Joint Discussion Paper Series in Economics

by the Universities of

Aachen · Gießen · Göttingen Kassel · Marburg · Siegen

ISSN 1867-3678

No. 04-2020

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Forward Guidance Under the Cost Channel*

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January 24, 2020

Abstract

A common finding in the literature is that forward guidance cannot be credible under discretionary policy as long as the zero lower bound is an one-off event. However, this is not the case when recurring episodes of zero interest rates are possible. In this paper, we contribute to this new result and assess the sustainability of forward guidance under the cost channel. We find that forward guidance can be sustainable under the cost channel. However, we show that it is less credible compared to a standard New Keynesian model. Our results show that this finding also depends on the strength of the cost channel. Furthermore, provide evidence that ignoring the presence of a cost channel can be costly in terms of steady-state consumption.

Keywords: Forward Guidance, Sustainability, Cost Channel, Discretion

JEL classification: E12, E43, E52, E58, E61

*I thank Peter Tillmann and Carl Walsh for helpful comments and suggestions.

1 Introduction

The relevance of the zero lower bound is a hot topic and much debated, both among policymakers and researchers alike. In an environment of low interest rate, central banks have in fact not much ammunition left to fight deflationary pressures. When short-term interest rates are stuck at the zero lower bound (ZLB), central banks typically rely on either large asset purchase programs or forward guidance, i.e., based on its assessment of the outlook for price stability, the central bank commits to future policy actions. One common result is that the ZLB is not a serious constraint on the implementation of optimal monetary policy as long as the central bank can commit to future actions (Eggertsson and Woodford, 2003; Nakov, 2008; and Jung et al., 2005). For example, Adam and Billi (2007) show that losses in terms of steady-state consumption increase by 65% under discretionary policy as against under commitment (see also, Adam and Billi, 2006). From the perspective that monetary policy is indeed discretionary, it follows that ZLB episodes have substantial adverse effects.

However, most of the papers that deal with forward guidance and its credibility assume that ZLB episodes are one-off events. More precisely, they rely on the assumption that, once the ZLB episode is over, it never reoccurs in the future. In essence, this is precisely why forward guidance in a discretionary fashion lacks credibility. That is, there is no benefit to fulfilling promises that were made in the past because these promises will never be honored. As such, following the insinuated path is of no further use if ZLB episodes cannot reoccur in the future. Recent models have been extended in a way to allow for frequent ZLB episodes. According to Nakata (2018), an optimal Ramsey plan can credibly sustain as long as there is only a marginal probability that a binding zero lower bound will reoccur in the future.[1] Walsh (2018) shows that also under discretionary policy, forward guidance can be sustainable as long as there are recurring episodes of binding constraints on interest rates. This result is new and important because (i) it is easier to think of discretionary policy to be the more realistic assumption than any time-inconsistent counterpart and (ii), as pointed out by Walsh (2018), discretionary policy might be way easier to communicate than a Ramsey plan, as investigated in Nakata (2018). As such, the sustainability concept of Walsh (2018) may serve as an essential benchmark for the analysis of monetary policy at the zero lower bound when alternative equilibria are possible.

[1] In a more recent paper, Bilbiie (2019) derives closed-form solutions for the optimal duration of forward guidance. In his framework, the duration of the policy is stochastic in a sense that, after exiting the zero lower bound, the central bank keeps the nominal interest rate at zero with a constant probability. However, also in his model, it is assumed that future episodes at the zero lower bound never reoccur.

However, the desire to analyze the sustainability of forward guidance in a framework of reoccurring ZLB episodes is not only born out of theoretical curiosity but is, in part, grounded on recent evidence of movements in financial markets. Laubach and Williams (2016) find that the real interest rate for the US dramatically declined over the past decade.[2] Moreover, there is evidence that markets also expect further ZLB episodes in the near future. For example, in the latest New York Fed's survey of primary dealers, respondants attached a median 35% chance of returning to the ZLB until the end of 2022. The 75th percentile was even at 43%. This allows for the assumption that periods may occur in the future, in which interest rates are pushed to the ZLB, saying that the zero lower bound will continue to be highly relevant in the future.

