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Cryptocurrencies: A Copula based approach for asymmetric risk marginal allocations

Vahidin Jeleskovic[i], Mirko Meloni[i] & Zahid Irshad Younas[ii]

Abstract

Given the increasing interest in cryptocurrencies shown by investors and researchers, and the importance of the potential loss scenarios resulting from investment/trading activities, this research provides market operators with a dynamic overview on the short-term portfolio tail risk contribution of six widely-traded cryptocurrencies. Considering the high volatility dynamics of the cryptocurrency market, realized volatility measures computed from different frames (1m, 5m, 15m, 30m, 1h) are included in the estimation of univariate GARCH models, to be used in combination with copula functions for VaR/ES Monte Carlo simulations. Even if results lack data frequency ordinality in terms of out-of-sample goodness, Bitcoin and Litecoin are generally recognized as the safest and riskiest currency respectively on an equally-weighted framework, reflecting how the contribution to portfolio returns is not representative of the real grade of risk diversification.

Keywords: cryptocurrency tradiing, tail risk, realized volatility, copula, portfolio optimization.
JEL classification: C15, C53, G17.

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1. Introduction

Considering cryptocurrency as a real asset is still a controversial topic (Cobert *et al*., 2018). However, in recent years the cryptocurrency market has observed a tremendous growth and quick rise in its market capitalization from $19 billion in February 2017 to $800 billion in December 2017. Due to this explosive growth, cryptocurrencies are considered to be a new investment avenue (Aggarwal *et al*., 2019; Ankenbrand & Bieri, 2018; Demir et al., 2018; Catania *et al*., 2019). Economists believe this growth to be a bubble based on sentiments and price fluctuations (Krugman, 2018, January). A shift in these factors coupled with a change in market trends have subsequently resulted in a drastic drop in bitcoin prices (Popken, 2018; Makadiya, 2018). According to previous research, the lack of intrinsic value, an unregulated cryptocurrency market and major cryptocurrency holdings by market participants are significant volatility factors in cryptocurrencies (Iinuma, 2017).

This study aims to provide market operators with an evaluation of the daily risk of six widely-traded cryptocurrencies (Bitcoin, Binance, Ethereum, Litecoin, Ripple and Eos) based on the maximum possible short-term loss, i.e., 30 days. This kind of risk is also called "tail risk", i.e., asymmetrical distributions which are more probable to have extreme results. The traditional standard deviation, even if easily applied, suffers from a variety of limitations which are not in line with the main findings on the cryptocurrency market. In fact, Alvarez-Ramirez (2017), Bariviera *et al*. (2017) and Phillip *et al*. (2018) found this type of market to have the following characteristics: fat-tails and skewness, autocorrelation, long memory in volatility, volatility heteroskedasticity and clustering and leverage effect.[1] All these features are not supposed to be correctly captured by standard deviation which is only a model for elliptical distributions[2]. Moreover, standard deviation expresses the variability resulting from both downside and upside movements of returns, thus it is not able to provide investors with a concrete measure of losses (Basile & Ferrari, 2016). Considering the volatility in cryptocurrency prices, this analysis is conducted on a short-term framework, estimating the tail risk of daily volatility models optimized by the inclusion of high-frequency data (namely 1m, 5m, 15m, 30m and 1h). An intra-day perspective is justified by the fact that a higher frequency is rationally able to carry a higher grade of information which is more accurate in predicting the volatility dynamic, especially when it tends to change significantly and quickly (Summinga-Sonagadu & Narsoo, 2019). Given these circumstances, a model which just relies on low frequency data will return a dynamic which is too slow with respect to the dynamic actually present in the market. This study firstly discusses the risk of loss from a vertical point of view, namely considering the risk of the same asset with respect to different investment time horizons. Secondly, the risk of loss from a horizonal point of view is examined, i.e. the risk resulting from other cryptocurrencies. This study uses an equally weighted portfolio comprised of

[1] Leverage effect denotes a different volatility response to different type of shocks.
[2] Distributions with symmetrical frameworks and linear relationships between variables.

cryptocurrencies which gives each cryptocurrency the same relevance in terms of returns. A buy and hold strategy[3] is applied to provide a more objective comparison while preventing managerial bias that could affect the portfolio risk-return profile. The marginal risk contribution of each asset is computed by using the Value at Risk (VaR) and Expected Shortfall (ES). They measure how much each crypto currency affects the overall tail risk given the contribution of the other assets within the same portfolio. Both VaR and ES are computed with a Monte Carlo simulation which sets up the use of copula functions and univariate ARMA-real GARCH models.. The contribution of this research is a clear understanding of all the risk conditions and diversification opportunities within the cryptocurrency market before entering it. This work is pertinent to banks, hedge funds and individual investors who want to understand the risk opportunities related to the cryptocurrency market in order to optimize/diversify their investments. Furthermore, the results of this study could be of use to entities forced to invest in risky assets in order to accumulate a certain amount of capital by a future date, e.g., pension funds. The before-mentioned investors might prefer to evaluate a risk measure that indicates the potential loss resulting from a long position rather than a measure which just expresses the variability around the expected performance, i.e., the standard deviation. Moreover, at the same time, they might want to know which cryptocurrency is the most suitable considering a defined time horizon. Marginal allocations can provide them with information about the level of risk from one asset compared to another. The proposed model can be used for simulation and risk management purposes, but also for asset allocation frameworks.

The previous literature deals with either a few assets or with interactions between cryptocurrencies and other asset classes assuming often basic models. For example, Katsiampa (2018) analyzed the co-movements between Bitcoin and Ethereum, modelling their dependence with a bivariate BEKK model. Bouri *et al*. (2017) and Bouri *et al*. (2017b) investigate the relationship between Bitcoin and respectively three commodity indices and indices representative of all the other asset classes (e.g. stocks, bonds, real estate) with ARMA and DCC-GARCH models (1,1). Aslanidis *et al*. (2019) applied a DCC-GARCH methodology (1,1) to model the joint dynamic of several cryptocurrencies with stocks, bonds and commodity prices from 2014 to 2018. Considering the different approaches just mentioned, the inexistence of a common practice with respect to multivariate models can be seen even though Aslanidis *et al*. (2019) found a DCC model to outperform the basic BEKK. This is due to the fact that dynamic models are able to capture the potential change in correlations resulting from both negative and positive market shocks. As for the employment of copulas, Boako et al. (2019) used vine-copulas to model the dependence between Bitcoin, Dash, Ether, Litecoin, Ripple and Stellar with data from 2015 to 2018. Furthermore, they also considered an AR (1) specification for the mean of each cryptocurrency and found that Archimedean copulas perform better than the Gaussian copula. Similar evidence (with a predominance of two specific types of Archimedean copulas) was found by Saha et al. (2018) which

[3] Once the starting weights are set, they are allowed to change over time depending on the new price level.

used vine-copulas to study the co-movements among Bitcoin, Ether, Litecoin and Ripple from 2015 to 2018.

The approach proposed here, called quasi-maximum likelihood, involves the usage of a given distribution for the GARCH computation and the subsequent fitting of alternative distributions on the residuals obtained. The distribution chosen for the first step is the Ged distribution. This choice was due to the fact that, in this way, it was possible to model residuals given the presence of fat tails, which characterize the crypto-market as demonstrated by the already-mentioned Alvarez-Ramirez (2017), Bariviera *et al*. (2017) and Phillip *et al*. (2018) papers. The Ged distribution is then retained asa good proxy of the real distribution.

It is important to note that the model selection concerning both GARCH (in terms of number of lags) and copulas (in terms of copula family) is implemented through procedures based on out-of-sample goodness measurements (e.g. walk forward cross validation), which are less likely to return overfitted results as they are not calculated on the training set. Finally, a copula-based model is expected to return highly flexible results with a good level of out-of-sample forecasts fitting. As for the contribution of each cryptocurrency with respect to the overall risk, they are supposed to reflect a different percentage of relevance (also different from the relevance that they have with respect to the portfolio return) and to change even considering a different time frame (testifying the importance of choosing the correct framework). Returns from lower frequencies, indeed, do not take into account what happens in the meantime, but, at the same time, might present a more stable intra-day variability, which could affect forecasts in a positive way. For this reason, expecting to have a unique result in term of frame might not be convenient. Also, the time horizon is retained to be a key factor, able to affect contributions and reveal smoothing effects (i.e. reducing the risk concentration referred to the riskiest cryprocurrencies or providing daily VaR/ES increasing less than proportionally).

2. Methodology

2.1. Var/ ES Term

The two asymmetric risk measures used for this analysis are Value at Risk (VaR) and Expected Shortfall (ES). In particular, VaR quantifies the maximum achievable loss given a certain level of confidence and a specified time horizon (Sironi & Resti, 2007; Christoffersen, 2012). On the other hand, ES measures the expected return given that the return is more extreme than VaR. It is important to note that ES has a higher informative content since two distributions can have the same VaR, but a different probability to reach extremes quantiles.

To determine how cryptocurrencies affect the VaR/ES of a portfolio, it is necessary to build a dynamic model which accounts for how the two indices vary over different time horizons. This instrument is called *Term structure* and can be built by crossing VaR or ES with the relative maturity.

For Term structure, a Monte Carlo simulation is chosen out of all the possible model due to the fact that simulation methods are generally highly accurate as they tend to smooth forecasts, increasing the reliability of long-term predictions.[4]

Returns are simulated using a location scale model:

$$R_{t+1} = \mu_{t+1} + \sigma_{t+1} Z_{t+1} \qquad (2.1)$$

Here, both μ_{t+1} and σ_{t+1} need to be predicted. Considering the heteroskedasticity phenomenon and volatility clustering, it is not convenient to work with unconditional distributions (where moments remain fixed over time; Christoffersen, 2012). This model was chosen for two main reasons: 1) It is easy to compute; 2) It is particularly useful if combined with a univariate ARMA-GARCH model, which allows both the conditional volatility and the mean to be used as inputs; 3) It is possible to model the shape and the kurtosis of the return series by modifying the distribution of Z.

2.2. Conditional mean and conditional volatility.

From a univariate point of view, it is necessary to create a model which is able to predict both the conditional mean and the conditional volatility for all the cryptocurrencies within the sample portfolio. This study focuses on thhe ARMA-real GARCH model from (Hansen *et al*., 2012; Equation 2.2), that is not only able to predict daily volatility by exploiting intra-day information, but is also able to include the leverage effect:

$$\begin{gathered} R_t = \mu_t + \sigma_t z_t, z_t \sim i.i.d(0,1) \\ \log(\sigma_t^2) = \omega + \sum_{k=1}^{q} \alpha_k \log(r_{t-k}) + \sum_{k=1}^{p} \beta_k \log(\sigma_{t-k}^2) \\ \log(r_t) = \xi + \delta \log(\sigma_t^2) + \tau(z_t) + u_t, u_t \sim N(0,\lambda) \end{gathered} \qquad (2.2)$$

$$\tau(z_t) = \eta_1 z_t + \eta_2 (z_t^2 - 1)$$

[4] The simplest model is the non-parametric one which assumes that historical returns are perfectively representative of the future ones. The semi-parametric approaches are aimed at estimating the parameters of a portion of the return distribution. Full-parametric models are based on the whole return distribution and make it possible to compute risk measures through closed formula. Simulation methods require the most computation but can lead to the most accurate results (Christoffersen, 2012).

