC. Welfare is given by the sum of the trade probabilities times the overall trade surplus plus the sum of the weighted monetary benefits.

Of course, welfare increases if there is a liquidity shortage in an SR, $\mu \to 0$. In this case, trade probability is low and the goods market is highly competitive, $q > q^*$. Since the emission of a CBDC increases money supply, trade probability increases and the goods market is less competitive so that $q > q_d > q_c > q^*$, at least if the increase in money supply and the gap in the monetary benefits is limited.

Hence, the question is whether a CBDC increases welfare even if there is no liquidity shortage in an SR. For the following it is assumed that $\mu = 1/2$ and $\rho > 1$. Welfare increases if $W^{DR} > W^{SR}$. Implementing $\mu_s = (1-\lambda)(1-\mu)$, $\mu_c = (1-\lambda)\mu$ and $\mu_d = \lambda$ in W^{DR} and rearranging $W^{DR} > W^{SR}$ yields

$$g(\lambda) \equiv -\lambda^2 + \chi_1\lambda - \chi_2 > 0,$$
$$\text{with} \quad \chi_1 \equiv \frac{2(\Delta_d - \Delta_c + \delta + \gamma_d)}{2\Delta_d - \Delta_c} \quad \text{and} \quad \chi_2 \equiv \frac{\Delta - \Delta_c}{2\Delta_d - \Delta_c}.$$

A positive χ_1 describes the benefit of using a better payment system which bears interest. The interest benefit is multiplied by the share of CBDC holders. In addition, a positive χ_2 describes the loss of cash holders who face higher prices and opportunity costs now. In this way, $\Delta_c'(\lambda) < 0$ always holds: the more CBDC holders there are, the higher the amount of money and therefore the price level in transactions with cash.

In general, a CBDC increases welfare if $g(\lambda) > 0$, see figure 2. First of all, $\lambda = 0$ implies $\Delta = \Delta_c(0)$ and therefore $\chi_2 = 0$. If nobody uses a CBDC, there are no opportunity costs for cash holders and the overall trade surplus in transactions with cash between an SR and a DR is equal. This implies that there is no difference in welfare, $g(0) = 0$ holds.

As soon as $\lambda > 0$, welfare increases since the additional benefit of having a payment system which bears interest, $\delta > 0$, outweighs the additional costs of having a higher money supply and thus a higher price level in transactions with cash. The gain of CBDC holders exceeds the loss of cash holders. The welfare difference increases in λ up to the optimum $\lambda^* \equiv \frac{\chi_1}{2}$. If χ_1 decreases, λ^* does too, see the dashed curve.

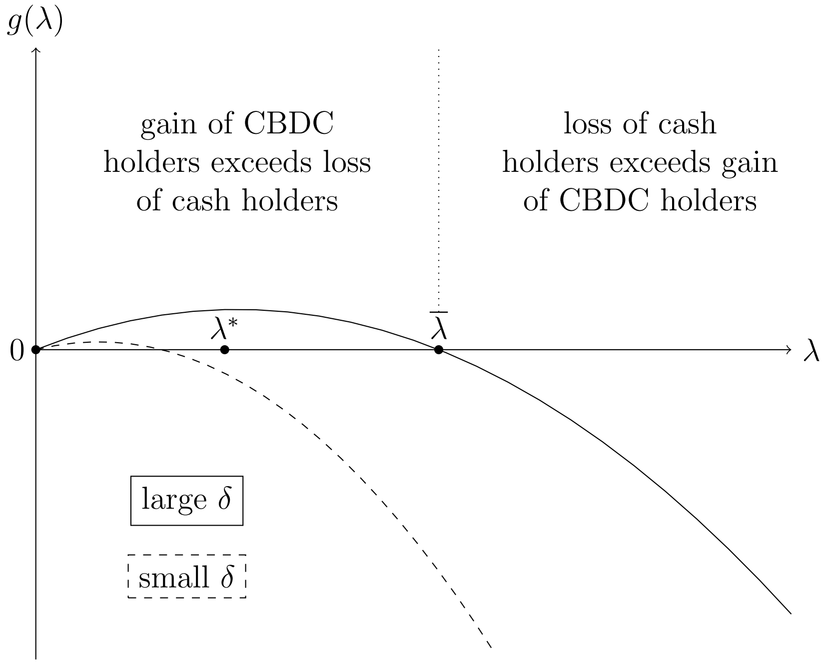

Figure 2: Solution to $g(\lambda) > 0$

Afterwards, the welfare difference is still positive but decreasing in λ since the price level in transactions with cash increases further, while the trade probability and thus consumption decreases. For $\lambda = \overline{\lambda} \equiv \frac{\chi_1}{2} + \sqrt{\frac{\chi_1^2}{4} - \chi_2}$, welfare between an SR and a DR is equal again. In this case, the gain of CBDC holders is equal to the loss of cash holders. Again, if χ_1 decreases, $\overline{\lambda}$ shifts to the left and the interval for a welfare improvement decreases since the additional benefit of the better payment system is smaller. Finally, for $\lambda > \overline{\lambda}$, there are too many CBDC holders and the price level is too high, trade probability and thus consumption decrease further. Welfare decreases since the loss cannot be compensated by an interest-bearing CBDC.

Proposition 6: *A CBDC increases welfare if the share of agents receiving a CBDC is below the threshold,* $\overline{\lambda} \equiv \frac{\chi_1}{2} + \sqrt{\frac{\chi_1^2}{4} - \chi_2}$. *Otherwise, the price level is too high and trade probability and thus consumption are too low.*

5 Conclusion

The task described in this paper was to investigate how a CBDC affects inefficiencies and welfare, even if there is no liquidity shortage in a single currency regime. To do this, the search model of Trejos and Wright (1995) was extended by a CBDC. First of all, we show that a monetary benefit provides an easy solution for overcoming the discount problem. In this case, there is no trade off between optimizing the traded quantity and maximizing the number of trades. Even if the goods market is less competitive, the welfare-maximizing quantity of goods is traded since sellers get compensated by a subsidy. Moreover, the Trejos and Wright environment is well suited for investigating whether a CBDC reinforces inefficiencies in transactions with cash. We show that inefficiencies increase if the money supply and interest rate for a CBDC are sufficiently large. Nevertheless, a CBDC is able to increase welfare if the share of CBDC holders is limited. Although prices in transactions with cash increase due to a higher money supply, the net welfare effect is positive since a CBDC is interest-bearing. Thus, the CBDC supply and the interest rate have to be chosen carefully.

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