In this paper, we build on the approach of Walsh (2018) and explore the sustainability of forward guidance in a calibrated New Keynesian model that features financial frictions. We investigate whether and, if so, how the sustainability of forward guidance is affected under the cost channel of monetary policy. A cost channel is present when firms' marginal cost depend on the nominal rate of interest. This can be motivated under the assumption that firms need to hold working capital in order to pay factors of production before receiving revenues. Investigating how a cost channel affects the credibility of forward guidance is based on three reasons. First, one drawback of the canonical New Keynesian model is that in its simplest form, i.e. without financial frictions, there is no tradeoff between the stabilization of output and inflation following any sort of shocks on the demand side. That is, a central bank can always perfectly stabilize these shocks which is known as the 'divine coincidence'. Contrary to that, a cost channel introduces a supply-side effect as firms' marginal cost depend on the nominal interest rate because firms need to raise nominal loans to finance production.[3] As a result, a cost channel always creates a tradeoff between the stabilization of output and inflation, which is contrary to the standard textbook model, as in Clarida et al. (1999). From a technical perspective, this should have a substantial effect on the sustainability of forward guidance, as the central bank cannot perfectly stabilize shocks on the demand side. Since equilibrium outcomes away from the ZLB will depend on the equilibrium at the ZLB, the supply-side effect of the cost channel must also have an effect when the economy is stuck at the ZLB. As the central bank aims to minimize both output and inflation variance in each period, the cost channel also introduces a direct effect on

[2] Given the assumption that transitory shocks to aggregate supply and aggregate demand have only diminishing effects, Laubach and Williams (2016) define the natural rate as the real short-term interest rate that is consistent with the economy working under full potential.

[3] The interest also plays a role if the production is not financed by a loan but financed internally, e.g. from retained earnings. In this case, the interest rate is equal to the opportunity cost of internal financing.

the specific targeting rule, that is the optimal relationship between output and inflation that must hold when the central bank adjusts its interest rate. Second, the cost channel as proposed by Ravenna and Walsh (2006) is easy to implement and nests the textbook model. In other words, this means that the cost channel technically consists of one additional term on the supply side. Therefore, it is easy to manually 'switch off' the cost channel and compare the outcomes across models. This is even more interesting from the standpoint that at the ZLB as well as during promises of forward guidance upon the lift-off date, any equilibrium condition in the standard New Keynesian model as well as under the cost channel will perfectly coincide which makes it easy for us to trace back the difference in results among both models. Third, the cost channel is empirically well documented for the US (see, for instance, Ravenna and Walsh, 2006; Chowdhury et al., 2006 and Tillmann, 2008). Ignoring the presence of the cost channel can therefore lead to misleading results. We tackle this possibility and, in a poor man's approach, investigate how costly ignoring the cost channel is in terms of steady-state consumption. Our results indicate that ignoring the cost channel can snap off up to one fourth of the improvement of losses at the ZLB.

We proceed as follows. Because the short-term interest rate is already determined at the ZLB and during the forward guidance episode, we first analyze how the cost channel affects the model dynamics when the economy is away from the ZLB. We then subsequently introduce forward guidance in the one-period case and extend it to the more general multiperiod case later on. We further examine the role of the 'strength' of the cost channel. This can be motivated by the assumption that firms only have to rely on financial intermediaries in part when financing factors of production. By the same token, one could argue that the pass-through from the central bank's policy rate to the actual lending rate is incomplete such that firms have to pay a markup on the policy rate. Finally, we compare our results in the model with reoccuring ZLB episodes with the standard framework of Eggertsson and Woodford (2003).