As it can be noticed by the first line in Equation 2.2, a return in time "t" follows the location-scale model, while the conditional volatility is treated in logarithmic terms (second line). Here, the current state of innovation (or market shock) is not measured by the squared daily return (like in the standard GARCH), but by the log of r_{t-k}, where r_{t-k} is a measure of realized volatility. The realized volatility (which can be denoted by "RV") of a generic day t, given regular-framed intra-day data with the frequency m, can easily be computed as shown in Equation 2.3:[6]

$$RV_t = \sum_{m=1}^{M} R_{m,t}^2 \quad (2.3)$$

$R_{m,t}$ is the return of the sub-period m in day t and M is the total number of sub-periods for that day.

For this research, 1m, 5m, 15m, 30m and hourly data related to the period from 31 May 2018 to 30 July 2019 are used to create different series for the realized volatility, to be exploited in an anchored walk forward cross validation procedure. All the GARCH are estimated considering the Ged distribution as starting assumption (since the distribution for standardized quantiles is fitted in a second step, it is not convenient to try several variants) and a maximum order of (5,5) for both the mean and volatility processes (to optimized during the process). Ged is retained to be able to return less-biased GARCH residuals (on which the real fitting procedure is later carried out), modelling the parameter that describe fat-tails. The walk forward procedure is implemented generating 5 different series of training and validation sets (as shown in Table 2.1).

Insert here, table [2.1].

The conditional volatility forecasted through the models fitted on the training sets is compared with the one observable in the related validation set though the RMSE loss function:

$$RMSE = \sqrt{\frac{1}{n_{val}} \sum_{i=1}^{n_{val}} \left(vol - E(vol)\right)^2} \quad (2.4)$$

The walk forward process implies that the RMSE values resulted from each pair of training and validation set shown in the previous Table need to be averaged, so that the best model could be considered the one minimizing the mean of all the past value of the loss function. This model will be optimized considering the final training set (31/05/2018 – 30/06/2019) to generate the forecasts needed to answer the research question. This approach is aimed at reducing the overfitting risk (potentially

[6] See Christoffersen (2012).

resulting from an in-sample selection criteria).

2.4. Copula function and multivariate modelling

On a multivariate point of view, it is necessary to estimate a dependence structure between the best-fitting cryptocurrency distributions. This research exploits the informative content of copulas[7]. The combination between different marginal univariate distributions and the multivariate copula makes it possible to obtain new distributions with respect to the traditional ones (Delatte & Lopez, 2013). Inputs of copulas are represented by probabilities, which are uniformly distributed, and not by quantiles, thus they are not required to be normally distributed even setting a particular copula (Simard & Rémillard, 2015).

This study compares several types of copulas[8] – Normal, t-Student, Gumbel, Clayton, Frank, Joe – in order to assess which one returns the best out-of-sample results (in terms of RMSE function between the predicted returns of each asset resulting from 10,000 simulations for each day included in the validation set and the effective ones:

$$RMSE_c = \sqrt{\sum_{asset=1}^{N_{asset}} \sum_{i=1}^{n_{val}} \left(r_{ai} - E(r_{ai})\right)^2} \quad (2.5)$$

r_{ai} is the return of the a-th asset within the portfolio of the j-th day included in the validation set.

The procedure follows a walk forward approach, like the GARCH models. After having fixed all the pairs of training and validation sets (already shown in Table 2.1), the RMSE resulting from each of them is averaged to the ones coming from the others. The best copula is the one which minimizes the final mean.

Please note that all these models are estimated under the hypothesis of constant conditional correlations, i.e. assuming that the correlation between assets does not change over time. Copulas are then estimated just once (this assumption can be justified by the fact that this research deals with the very short term, but it was also due to limitations on the computational power exploitable). The models were computed through the R package *copula* (Hofert *et al*., 2019).

[7] Copula functions are cumulative probability functions which link any given set of marginal distributions to construct a joint dependence structure (Patton, 2009; Jaworski et al., 2010; Christoffersen, 2012; Durante & Sempi, 2015)

[8] There are several types of copula families, but the ones considered for this research are (see Appendix A.2 for mathematical specifications): 1) Elliptical copulas and 2) Archimedean copulas. The first family is made up of Normal and t-Student copulas, which, being symmetrical, might not model in a proper way the tail of the multivariate distribution between assets. Moreover, elliptical distributions assume a linear dependence between two or more objects, so that the unknown parameters can be represented by Pearson's correlations. This means that it could be hard to reach a convergence in a high dimensional multivariate framework (in presence of a high number of assets in the portfolio of interest). Archimedean copulas, on the other hand, can not only treat the symmetry differently according to the type of function chosen, but also simplify the optimization as they depend on a unique parameter λ (independently from the number of assets), which measures the strength of the dependence between series (not necessarily linear). This property, however, could also make them inappropriate in case of very high dimensions as the quality of final approximation could not be sufficiently satisfactory (Nelsen, 2007).

2.6. Marginal risk allocation: Euler and Kernel Methods

The research question of this study is related to how much each cryptocurrency affect the overall risk of a loss. To provide investors with an answer, it is necessary to compute marginal risk allocations:[9]

$$Risk\ Mesaure_p = \sum_{i=1}^{N} Marginal\ Risk_i \quad (2.6)$$

In literature, the main model used for such a purpose in the Eulerian one, which states that if the risk measure is a continuously differentiable function of the weights of each asset, it is possible to compute the risk measure as following:[10]

$$Risk\ Mesaure(w) = \sum_{i=1}^{N} w_i \frac{\partial Risk\ Mesaure}{\partial w_i} \quad (2.7)$$

w is intended to be the marginal risk allocation related to the generic i-th asset. As it can be noticed, this approach relies on the computation of the derivatives of the risk measure with respect to each weight. In case of standard deviation, they are quite easy to compute, but, in case of VaR and ES (calculated through a discrete simulation approach), it is impossible to obtain reliable results. First of all, a discrete approach does not guarantee a continuous function, so that derivatives should be approximated by discrete differences. Discrete differences, in turn, do not guarantee the respect of the additivity rule. Moreover, results of simulated discrete calculations are too volatile, meaning that a re-simulation could change them significantly. Therefore, Eulerian marginal allocations cannot be used in combination with Monte Carlo simulations. Another approach could be taking the return (profit or loss) in correspondence to the portfolio VaR and ES as marginal allocations for each cryptocurrency:[11]

$$mVaR_i = w_i E(PL_i | PL_p = VaR_p) \quad (2.8)$$

$$mES_i = w_i E(PL_i | PL_p \leq VaR_p) \quad (2.9)$$

Considering an equally-weighted portfolio, this would guarantee the additivity rule for both the risk measures, but results would be still too volatile with respect to different simulations. In this sense Epperleine & Smillie (2006) and Epperleine & Smillie (2016) proposed two different kernel approaches aimed at smoothing the results and giving them a more consistent stability. Kernel functions are non-parametric tools used to estimate a conditional expectation not knowing its functional form (Fan, 1993). Epperleine & Smillie (2006) suggests using the Nadaraya-Watson kernel regression (considering a

[9] This is also called additivity rule (Tasche, 2007).
[10] See Braga (2015).
[11] This result can be obtained by extending the Euler approach (Tasche, 2007).

locally constant conditional expectation)[12] and starting from the loss function shown in Equation 2.10. The final marginal VaR estimator is the value of the unknown conditional expectation (A) which minimized the loss function given here by the following equation 2.11:

$$Loss(A) = \sum_{j=1}^{N} \left(PL_i^j - A\right)^2 K\left(\frac{PL_p^j - VaR_p}{h}\right) \quad (2.10)$$

$$mVaR_i = \frac{\sum_{j=1}^{N} K\left(\frac{PL_p^j - VaR}{h}\right) PL_i^j}{\sum_{j=1}^{N} K\left(\frac{PL_p^j - VaR}{h}\right)} \quad (2.11)$$

N is the number of simulations, K(./h) is the kernel function, PL_p^j the absolute portfolio profit/loss related to the j-th simulation, $PL_p^j$the absolute i-th cryptocurrency profit/loss related to j-th simulation and h the smoothing parameter of the kernel chosen (equal to $2{,}275\sigma N^{-\left(\frac{1}{5}\right)}$. As it can be noticed, Equation 2.10 is a kernel-weighted average since PL_i has a weight equal to K, which is higher as the j-th simulation returns a profit profit/loss near the portfolio VaR. This kind of function has a smoothing power basing on the weighted average among different simulations, but it does not satisfy the additivity rule (depending on h). It is then necessary to apply a rescaling factor, equal to the portfolio Value at Risk as follows:

$$mVaR_i = VaR_p \frac{\sum_{j=1}^{N} K\left(\frac{PL_p^j - VaR}{h}\right) PL_i^j}{\sum_{j=1}^{N} K\left(\frac{PL_p^j - VaR}{h}\right)} \quad (2.12)$$

In light of the above, marginal Expected Shortfalls can be computed as the weighted average of all the profit/loss values in correspondence of scenarios leading to a more extreme loss than the portfolio VaR:

$$mES_i(\alpha) = \sum_{j=1}^{k\sim VaR_p} \frac{w_j E(PL_i | PL_p = PL_p^j)}{1-\alpha} \quad (2.13)$$

Substituting $E(PL_i|PL_p = PL^j$) with the kernel-averaged $mVaR_i$ (considering at each j the j-th more extreme scenario as portfolio VaR), it is possible to obtain the following result:

$$mES_i(\alpha) = \sum_{j=1}^{k\sim VaR_p} \frac{w_j mVaR_i}{1-\alpha} \quad (2.14)$$

[12] Where h is a smoothing parameter A kernel with subscript h is called "scaled kernel" and is defined as Kh(x) = 1/h K(x/h). Please note that the choice of h can strongly influence the final estimation (the lower h is, the less smoothed the resulting kernel distribution might be).