Overall, we find that the cost channel plays an important role for forward guidance and its credibility. Our results are threefold. First, we find that past promises might be honored, which is, however, only the case when fulfilling past promises is the central banks' best strategy at any point until the promised lift-off date. Across both models, we observe that the reason for forward guidance to be sustainable is that the promise to keep the interest rate at zero once the economy leaves the ZLB improves outcomes at the ZLB by raising expectations of inflation and the output gap after exiting the ZLB. Nevertheless, in our calibrated model, we find that the supply-side effect makes forward guidance less credible compared to the textbook model.

In other words, our results imply that under the cost channel, the horizon for forward guidance, which results in an improvement of outcomes at the ZLB, is shorter than in the textbook model. Second, as the central bank away from the ZLB cannot perfectly stabilize real rate shocks and since these shocks create a tradeoff between the stabilization of output and inflation, we find that for the optimal promise to keep the interest rate at zero, the improvement at the ZLB is larger under the textbook model. Finally, we find that these effects depend on the strength of the cost channel. We, therefore, show how costly ignoring the cost channel is. We find evidence that if the cost channel is present but ignored by the central bank and if the central bank behaves optimally under the perceived model it believes to be the true one, this can result in foregone improvements at the ZLB.

The remainder of this paper is structured as follows. In section 2, we introduce the model and show how to solve it with numerical techniques. In section 3, we investigate how the cost channel affects equilibrium outcomes away from the ZLB in the absence of forward guidance. The latter is introduced in section 4, where we first present the results for one-period forward guidance and subsequently extend the model to the multiperiod case. A battery of robustness checks and further results are presented in section 5. Section 6 concludes.

2 Methodology

A. *The model*

Consider a simple New Keynesian model that features the cost channel, as proposed in Ravenna and Walsh (2006). The private sector can be summarized by

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t \right) \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[\left(\sigma + \eta \right) x_t + \delta i_t \right], \tag{2}$$

where x_t is the output gap in period t, i_t is the short-term nominal interest rate, π_t is the current inflation rate, and r_t is an exogenous stochastic process. Contrary to the standard New Keynesian model, the Phillips curve now also includes a supply-side effect, δi_t, such that the short-term nominal interest rate appears both on the demand and the supply side. This effect relies the assumption that firms' marginal costs depend on the nominal interest rate because firms need to raise nominal debt to finance production. Importantly, the Phillips curve as in (2) nests the conventional Phillips curve for $\delta = 0$.

An appropriate specification of monetary policy closes the model. Lastly, we assume that the nominal interest rate must not be smaller than zero. That is, it must hold that[4]

$$i_t \geqslant 0. \tag{3}$$

The shock r_t plays an important role in this framework. We assume a two-state (states n and z) Markov chain process, so r_t can have two different values in each state, namely $r_z < 0$ or $\rho > 0$. Define $\mathbf{P}$ as the matrix of transition probabilities which is equal to

$$\mathbf{P} \equiv \begin{bmatrix} Pr(n \mid n) & Pr(n \mid z) \\ Pr(z \mid n) & Pr(z \mid z) \end{bmatrix} = \begin{bmatrix} s & 1-q \\ 1-s & q \end{bmatrix}.$$

In particular, we assume that in state z (ZLB is binding) when $r_t = r_z$, then $r_{t+1} = r_z$ with probability q and $r_{t+1} = \rho$ with $1-q$. We can think of q as the probability of staying at the ZLB. Accordingly for state n , when $r_t = \rho$ (ZLB not binding), then $r_{t+1} = \rho$ with probability s and $r_{t+1} = r_z$ with $1-s$. In other words, once we exit the ZLB, we assume that a non-negative probability $1-s$ exists that periods can occur where the ZLB becomes binding (i.e. a reversion to the ZLB). This is the crucial difference to the Eggertsson and Woodford (2003) framework, where state z is a one-off event, i.e. where the probability of staying away from the ZLB is equal to $s = 1$.