Epperleine & Sneille (2016), in order to keep the rescaling factor from affecting the statistical estimations, proposed another approach, in which the conditional expectation is no longer considered locally constant, but locally linear with respect to the portfolio profit/loss function:

$$Loss(A,B) = \sum_{j=1}^{N} \left(PL_i^j - \left(A + BPL_p^j\right)\right)^2 K\left(\frac{PL_p^j - VaR_p}{h}\right) \quad (2.15)$$

Once computed A and B which minimize the loss function (Fan, 1993), marginal VaR can be computed as shown in Equation 2.16 and marginal ES as shown in Equation 2.14.

$$mVaR_i = A_i^* + B_i^* VaR_p \quad (2.16)$$

$$A_i^* = \sum_{j=1}^{N} \frac{K_j S_2 - K_j PL_p^j S_1}{S_0 S_2 - S_1^2} PL_i^j \quad (2.17)$$

$$B_i^* = \sum_{j=1}^{N} \frac{K_j PL_p^j S_0 - K_j S_1}{S_0 S_2 - S_1^2} PL_i^j \quad (2.18)$$

$$K_j = K\left(\frac{PL_p^j - VaR_p}{h}\right) \quad (2.19)$$

$$S_0 = \sum_{j=1}^{N} K_j \quad (2.20)$$

$$S_1 = \sum_{j=1}^{N} K_j PL_p^j \quad (2.21)$$

$$S_2 = \sum_{j=1}^{N} K_j \left(PL_p^j\right)^2 \quad (2.22)$$

Considering Equation 2.16 and that the sum of the profit/loss of all the cryptocurrencies for the j-th simulation is equal to the j-th portfolio profit/loss by definition, it can be easily verified that:

$$\begin{gathered} \sum_{i=1} A_i = 0 \\ \sum_{i=1} B_i = 1 \end{gathered} \quad (2.23)$$

Combining Equation 2.23 with Equation 2.16, it is possible to verify the additivity rule on VaR (and on ES as extension; Equation 2.24) without the usage of rescaling factors.

$$\sum_{i=1} mVaR_i = VaR_p \quad (2.24)$$

The two approaches proposed by Epperlein & Smillie (which considered the Gaussian kernel in their estimations) are used and compared in terms of variability of results:

$$K(x,h) = \frac{1}{\sqrt{2\pi}h} e^{-\frac{1}{2}\left(\frac{x}{h}\right)^2} \quad (2.25)$$

The gaussian kernel with a zero mean, designed to model the distribution of the difference between the portfolio P/L and its VaR, is particularly suitable as it is applied to estimate the weights to be assigned to each simulation. When the two measures are equal, the difference between them is equal to 0, in correspondence to which the gaussian curve reaches its maximum value. The weights tend to decrease when simulations are far from the portfolio VaR.

The above-illustrated methodology is structured as to generate 5 different (one per frame) contributions for each cryptocurrency and with respect to VaR and ES at both 99% and 95% level of confidence.

3. Data and descriptive statistics

The current study takes into considerations six cryptoccurencies – Bitcoin, Binance, Ethereum, Litecoin, Ripple and Eos – chosen among the most traded ones on the binance platform considering the 24h volumes. For the analysis, 1m, 5m, 15m, 30m, 1h and 1d data (price against Tether, USDT[13]) – from 31 May 2018 to 30 July 2019 were exploited in order to assess the incidence of the time frame on the final marginal risk allocations (retrieved through a *Binance* API). Data were first analysed in order to detect the presence of repetitions or missing values (to be filled through an interpolation method), but they did not show irregularities. The presence and the relevance of extreme returns were instead studied considering several alternatives of fat-tailed distributions. Returns from 31 May 2018 to 30 June 2019 was chosen to represent the training set; July 2019 was chosen as test set, through which carry out the forward-looking backtesting and evaluate the forecasts goodness of fitting. This period of time is considered to be sufficiently representative of the short trend (avoiding taking into consideration the "bubble crash" of the early 2018) and consistent with this kind of analysis. Assuming a portfolio starting with an investment of 100€ in each individual cryptocurrency (Figure 3.1), it can be noticed that Bitcoin, Ethereum, Litecoin, Ripple and Eos have a similar trend; Binance, on the other hand, seems to diverge showing a stronger increasing positive trend starting from February 2019.

[13] According to cryptocompare.com (last access: 02-08-2019), the trading volumes for all the pairs with Tether are significantly higher than the pairs with respect to fiat currencies (e.g. US Dollar). Moreover, the usage of stablecoins as base currency has already become a common practice for crypto-traders as their stability can help to hedge against volatile market phases.

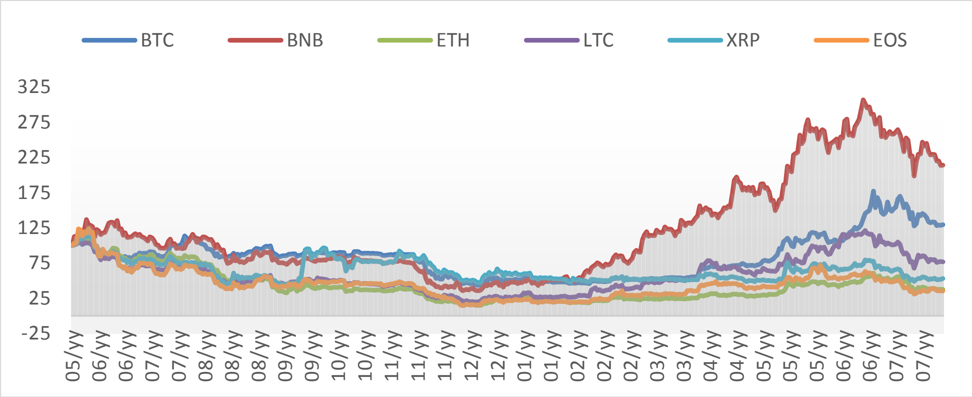

Figure 3.1. Cryptocurrencies daily prices May 2018-June2019

To have a clearer vision, Table 3.1 shows the unconditional descriptive statistics of the training set for each cryptocurrency (31 May 2018 – 30 June 2019). It is possible to note how the return distributions tend to present different characteristics (despite the similarity shown in Figure 3.1). This fact makes it possible to assume a different risk contribution in terms of potential losses. Going into more detail, all the cryptocurrencies (except Bitcoin and Binance) have a negative median and mean (no Litecoin). In terms of variability, Eos, Ripple and Litecoin present a higher standard deviation. Bitcoin, on the other hand, seems to be the less risky asset in terms of erratic movement around the expected value, with the lowest standard deviation and the lowest variability range (max-min). As for the other measures, all the distributions are not symmetrical. Bitcoin, Binance, Ethereum and Eos show a negative skewness (in presence of which there is a longest tail for the negative returns). Litecoin and Ripple, contrary, are positively skewed (with a longer right tail for extreme quantiles). Given the skewness, and that each coin has a kurtosis higher than 3, the Jarque-Bera test (last row) returns an almost null p-value for all the assets. As consequence, none of cryptocurrencies can be considered normally distributed. This result justifies the usage of a copula-based simulated VaR and ES as measures of risk. In presence of shape and fat tails, indeed, standard deviation cannot be considered the only relevant risk factor.

Insert here, table [3.1].

4. Results

4.1. Univariate GARCH models for volatility

Table 4.1 contains the order for both the volatility and mean processes and final RMSE value as out-of-sample goodness of fitting measurements.Unfortunately, there is not a standard result in terms of which frame is the most suitable in terms of volatility forecasts: 1) 1m data are the best solution for Ethereum, Litecoin, Ripple and Eos; 2) 1h for Bitcoin and 3) 30m for Binance. As for the worst results, on the contrary, 15m data seem to be the less accurate time frequency (followed by 5m ones). Despite these evidences, the RMSE assumed by each frame for each cryptocurrency remains at a low level, testifying a general adaptability of realized GARCH models with respect to conditional volatility predictions. This result, in accordance to literature, testifies how a combination between ARMA and GARCH can improve the quality of forecasts, even if extended to higher frequencies perspectives.

Insert here, table [4.1].

4.2. Marginal Distributions

Insert here, table [4.2].

Table 4.2 shows the significance of both the Kolmogorv-Smirnov test and the Uniformity test, highlighting how Ged and Skewed Ged are the most suitable density functions for all the frames considered. Indeed, they show the highest p-values: Skewed Ged for Bitcoin and Eos; Ged for Binance, Ethereum, Litecoin and Ripple. Even from a graphical point of view (QQ-plot[14]), these distributions seem to be the most similar to the empirical returns (although if in absence of an evident significant difference). Given their predominance, the above-mentioned probability functions were chosen as marginal distributions to be inserted in the multivariate copula.

4.3. Copula function.

The RMSE resulting from the out-of-sample forecasts are summarized in Table 4.3, where results tend to differ considering different time frames: 1) Gumbel copula for 1m and 15m data; 2) Joe copula for 5m; 3) Clayton copula for 30m and 4) Frank copula for 1h.

Archimedean copulas are always preferred to the Elliptical ones, testifying a higher accuracy for models which do not assume a priori linear dependencies among the assets within the same portfolio. Similar results, as already mentioned in the Introduction, have been already found in literature. Another important evidence is represented by the quality of 5m data. All the RMSE are indeed higher than the values obtained from the other frames. This could be related to the accuracy obtained for all the GARCH models (higher than the one resulting from a 15m frame, but still lower the other frequencies), capable to influence the multivariate forecasts. 1m data, contrary, have the lowest RMSE, showing a greater predictive power. However, while the RMSE for GARCH has been computed considering real out-of-sample realized volatilities, here it is computed considering the expected return for each day (mean of all the simulations per period).

Insert here, table [4.3].

4.4. Term Structure

[14] Please note that the plots are not shown given their huge numerosity (6 assets x 5 frames x 8 distributions = 240 plots).

After having optimized the copula models, 10,000 simulations have been carried out for each frame for the month of July 2019, in order to forecast daily multi-period Value at Risk and Expected Shortfall for both the 99% (Figure 4.1) and 95% (Figure 4.2) levels of confidence[15]. As can be seen, 5m data is generally the less defensive frame, returning lower asymmetrical risks for almost all the time horizons (indicating a reduced daily risk of losses for all the investment maturities). As for the other frequencies, 15m, 30m and 1h data tend to assume similar values (especially for VaR), while 1m converges with them from below, but still maintains a lower level. This finding testifies how important choosing the right framework could be.