Monetary policy is assumed to minimize the microfounded loss function

$$L_t = \frac{1}{2}\sum_{k=0}^{\infty} \beta^k \left(\pi_{t+k}^2 + \lambda x_{t+k}^2\right), \tag{4}$$

by means of discretionary policy, where λ is the relative weight the central bank places on the output gap.[5] Hence, we assume that the central bank re-optimizes the tradeoff between the stabilization of output and inflation and there is no mechanism that allows the central bank to commit to future actions. The central bank minimizes (4) subject to (1)-(3) and takes households' expectations as given. The resulting targeting rule reads

$$\lambda x_t + \kappa \left[\sigma(1-\delta) + \eta\right] \pi_t = 0. \tag{5}$$

[4]Another possibility would be to constrain the short-term nominal interest rate not to be smaller than the so-called 'effective lower bound' (ELB), i.e., the rate below which it becomes profitable to exchange reserves for cash. This is based on the observation that in many industrialized countries, nominal interest rates have already been below the zero lower bound because of storage costs. Since we abstract from storage costs entirely within our models, one can think of the ELB to be equal to zero for simplicity.

[5]Note that this loss function is the appropriate microfounded version for both, the textbook model as well as under the cost channel. Ravenna and Walsh (2006) show that the policymaker's loss function reads $L_t = \frac{1}{2}\sum_{k=0}^{\infty} \beta^k \left(\pi_{t+k}^2 + \lambda \left[x_{t+k} - (\sigma+\eta)^{-1}\hat{\gamma}_{t+k}\right]^2\right)$. However, they assume that government purchases are proportional to output and are equal to $G_t = (1-\gamma_t)Y_t$, where γ_t is stochastic and normally distributed. Since we abstract from modeling the government, it follows that $\hat{\gamma}_t = 0$ such that the corresponding loss functions both, under the cost channel and in the textbook model coincide.

That is, once the shock r_t occurs in state n, the central bank sets the nominal interest rate in a way that (5) always holds.

B. *Solving the model*

In the absence of forward guidance, we need to solve the model for two different states, namely for state n when the zero lower bound is not binding and for state z when the zero lower bound is binding. Recall that in state n, there is a non-negative probability $1 - s$ that the zero lower bound will bind in the subsequent period. Accordingly, we remain away from the zero lower bound with probability s. Rational agents are able to solve the model and form expectations accordingly. Hence, under discretion, expected outcomes for the output gap and inflation in state n are weighted averages of the equilibrium values in state n and z, respectively, and read

$$E_t(x_{t+1}|n) = sx_n + (1-s)x_z$$
$$E_t(\pi_{t+1}|n) = s\pi_n + (1-s)\pi_z.$$

Under optimal discretionary policy, the central bank implements the targeting rule (5) such that the resulting equilibrium condition in state n reads

$$\lambda x_n^d + \kappa\left[\sigma(1-\delta)+\eta\right]\pi_n^d = 0, \tag{6}$$

where the superscript d denotes outcomes under discretionary policy. The resulting equilibrium conditions for state n are obtained by using the expectations for output and inflation given we are in state n and read

$$x_n^d = \left[sx_n^d + (1-s)x_z^d\right] - \sigma^{-1}\left(i_n^d - \left[s\pi_n^d + (1-s)\pi_z^d\right] - \rho\right) \tag{7}$$
$$\pi_n^d = \beta\left[s\pi_n + (1-s)\pi_z\right] + \kappa\left[(\sigma+\eta)\,x_n^d + \delta i_n^d\right]. \tag{8}$$

We can apply the same procedure when the economy is stuck at the zero lower bound. In this case, the nominal interest rate is already determined and equal to zero. The model-consistent expected values of the output gap and inflation, given we are in state z, read

$$E_t(x_{t+1}|z) = qx_z^d + (1-q)x_n^d$$
$$E_t(\pi_{t+1}|z) = q\pi_z^d + (1-q)\pi_n^d.$$