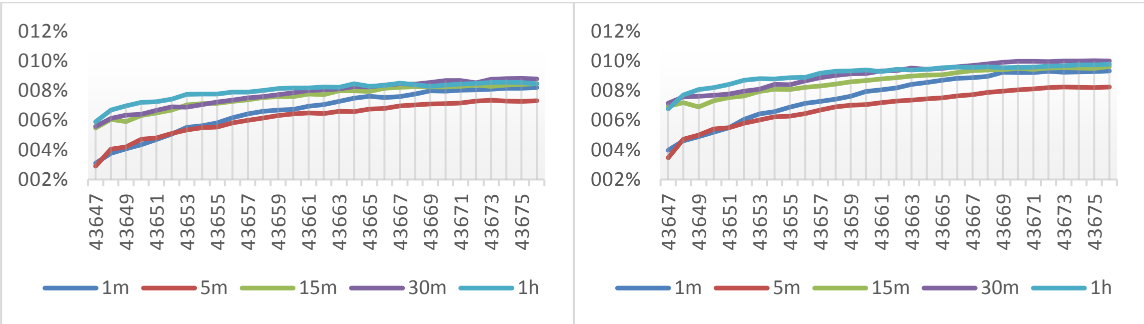

Figure 4.1. VaR and ES 99% Term Structure

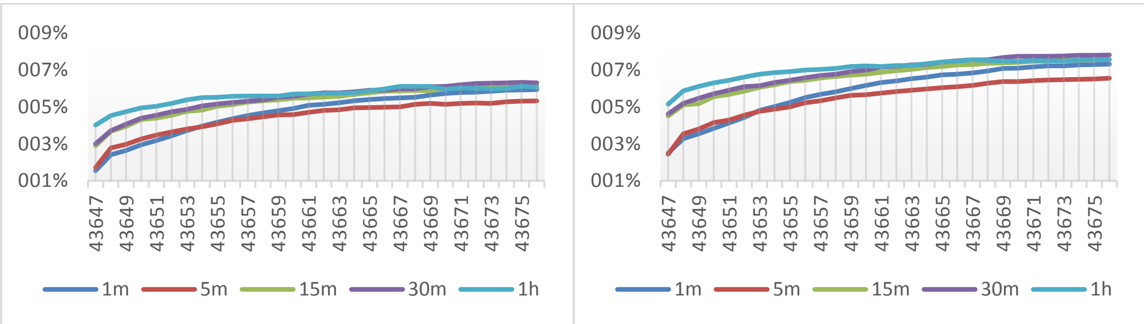

Figure 4.2. VaR and ES 95% Term Structure

Anyway, even if 1m data are the ones which best fit the prediction of the real returns from a multivariate point of view, they cannot be directly considered as the most reliable for asymmetrical risk prediction purposes. The reason is linked to the fact that the tails of a distribution could act differently from the mean according to the moments. The same evidence can be found in Table 4.4, which summarizes the risk measures week by week.

Insert here, table [4.4].

4.5. Marginal risk contributions

[15] Please note that VaR was back-tested using frequency tests. None of them rejected the model proposed by this article.

Before showing the marginal allocations, it is necessary to have an understanding of what happens to the expected weights (computed as the mean of all the simulations per day) for each cryptocurrency at each future investment maturity. As already explained in Sect. 2.6, a risk contribution depends directly on the weight of the related asset. Assuming a *ceteris paribus* framework for all the other parameters, an increase in the weight would lead to an increase in the risk contribution. Results are shown in Figure 4.3, where it is possible to note that all the weights tend to be stable over time for each frequency. This finding allows us to proceed with the analysis, making it possible to compare risk allocations.

Figure 4.3. Expected weights (July 2019)

4.6. Locally –Constant Kernel at various tail risks.

Figure 4.4 and Figure 4.5 in the (Figures Section)[16] presents how different frames tend to modify the marginal contributions of each cryptocurrency on the tail risk measures. Focusing first on the evolution

[16] Respectively related to a level of confidence of 99% and 95%.

of the contributions over time for each frame, it is possible to note how they tend to assume a more stable shape as the time horizon increases for both VaR and ES and for each tail risk level (99% and 95%).

Looking at the longest time horizon (1 month) in Table 4.5 and Table 4.6 (Tables Section)[17], even if all the frames return different contributions, they tend to respect the same ordinality for the riskiest cryptocurrencies. On a scale from the least to the riskiest one, cryptocurrencies generally follow this order (that is more stable in case of a 99% level of confidence): 1) BTC; 2) BNB; 3) XRP; 4) ETH; 5) EOS, 6) LTC. Looking at the shortest time horizon (overnight perspective), the ordinality seems to be completely different and more variable with respect to the different frequencies (and also with respect to different risk measures). However, the main difference between shorter and longer horizons is the level of concentration. It is also possible to note how the range between the least and the riskiest cryptocurrency tend to decrease after the overnight scenario. It means that time smooths the risk associated to each cryptocurrency, providing a portfolio with a risk time diversification. This is important evidence considering the standardized framework related to a trend in the weights, and it justifies the importance of studying the risk composition within a portfolio (being in line with the expectations): an asset might influence the portfolio return by a certain percentage, but affect the overall risk in a more severe way.

Another interesting finding is highlighted in the overnight scenario in Table 4.6 (95% level of confidence), 1m, 15m and 30m data presents some negative percentages both in terms of Value at Risk and Expected Shortfall. A negative contribution leads to a strong diversification effect between the related asset and the combination of the others as its inclusion in the portfolio is certainly reducing the overall tail risk[18].

After examining the asymmetric risk marginal allocation of each cryptocurrency, it is possible to split the six cryptocurrencies into two groups: the leastless risky and the riskiest ones. The former is made up of Ripple, Binance (which have a minimum risk for overnight frameworks) and Bitcoin (which seem to be the safest currency in terms of potential losses, especially for longer short-term horizons); the latter, on the other hand, is composed of Eos, Ethereum and Litecoin (which presents the highest risk for almost all the scenarios).

4.8. Locally – Linear Kernel at various tail risks

The contributions related to tail risk measures at a 99% and 95% level of confidence present a higher variability in comparison to the ones obtained with a locally constant methodology (as shown in Figure

[17] Respectively related to a level of confidence of 99% and 95%.

[18] These diversification possibilities are not shown in tail risk measures at a 99% level of confidence. This fact is certainly due to the evidence that an error component of 1% returns highly extreme negative returns, so that, even in presence of weak positive correlations, it would not be possible to have a positive return for one asset when all the others suffers losses.

4.6 and Figure 4.7 in the Figures Section[19]). Indeed, it is possible to note how their trend is not smoothed over different time horizons, leading to a non-clear ordinality among the less risky and riskier cryptocurrencies and to a difficultly interpretable time diversification effect. After having a look at Table 4.7 and Table 4.8 (Tables Section)[20], it becomes evident that the range between the riskiest asset and the safest one does not tend to reduce in the mean for all the frames as the investment maturity increases (like in the constant kernel framework). For example, Bitcoin appears to be the least risky investment for a great number of scenarios of a 99% level of confindence, but this result is not reliable. The results for a 95% level of confindece, however, seems to be more stable. Indeed, results for all the frames from a certain investment maturity follow almost the same ordinality in terms of risk. As for the riskiest asset, Litecoin has the highest percentage of contribution for all the scenarios summarized in Table 4.8. Bitcoin, on the other hand, is not always the least risky investment. This generally happens for frames with data that is higher than 1m and 5m data. This fact makes it difficult to rank the currencies with respect to their contribution to the overall risk. What is interesting is that 1m data still provides some strong diversification effects by returning negative contributions for the overnight scenario (Binance) and the 14d and 21d horizons (Ripple).

Such a model has to be carefully evaluated before its application as it tends to be more reactive to different intra-day frameworks, but it could be relevant to note how Bitcoin, Ripple and Binance are in general the coins which are chosen as safer investments. As for the riskier alternatives, Ethereum, Eos and Litecoin show, in the mean, generally higher contributions for both the tail measures.

5. Conclusion

The results show that from a univariate point of view, intraday data were considered to be most representative of the “real” level of volatility. The ARMA-realized GARCH model with GED residuals is able to capture the leverage effect and to exploit the informative content of high frequencies. Consequentially, the ARMA-realized GARCH with GED was picked as the most appropriate model to predict the dynamic of each cryptocurrency in terms of analysed models. Moreover, the fat-tailed distribution was preferred to the normal one given the evidence on the crypto-market as market with frequent extreme values. What is known is that the inclusion of realized volatility measures in the estimation of predictive models has highlighted the impact of the data frequency on predictions and was confirmed by the sensitivity to different time frames shown by results. Such a context demonstrates the usefulness of risk budgeting activities. In light of the above, this work is still believed to be a contribution to the knowledge on the cryptocurrency market, having returned results that, even if not in line with all the expectations, have suggested the presence of important effects. Certainly, the

[19] Respectively related to a level of confidence of 99% and 95%.
[20] Respectively related to a level of confidence of 99% and 95%.

expectation of a contribution to risk being different from the contribution to return was respected in all the investigated alternatives. With reference to the value of these contributions, however, the variability shown by the different frames combined with the disparity obtained by the two kernel approaches, makes it impossible to quantify precise percentages of incidence. What has generally emerged is a sort of constant ordinality among the analysed cryptocurrencies. In the short term (within 30 days), Bitcoin proved to be a generally low-risk investment in terms of potential losses. This result, in fact, belies the common beliefs born after the "bursting" of the bubble. Binance and Ripple, on the other hand, have manifested themselves as safe alternatives in the very short term (overnight and within a week), with percentages of contributions even below 1% (depending on the frame taken into account). This disparity in percentage suggests opportunities for risk diversification. In terms of greater incidence, Litecoin has revealed a certain concentration, especially in the very short term. The presence of concentration considerably increases the riskiness of a portfolio. This evidence, in absence of an investment strategy that focuses on risk, would suggest spreading the possibility of loss equally among the cryptocurrencies. Such a result, in any case, is obtained naturally with an increase of the time horizon. All contributions showed a kind of convergence at more uniform levels: time reduces concentration, making the portfolio actually less risky. The lower level of risk is also demonstrated by the term structures, whose slope made them a nonlinear function of time (whose presence would have testified the lack of time diversification). Their shape, indeed, highlighted how the daily VaR/ES grows but at decreasing rates with lower final tail risk levels. The resulting information could eventually be easily exploitable in designing investment strategies on the crypto-market. Unfortunately, the lack of equal percentages or equivalent out-of-sample measures of goodness for different frames prohibits establishing an order of preference in terms of frequencies. However, if one wanted to build a diversified portfolio without using particularly targeted strategies (such as risk parity approaches, i.e. those aimed at matching the contribution to the portfolio standard deviation of all the securities within it), the 30-day horizon would be able to offer a sufficiently satisfactory solution. Ethereum and Litecoin should be avoided in the very short term, especially in the presence of trading strategies characterized by conservative stop-losses. Investing in Bitcoin or Binance instead (which are the only securities to have a significant positive non-conditioned average and median) could actually lead to more efficient risk-return profiles (as they also generally have the lowest risk contributions).

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Tables

Table 2.1: Training and test sets for WF cross validation

Training		Validation	
Start	End	Start	End
31/05/2018	30/01/2019	31/01/2019	02/03/2019
31/05/2018	01/03/2019	02/03/2019	01/04/2019
31/05/2018	31/03/2019	01/04/2019	01/05/2019
31/05/2018	30/04/2019	01/05/2019	31/05/2019
31/05/2018	30/05/2019	31/05/2019	30/06/2019

Table 3.1 Daily unconditional descriptive statistics (training set)

Stats	**BTC**	**BNB**	**ETH**	**LTC**	**XRP**	**EOS**
min	-14,50%	-21,44%	-22,25%	-16,12%	-19,63%	-22,98%
1st q	-0,98%	-2,15%	-2,29%	-2,49%	-2,38%	-2,75%
median	0,22%	0,25%	-0,11%	-0,36%	-0,11%	-0,16%
mean	0,10%	0,24%	-0,16%	0,01%	-0,10%	-0,18%
3rd q	1,57%	2,94%	2,11%	2,79%	1,88%	2,42%
max	15,87%	17,77%	17,35%	25,75%	31,60%	21,90%
dev st	3,51%	4,94%	4,97%	5,10%	5,13%	5,96%
skew	-0,17	-0,21	-0,35	0,45	1,03	-0,04
kurt	6,55	4,83	5,59	6,04	9,94	5,46
J-B	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 3.2. Ljung-Box and LM-Arch tests (for each frame)

	Ljung-Box (returns)						Ljung-Box (realized volatility)					
	BTC	BNB	ETH	LTC	XRP	EOS	BTC	BNB	ETH	LTC	XRP	EOS
1m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
5m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
15m	0,00%	0,00%	0,03%	0,01%	0,00%	0,81%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
30m	0,01%	0,00%	0,00%	0,19%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
1h	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
1d	3,58%	7,35%	2,77%	7,36%	8,23%	3,66%	3,24%	4,45%	7,04%	9,49%	0,08%	6,40%

	LM arch					
	BTC	BNB	ETH	LTC	XRP	EOS
1m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
5m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
15m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
30m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
1h	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
1d	5,06%	8,94%	3,81%	9,82%	0,12%	3,30%

Table 4.1. Garch models summary (for each frame)

BTC						LTC					
Orders	1m	5m	15m	30m	1h	Orders	1m	5m	15m	30m	1h
garch	(2;1)	(3;1)	(1;1)	(1;1)	(2;2)	garch	(1;1)	(3;1)	(3;1)	(3;1)	(3;1)
arma	(1;1)	(1;1)	(1;1)	(1;1)	(1;1)	arma	(0;0)	(1;1)	(0;0)	(0;0)	(0;0)
RMSE	0,24%	0,27%	0,36%	0,22%	0,21%	RMSE	0,39%	0,51%	0,53%	0,46%	0,47%
BNB						XRP					
Orders	1m	5m	15m	30m	1h	Orders	1m	5m	15m	30m	1h
garch	(3;1)	(2;1)	(1;1)	(3;1)	(3;1)	garch	(1;1)	(1;1)	(2;1)	(2;1)	(1;1)
arma	(0;1)	(0;1)	(1;1)	(1;1)	(0;1)	arma	(1;1)	(0;0)	(1;1)	(0;1)	(0;0)
RMSE	0,31%	0,31%	0,32%	0,27%	0,28%	RMSE	0,39%	0,51%	0,53%	0,46%	0,47%
ETH						EOS					
Orders	1m	5m	15m	30m	1h	Orders	1m	5m	15m	30m	1h
garch	(1;1)	(2;1)	(2;1)	(3;3)	(1;3)	garch	(1;1)	(2;1)	(2;1)	(2;1)	(2;1)
arma	(1;1)	(0;1)	(0;1)	(0;1)	(0;1)	arma	(1;1)	(0;1)	(1;1)	(0;1)	(0;1)
RMSE	0,30%	0,34%	0,36%	0,32%	0,33%	RMSE	0,39%	0,51%	0,53%	0,46%	0,47%

Table 4.2. Kolmogorov-Smirnov and Uniformity tests

		Norm		sNorm		C-F		t-S		st-S 1		st-S 2		Ged		sGed	
		K-S	U	K-S	U	K-S	U	K-S	U	K-S	U	K-S	U	K-S	U	K-S	U
BTC	1m	0,36	0,00	0,36	0,00	0,59	0,00	40,90	9,60	90,30	47,32	57,41	1,77	98,33	63,46	98,33	66,42
	5m	0,28	0,00	0,36	0,00	5,22	0,21	31,61	10,01	80,79	34,20	51,63	0,01	98,33	96,63	98,33	97,39
	15m	0,36	0,00	0,47	0,00	9,07	0,38	46,11	20,35	90,30	16,67	63,36	0,05	99,80	91,75	99,80	91,75
	30m	0,28	0,00	0,36	0,00	7,58	0,09	51,63	11,78	93,89	16,36	63,36	7,94	99,34	91,44	99,34	91,44
	1h	0,28	0,00	0,28	0,00	12,78	1,20	46,11	7,61	93,89	5,34	63,36	0,05	99,34	79,11	99,34	79,11
BNB	1m	15,06	0,03	15,06	0,01	90,30	52,64	90,30	35,73	85,88	19,64	80,79	0,16	100,00	82,96	99,96	69,33
	5m	9,07	0,01	10,79	0,01	90,30	53,23	80,79	32,71	80,79	24,58	75,21	32,22	98,33	78,60	98,33	74,93
	15m	9,07	0,00	10,79	0,00	69,34	34,20	80,79	19,99	85,88	6,98	75,21	10,43	96,55	84,74	96,55	84,74
	30m	10,79	0,00	12,78	0,00	75,21	31,25	85,88	12,01	85,88	4,15	80,79	6,39	90,30	71,61	90,30	65,83
	1h	9,07	0,01	12,78	0,01	80,79	25,83	75,21	30,30	80,79	24,58	75,21	26,69	90,30	97,25	90,30	96,29
ETH	1m	1,20	0,00	0,59	0,00	5,22	4,66	80,79	13,52	85,88	9,21	93,89	17,30	99,80	89,80	96,55	76,53
	5m	0,59	0,00	0,28	0,00	4,29	0,68	57,41	17,30	63,36	14,33	85,88	10,86	96,55	97,52	93,89	95,72
	15m	0,76	0,00	0,36	0,00	5,22	2,39	63,36	14,89	69,34	12,01	80,79	11,78	96,55	96,46	93,89	98,01
	30m	0,47	0,00	0,22	0,00	3,52	2,11	51,63	2,00	51,63	5,46	75,21	0,01	90,30	97,90	85,88	92,90
	1h	0,47	0,00	0,22	0,00	4,29	1,44	51,63	2,57	51,63	1,95	75,21	9,80	90,30	96,63	85,88	95,53
LTC	1m	5,22	0,02	2,33	0,15	0,02	0,00	63,36	32,22	75,21	44,44	57,41	74,93	85,88	56,84	63,36	42,18
	5m	1,51	0,00	0,96	0,00	0,07	0,00	93,89	48,49	75,21	32,71	69,34	61,06	99,34	85,17	98,33	89,10
	15m	3,52	0,00	1,51	0,01	0,00	0,00	46,11	54,43	75,21	67,01	46,11	60,46	85,88	68,17	69,34	58,65
	30m	3,52	0,00	1,51	0,02	0,00	0,00	51,63	55,04	69,34	52,04	51,63	71,05	90,30	36,25	69,34	34,70
	1h	4,29	0,00	1,88	0,04	0,00	0,00	57,41	49,08	75,21	30,30	51,63	2,30	90,30	47,90	69,34	31,73
XRP	1m	1,20	0,00	0,96	0,00	0,47	0,00	75,21	55,64	93,89	83,41	75,21	56,24	96,55	86,82	90,30	76,00
	5m	1,20	0,00	1,51	0,00	2,87	0,00	75,21	67,01	69,34	61,05	76,33	0,73	78,11	64,44	70,90	62,74
	15m	0,96	0,00	0,76	0,00	2,87	0,83	63,36	62,86	90,30	47,90	69,34	44,44	85,88	95,11	85,88	94,43
	30m	0,76	0,00	0,76	0,00	5,22	1,72	57,41	61,06	85,88	78,09	63,36	49,67	96,55	80,59	90,30	74,39
	1h	3,52	0,00	4,29	0,00	23,88	1,40	72,55	22,98	69,34	41,62	68,33	0,84	75,21	41,07	63,36	40,20
EOS	1m	0,47	0,00	0,59	0,00	93,89	75,47	57,41	1,95	75,21	3,96	69,34	1,33	99,80	95,53	99,80	95,53
	5m	0,96	0,00	0,96	0,00	99,34	77,06	51,63	5,46	63,36	4,88	63,36	3,87	97,73	98,01	98,33	98,61
	15m	0,36	0,00	0,47	0,00	75,21	48,49	15,06	2,51	36,06	3,52	46,11	0,01	63,36	19,29	63,36	19,29
	30m	0,36	0,00	0,47	0,00	75,21	63,46	23,88	2,05	36,06	0,15	40,90	0,09	80,79	74,93	85,88	77,06
	1h	0,47	0,00	0,47	0,00	63,36	15,18	17,65	0,44	27,55	0,07	31,61	0,42	69,34	27,57	69,34	30,78

Table 4.3 Copula RMSE for each frame

Copula	Frame				
	1m	5m	15m	30m	1h
Normal	5,2031%	5,2622%	5,2237%	5,2219%	5,2157%
t-Student	5,2051%	5,2609%	5,2209%	5,2191%	5,2246%
Gumbel	5,1980%	5,2572%	5,2063%	5,2058%	5,2295%
Clayton	5,2103%	5,2594%	5,2175%	5,2158%	5,2291%
Frank	5,2075%	5,2655%	5,2216%	5,2162%	5,2142%
Joe	5,2045%	5,2559%	5,2226%	5,2159%	5,2193%

Table 4.4. Term Structures in % (weeks)

	VaR 99%					ES 99%				
Time	1m	5m	15m	30m	1h	1m	5m	15m	30m	1h
01/07	3,10	2,90	5,48	5,59	5,90	3,98	3,47	6,95	7,16	6,77
07/07	5,51	5,35	7,05	6,89	7,77	6,43	6,03	7,96	8,10	8,81
14/07	6,74	6,44	7,60	7,87	8,18	7,96	7,07	8,70	9,17	9,40
21/07	7,60	6,98	8,24	8,48	8,53	8,89	7,74	9,36	9,70	9,58
30/07	8,22	7,33	8,48	8,80	8,47	9,34	8,25	9,63	10,01	9,79
	VaR 95%					ES 95%				
Time	1m	5m	15m	30m	1h	1m	5m	15m	30m	1h
01/07	1,54	1,72	2,90	3,01	4,01	2,50	2,44	4,50	4,62	5,15
07/07	3,72	3,80	4,76	4,86	5,38	4,81	4,75	6,05	6,14	6,76
14/07	4,90	4,58	5,46	5,58	5,70	6,15	5,66	6,76	6,99	7,22
21/07	5,48	5,00	5,86	5,97	6,12	6,83	6,17	7,29	7,54	7,55
30/07	5,93	5,32	6,14	6,30	6,06	7,31	6,55	7,57	7,80	7,55