As a result, the equilibrium conditions as implied by the supply and demand side are given by

$$x_z^d = \left[q x_z^d + (1-q) x_n^d\right] + \sigma^{-1}\left(\left[q \pi_z^d + (1-q)\pi_n^d\right] + r_z\right) \tag{9}$$

$$\pi_z^d = \beta\left[q\pi_z^d + (1-q)\pi_n^d\right] + \kappa\left(\sigma + \eta\right) x_z^d. \tag{10}$$

We summarize the five unknowns in a vector $\mathbf{y} = [x_n^d\ \pi_n^d\ i_n^d\ x_z^d\ \pi_z^d]'$ and solve the model

$$\mathbf{Ay} = \mathbf{c},$$

where $\mathbf{A}$ contains the reduced-form coefficients and $\mathbf{c}$ is an appropriate vector that contains the corresponding entries of the stochastic process r_t, i.e. the corresponding entries of either ρ and r_z or zero otherwise.[6]

C. *Calibration*

The baseline calibration for the structural parameters and the shock is summarized in table (1).

Parameter	β	σ	η	κ	λ	r_z	ρ
Value	0.99	2	2	$0.02(\eta+\sigma)^{-1}$	0.03	-0.005	0.01

Table 1: *Baseline Calibration*

The values of the deep structural parameters for $\beta, \sigma, \eta, \lambda$ are commonly used in the literature (see Eggertsson and Woodford, 2003; McKay et al., 2016; and more recently, Walsh, 2018). While r_z is calibrated to model a shock with an annualized size of -2%, the value of β implies a long run real interest rate of $\rho = \beta^{-1} - 1 = 0.01$. It is worth noting that we need to multiply the value of κ in our model with $(\sigma + \eta)^{-1}$ to get a value of $\kappa = 0.02$ which corresponds to the slope of the Phillips curve as in the papers cited above. Throughout the paper, we compare our results from the textbook model, that is the New Keynesian model, as in Clarida et al. (1999), with the model that features the cost channel as in Ravenna and Walsh (2006). For the former case, this amounts to setting $\delta = 0$, such that the model collapses to the textbook model. However, when the cost channel is considered, we use $\delta = 1$.[7]

[6] In the appendix it is shown how to numerically solve the model. Because the system is purely forward-looking, we use this procedure throughout the paper. Note that many papers (see, for instance, Adam and Billi, 2006; Adam and Billi, 2007 and Nakata, 2018) model both a zero lower bound on nominal interest rates as well as a normal distribution for stochastic disturbances, which in combination renders the model highly non-linear. This is in part circumvented throughout this paper because we assume that the natural rate of real interest follows a two-state Markov process and we rather assign different states to equilibrium outcomes at and away from the ZLB. This simplifies our solution method.

[7] This is the empirically estimated value for δ for the US. In order to test the sensitivity of our results with respect to this parameter, we will also try different values for δ.

Finally, we need to calibrate benchmark values for s and q. We follow Walsh (2018) and Nakata (2018) and confront the transition probabilities for states n and z with historical data for the effective federal funds rate. We extend the model to the most recent data available which amounts to a sample running from 1960:I–2019:IV. For this sample, the federal funds rate has been below 25bp for 11.81% of the time, while for 88.19% it was above or equal to 25bp. Thanks to the nature of Markov Chains, we can derive the steady-state behavior of our chain. For the limiting case $\lim_{k\to\infty} \mathbf{P}^k$, it must hold that

$$\Psi = \Psi \mathbf{P},$$

where Ψ contains the long-run fractions of being in either state n or state z, respectively. After some algebra, one can show that

$$\Psi(n) = \frac{1-q}{2-s-q} \overset{!}{=} 0.8819, \qquad \Psi(z) = \frac{1-s}{2-s-q} \overset{!}{=} 0.1181.$$