Table 4.5. Constant Kernel marginal tail risk allocations 99%

		VaR 99%						ES 99%					
	Time	BTC	BNB	ETH	LTC	XRP	EOS	BTC	BNB	ETH	LTC	XRP	EOS
1d	1m	13,00%	0,03%	34,77%	29,10%	2,78%	20,31%	15,04%	0,99%	35,34%	25,55%	3,94%	19,15%
	5m	10,53%	4,63%	31,59%	22,07%	11,35%	19,83%	10,16%	4,39%	32,00%	22,76%	9,87%	20,82%
	15m	17,52%	9,32%	25,63%	28,97%	0,26%	18,31%	18,11%	9,81%	24,84%	27,04%	0,82%	19,38%
	30m	20,61%	20,67%	17,03%	20,73%	0,45%	20,52%	21,61%	18,81%	16,83%	20,05%	1,33%	21,37%
	1h	18,00%	14,00%	20,92%	22,29%	5,82%	18,97%	18,26%	13,49%	21,19%	20,40%	5,29%	21,37%
7d	1m	10,99%	9,16%	24,01%	24,19%	13,70%	17,94%	11,75%	9,31%	24,08%	23,08%	13,54%	18,24%
	5m	11,11%	13,49%	20,69%	24,00%	12,69%	18,02%	11,34%	13,28%	20,88%	24,07%	12,14%	18,29%
	15m	14,27%	12,88%	19,19%	24,54%	10,93%	18,19%	14,45%	12,70%	19,30%	24,48%	10,79%	18,27%
	30m	14,82%	17,02%	18,24%	21,92%	10,37%	17,62%	14,68%	17,63%	17,79%	21,62%	10,17%	18,10%
	1h	15,06%	16,07%	18,97%	20,90%	10,43%	18,58%	15,04%	16,19%	19,14%	20,75%	10,24%	18,65%
14d	1m	9,81%	12,35%	20,74%	22,35%	15,92%	18,84%	10,57%	12,70%	20,68%	21,56%	16,19%	18,30%
	5m	10,26%	14,64%	19,98%	23,00%	14,49%	17,63%	10,16%	14,89%	20,12%	22,96%	14,32%	17,55%
	15m	12,42%	13,75%	18,18%	23,48%	14,56%	17,60%	12,56%	14,09%	18,14%	23,16%	14,06%	18,00%
	30m	12,66%	16,35%	17,33%	21,21%	14,88%	17,57%	12,71%	16,68%	17,46%	21,00%	14,56%	17,60%
	1h	13,70%	15,77%	17,49%	21,39%	13,17%	18,49%	13,81%	16,22%	17,40%	21,36%	13,11%	18,10%
21d	1m	9,81%	13,86%	18,86%	21,80%	17,18%	18,49%	10,11%	14,31%	18,92%	21,12%	17,15%	18,38%
	5m	10,16%	15,11%	19,28%	21,85%	15,29%	18,30%	10,39%	15,18%	19,60%	21,32%	15,29%	18,22%
	15m	11,27%	14,58%	17,64%	22,24%	16,26%	18,00%	11,54%	14,87%	17,88%	21,41%	16,01%	18,28%
	30m	11,72%	16,37%	16,86%	21,07%	15,80%	18,18%	12,20%	16,36%	17,06%	20,47%	15,63%	18,28%
	1h	12,51%	15,88%	17,47%	21,33%	14,73%	18,09%	12,78%	15,87%	17,29%	21,19%	14,78%	18,08%
30d	1m	9,50%	14,44%	18,63%	20,85%	17,62%	18,96%	9,95%	14,65%	18,60%	20,08%	17,23%	19,49%
	5m	9,83%	14,93%	18,39%	22,07%	15,75%	19,03%	9,86%	15,26%	18,64%	22,01%	15,48%	18,73%
	15m	10,87%	14,41%	17,48%	21,50%	16,86%	18,89%	11,23%	14,62%	17,48%	21,03%	16,59%	19,06%
	30m	10,92%	16,38%	16,64%	20,80%	17,19%	18,07%	11,32%	16,42%	16,75%	20,43%	17,03%	18,05%
	1h	11,84%	15,97%	17,42%	21,07%	15,30%	18,39%	12,16%	16,64%	16,92%	20,40%	15,64%	18,23%

Table 4.6. Constant Kernel marginal tail risk allocations 95%

		VaR 95%						ES 95%					
	Time	BTC	BNB	ETH	LTC	XRP	EOS	BTC	BNB	ETH	LTC	XRP	EOS
1d	1m	9,37%	-7,40%	36,39%	44,63%	-2,21%	19,21%	12,48%	-1,83%	35,08%	32,72%	1,86%	19,68%
	5m	9,84%	1,69%	32,07%	19,36%	18,05%	18,99%	10,30%	3,74%	31,59%	21,39%	13,18%	19,80%
	15m	17,81%	8,73%	22,91%	37,91%	-3,95%	16,60%	17,94%	9,47%	24,37%	31,01%	-0,84%	18,05%
	30m	18,50%	19,57%	16,49%	29,22%	-2,92%	19,14%	20,20%	19,93%	16,70%	23,24%	-0,31%	20,23%
	1h	16,69%	14,01%	19,44%	25,07%	7,40%	17,38%	17,58%	13,94%	20,38%	22,93%	6,34%	18,83%
7d	1m	9,89%	8,29%	23,62%	26,42%	14,24%	17,53%	10,76%	8,91%	23,86%	24,80%	13,86%	17,83%
	5m	10,80%	12,87%	20,96%	24,05%	13,52%	17,79%	11,06%	13,21%	20,83%	23,99%	12,93%	17,98%
	15m	13,49%	12,57%	19,05%	26,33%	11,29%	17,28%	13,97%	12,74%	19,21%	25,23%	11,05%	17,81%
	30m	14,20%	16,80%	17,84%	23,26%	10,91%	16,98%	14,61%	17,11%	17,99%	22,30%	10,50%	17,50%
	1h	14,62%	15,11%	18,98%	22,09%	11,32%	17,87%	14,90%	15,66%	19,04%	21,32%	10,77%	18,31%
14d	1m	9,10%	11,52%	20,39%	24,03%	16,71%	18,24%	9,75%	12,14%	20,60%	22,75%	16,23%	18,54%
	5m	9,90%	13,60%	19,92%	23,22%	15,10%	18,26%	10,14%	14,27%	19,96%	23,02%	14,72%	17,90%
	15m	11,53%	12,94%	18,17%	24,94%	15,48%	16,93%	12,10%	13,52%	18,20%	23,96%	14,82%	17,40%
	30m	11,70%	15,87%	17,01%	22,90%	15,27%	17,26%	12,31%	16,25%	17,19%	21,84%	14,94%	17,47%
	1h	12,52%	15,15%	17,96%	22,44%	13,99%	17,95%	13,26%	15,61%	17,72%	21,70%	13,46%	18,26%
21d	1m	8,86%	12,86%	19,04%	23,16%	17,80%	18,27%	9,54%	13,62%	18,95%	22,13%	17,37%	18,39%
	5m	9,47%	14,07%	18,81%	23,46%	15,42%	18,76%	9,93%	14,71%	19,14%	22,42%	15,33%	18,47%
	15m	10,62%	13,50%	17,61%	24,21%	16,80%	17,26%	11,07%	14,22%	17,67%	22,88%	16,37%	17,79%
	30m	10,70%	15,56%	16,54%	22,79%	16,92%	17,49%	11,45%	16,11%	16,79%	21,56%	16,15%	17,93%
	1h	11,56%	15,43%	17,71%	22,35%	15,14%	17,81%	12,14%	15,72%	17,59%	21,67%	14,86%	18,02%
30d	1m	8,55%	13,14%	18,13%	22,58%	18,80%	18,80%	9,22%	13,99%	18,40%	21,38%	18,06%	18,95%
	5m	8,98%	14,36%	18,39%	22,99%	15,99%	19,29%	9,53%	14,77%	18,42%	22,37%	15,82%	19,09%
	15m	9,97%	13,94%	17,20%	23,35%	17,83%	17,73%	10,59%	14,30%	17,36%	22,10%	17,17%	18,48%
	30m	9,95%	15,54%	16,39%	22,37%	17,99%	17,76%	10,61%	16,09%	16,56%	21,33%	17,43%	17,98%
	1h	11,09%	15,04%	17,86%	22,56%	15,69%	17,76%	11,64%	15,73%	17,54%	21,47%	15,47%	18,14%

Table 4.7. Linear Kernel marginal tail risk allocations 99%

		VaR 99%						ES 99%					
	Time	BTC	BNB	ETH	LTC	XRP	EOS	BTC	BNB	ETH	LTC	XRP	EOS
1d	1m	13,78%	0,76%	35,25%	26,83%	3,09%	20,29%	20,01%	1,25%	34,46%	22,38%	1,53%	20,37%
	5m	10,92%	9,62%	25,63%	22,76%	12,82%	18,25%	12,60%	8,51%	22,01%	21,87%	13,57%	21,44%
	15m	9,88%	12,78%	20,61%	21,68%	16,35%	18,69%	11,79%	10,86%	20,32%	19,30%	19,28%	18,46%
	30m	11,06%	14,30%	18,56%	21,51%	16,93%	17,66%	12,00%	9,97%	20,83%	21,58%	17,30%	18,31%
	1h	10,92%	13,52%	19,16%	20,38%	16,58%	19,44%	8,72%	7,48%	19,13%	23,25%	26,44%	14,98%
7d	1m	10,38%	5,01%	32,59%	22,99%	8,70%	20,32%	16,41%	2,98%	31,30%	17,98%	8,13%	23,20%
	5m	11,44%	14,55%	20,15%	24,36%	11,75%	17,76%	11,01%	16,69%	19,78%	21,50%	12,10%	18,93%
	15m	10,12%	15,52%	21,10%	23,35%	13,82%	16,10%	10,47%	16,01%	21,76%	18,17%	15,78%	17,81%
	30m	10,71%	15,23%	20,27%	20,96%	15,42%	17,41%	10,90%	16,46%	22,30%	17,13%	13,59%	19,62%
	1h	10,03%	15,42%	19,22%	21,40%	15,04%	18,89%	8,90%	18,80%	15,00%	18,38%	15,17%	23,75%
14d	1m	17,16%	9,25%	26,04%	28,12%	0,79%	18,65%	15,45%	11,92%	26,36%	28,04%	2,56%	15,66%
	5m	14,64%	13,27%	18,21%	23,61%	11,24%	19,04%	13,10%	12,56%	19,51%	24,73%	11,11%	19,00%
	15m	12,64%	14,57%	17,73%	22,83%	14,22%	18,01%	11,43%	13,85%	19,75%	21,99%	12,90%	20,09%
	30m	12,26%	15,50%	16,97%	22,13%	16,04%	17,09%	11,45%	15,41%	19,06%	18,87%	15,76%	19,46%
	1h	11,59%	13,66%	18,45%	21,04%	15,99%	19,28%	14,33%	10,61%	15,20%	25,36%	15,67%	18,83%
21d	1m	20,94%	20,07%	17,29%	19,81%	0,84%	21,05%	15,32%	20,01%	23,37%	22,10%	0,43%	18,78%
	5m	14,62%	17,18%	18,27%	21,71%	10,34%	17,89%	13,03%	20,12%	18,75%	22,60%	8,39%	17,11%
	15m	12,57%	15,67%	18,55%	21,26%	14,21%	17,74%	13,30%	18,14%	18,52%	19,20%	13,84%	17,00%
	30m	11,78%	16,23%	17,90%	19,33%	16,08%	18,68%	13,01%	16,08%	18,52%	19,54%	15,06%	17,79%
	1h	11,13%	17,23%	17,79%	19,88%	16,59%	17,38%	11,25%	21,69%	16,07%	14,43%	20,11%	16,46%
30d	1m	18,44%	13,77%	21,53%	20,76%	5,06%	20,44%	13,64%	14,00%	21,39%	18,12%	5,90%	26,95%
	5m	15,18%	16,68%	18,55%	20,65%	10,02%	18,92%	13,85%	15,94%	20,39%	22,48%	12,20%	15,14%
	15m	14,18%	16,03%	17,14%	21,42%	13,05%	18,17%	13,40%	15,77%	18,86%	19,74%	16,68%	15,54%
	30m	12,99%	15,80%	17,64%	20,30%	15,16%	18,11%	12,00%	15,64%	17,54%	20,35%	14,83%	19,64%
	1h	11,50%	15,83%	16,49%	21,64%	15,38%	19,16%	11,27%	22,50%	10,55%	26,34%	8,99%	20,36%

Table 4.8. Linear Kernel marginal tail risk allocations 95%

		VaR 95%						ES 95%					
	Time	BTC	BNB	ETH	LTC	XRP	EOS	BTC	BNB	ETH	LTC	XRP	EOS
1d	1m	11,08%	-3,89%	34,88%	37,42%	0,67%	19,83%	15,92%	0,81%	30,23%	33,97%	1,96%	17,11%
	5m	10,13%	9,02%	24,30%	25,39%	13,65%	17,52%	12,14%	10,16%	20,25%	24,04%	16,84%	16,56%
	15m	9,26%	12,31%	20,56%	22,35%	16,56%	18,97%	11,24%	13,74%	20,13%	21,12%	16,83%	16,93%
	30m	9,99%	14,28%	18,74%	21,73%	15,65%	19,61%	10,93%	14,90%	18,92%	20,99%	17,51%	16,75%
	1h	9,52%	13,75%	18,59%	21,32%	19,11%	17,71%	13,13%	16,94%	18,26%	22,13%	16,45%	13,09%
7d	1m	11,04%	5,00%	30,22%	21,75%	12,49%	19,51%	8,60%	6,75%	28,18%	21,10%	10,82%	24,55%
	5m	11,77%	12,84%	20,53%	23,93%	13,27%	17,66%	10,11%	14,10%	20,66%	22,23%	13,98%	18,91%
	15m	10,70%	13,93%	19,61%	22,45%	15,03%	18,29%	9,77%	15,03%	19,53%	22,61%	15,56%	17,50%
	30m	9,38%	13,86%	19,07%	23,10%	15,01%	19,58%	9,67%	15,54%	19,07%	22,01%	15,53%	18,18%
	1h	9,39%	15,01%	17,67%	22,84%	15,71%	19,38%	10,73%	14,24%	18,23%	22,41%	15,08%	19,31%
14d	1m	17,96%	9,64%	23,57%	33,54%	-1,88%	17,17%	18,77%	8,53%	23,44%	32,81%	1,14%	15,31%
	5m	13,68%	13,17%	19,57%	25,43%	11,22%	16,94%	14,25%	12,45%	21,90%	24,33%	10,06%	17,00%
	15m	12,13%	13,24%	18,79%	23,45%	15,43%	16,97%	12,11%	14,29%	18,74%	24,39%	13,72%	16,74%
	30m	10,98%	13,86%	18,00%	23,23%	16,34%	17,59%	10,68%	15,61%	18,79%	23,02%	15,80%	16,10%
	1h	10,72%	14,11%	17,43%	22,27%	17,37%	18,10%	13,31%	12,03%	14,32%	20,48%	18,34%	21,52%
21d	1m	19,13%	20,32%	16,56%	25,67%	-1,27%	19,58%	19,12%	17,86%	17,92%	23,21%	0,17%	21,72%
	5m	14,99%	16,92%	18,14%	21,95%	10,33%	17,68%	14,93%	16,74%	18,64%	22,78%	9,37%	17,55%
	15m	12,31%	16,38%	16,94%	22,07%	14,57%	17,73%	12,61%	17,77%	16,93%	21,27%	14,12%	17,29%
	30m	11,08%	16,57%	16,74%	21,37%	16,24%	18,01%	10,75%	16,80%	17,66%	20,75%	15,61%	18,43%
	1h	10,05%	16,79%	16,32%	21,60%	17,52%	17,71%	11,08%	16,88%	15,28%	20,80%	16,66%	19,31%
30d	1m	17,37%	14,24%	19,97%	23,87%	6,81%	17,75%	16,93%	14,75%	21,80%	21,08%	6,26%	19,17%
	5m	15,14%	14,95%	19,40%	21,40%	11,16%	17,94%	15,10%	15,80%	18,87%	21,08%	10,65%	18,50%
	15m	12,44%	15,54%	18,33%	21,83%	13,45%	18,41%	13,34%	16,44%	19,12%	20,88%	12,86%	17,36%
	30m	11,25%	15,20%	18,24%	22,06%	14,06%	19,19%	13,09%	16,78%	18,41%	20,75%	13,15%	17,83%
	1h	12,38%	15,53%	17,80%	21,41%	14,85%	18,03%	12,24%	17,60%	17,72%	19,19%	14,15%	19,10%

Figures

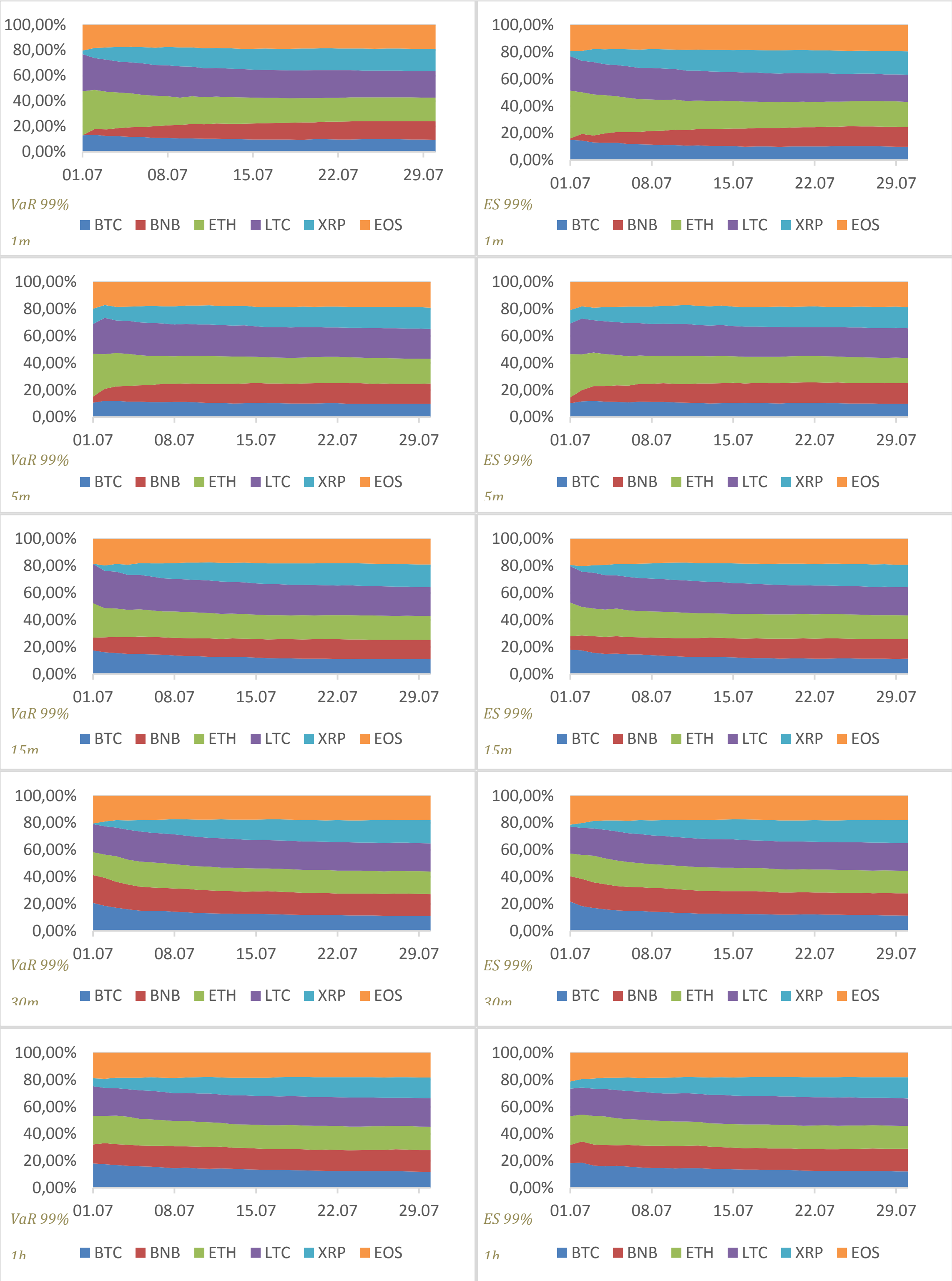

Figure 4.4. Constant Kernel marginal tail risk allocations 99%

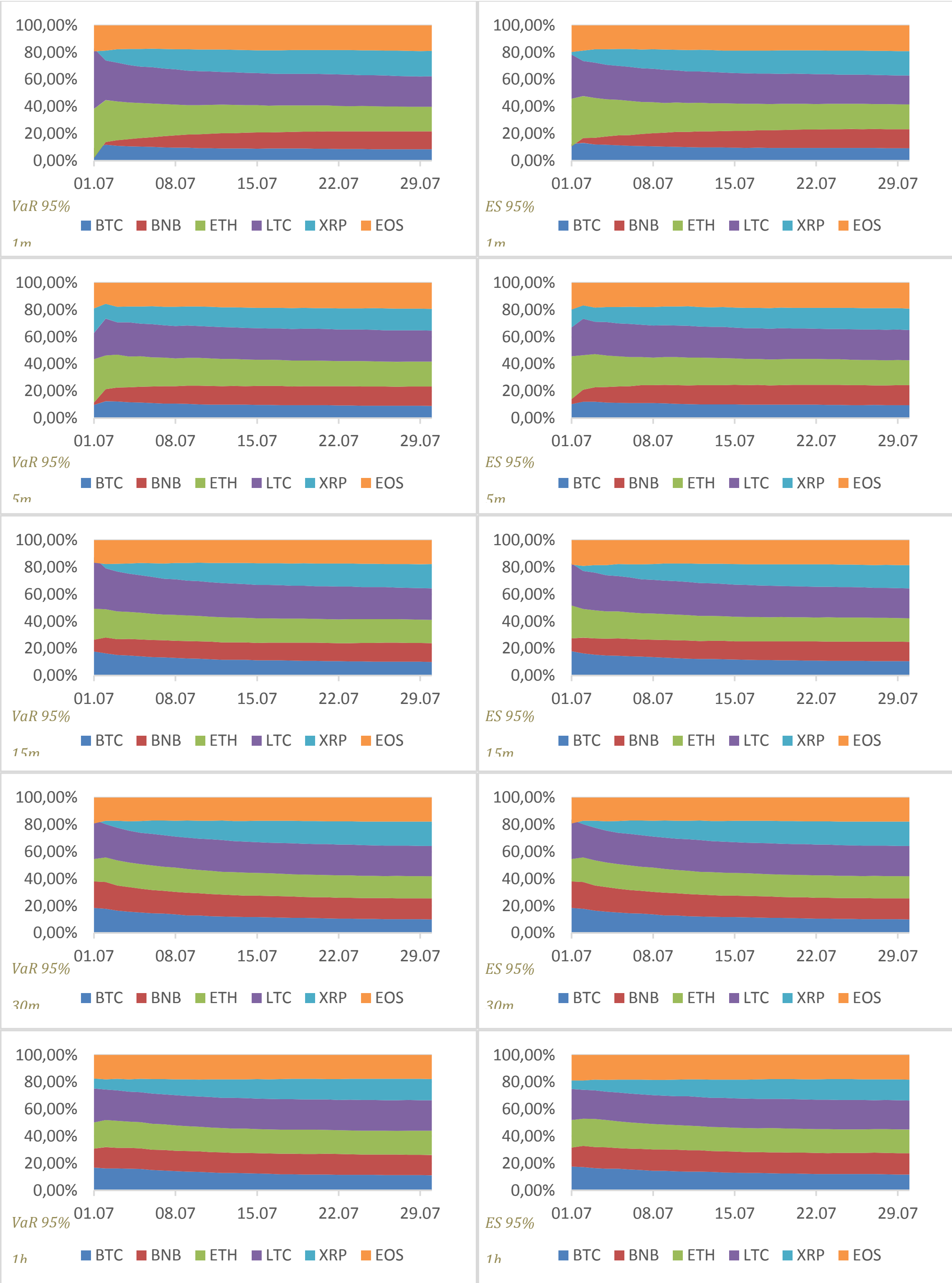

Figure 4.5. Constant Kernel marginal tail risk allocations 95%

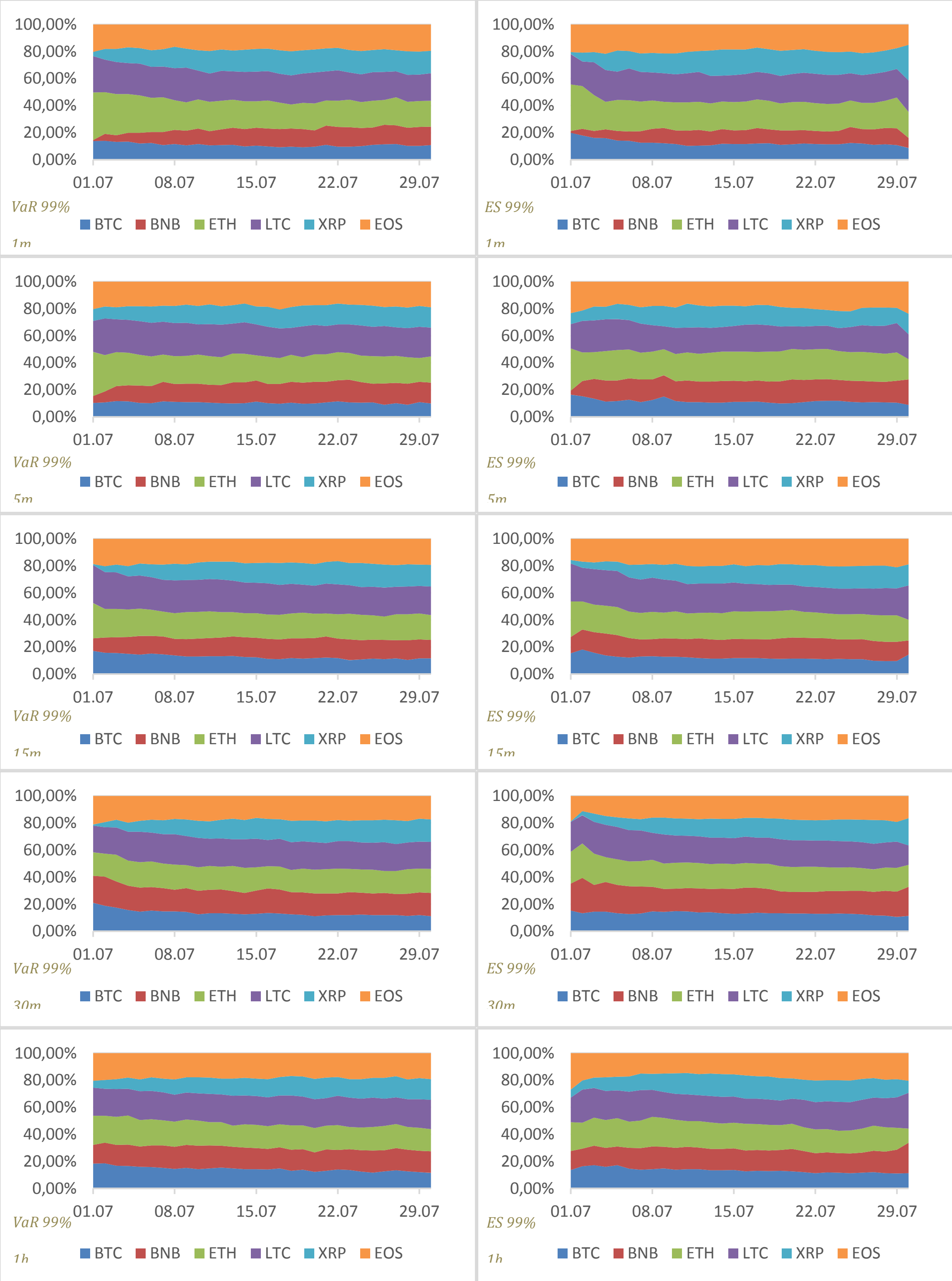

Figure 4.6. Linear Kernel marginal tail risk allocations 99%

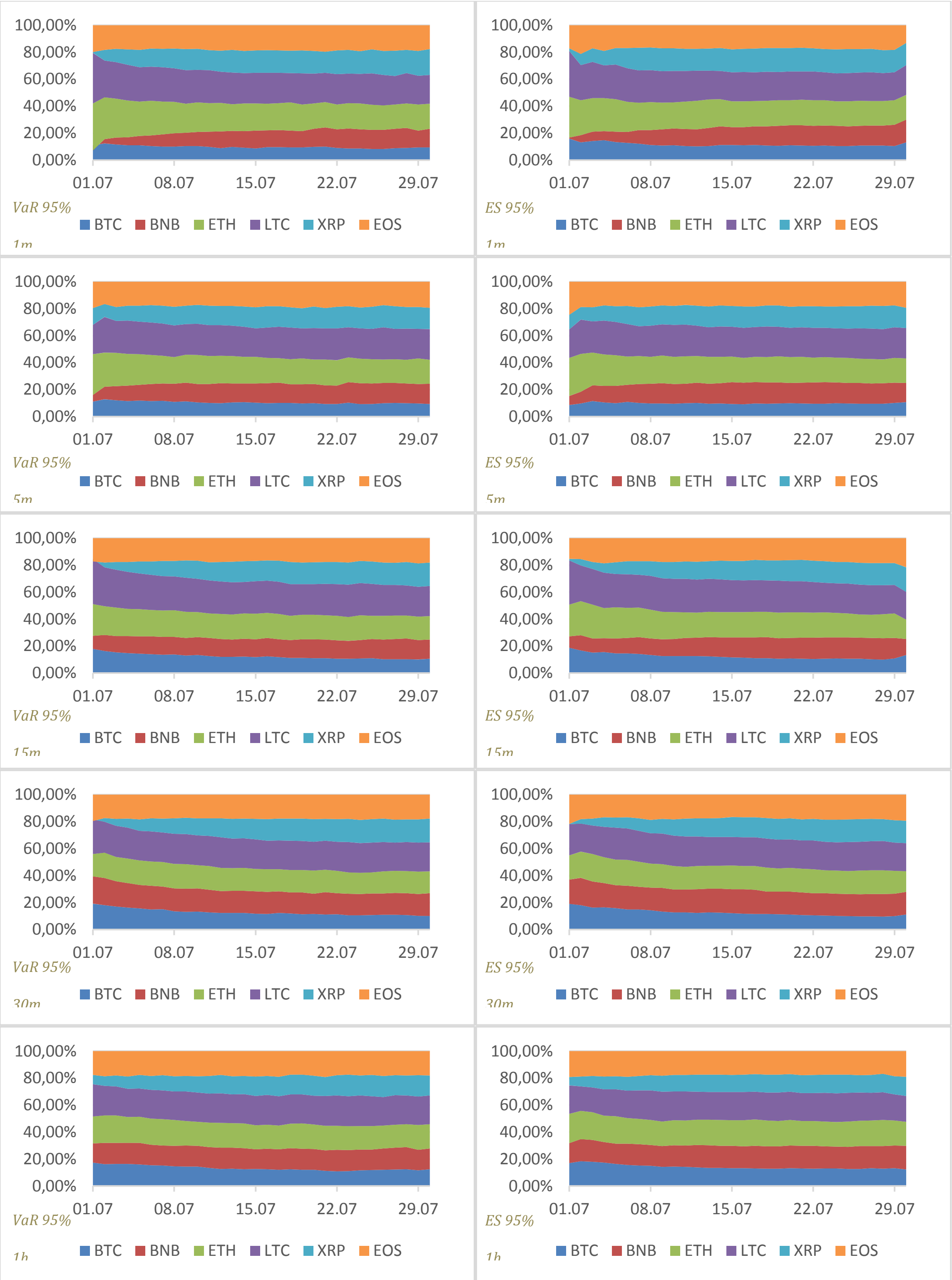

Figure 4.7. Linear Kernel marginal tail risk allocations 95%