Following Walsh (2018) and Eggertsson and Woodford (2003) we choose values for q that are close to 0.9 and satisfy both conditions above, in particular $q = 0.875$ and $q = 0.85$. That is, once the economy is stuck at the ZLB, there is a 87.5% (85%) chance of staying at the ZLB the next period. This in turn translates into a value of $s = 0.98326$ ($s = 0.97991$). Recall that in state n, when the zero lower bound is not binding, the nominal interest rate needs to satisfy the non-negativity constraint, i.e. it must hold that $i_n \geqslant 0$. Hence, we need to ensure that our benchmark calibration does not imply an interest-rate which does not satisfy conditions (3).[8]

Figure (1) plots the equilibrium values i_n (multiplied by 100) for different combinations of s and q. More precisely, the upper panel plots i_n in the absence of the cost channel, i.e., when $\delta = 0$, whereas in the bottom panel, the cost channel is included with $\delta = 1$. Although we will discuss the effects of the cost channel later on in detail, it stands out that under our baseline calibration of $s = 0.98326$ and $q = 0.875$, the interest rate in both models is positive (0.794 without and 0.7598 with the cost channel) and, thus, does not violate condition (3). Assuming a lower probability of staying at the ZLB instead, with q being equal to 0.85, the interest rate is also positive in both models. Figure (1) also presents the schedule for i_n when $q = 0.89$. This is only presented because this value for the probability of staying at the ZLB is very close to Eggertsson and Woodford (2003). Nevertheless, throughout the paper, we mainly present our results for $q = 0.875$ and $q = 0.85$.

[8] This is why we cannot use a value of $q = 0.9$, as a 90% probability of staying at the ZLB violates the non-negative constraint.

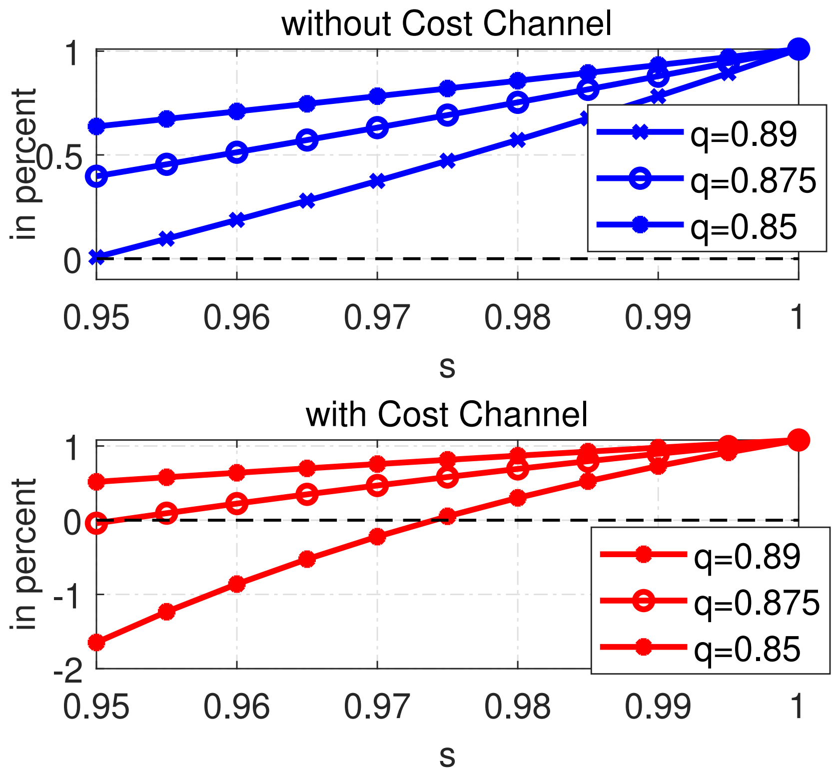

Figure 1: *Short-term interest rate i_n in equilibrium for different combinations of s and q. Values are multiplied by 100.*

To sum up, the baseline calibration ensures that the non-negative constraint is satisfied for both models. However, we might be cautious when trying different combinations of s and q, as the equilibrium outcomes for i_n are very sensitive to s when the probability of staying at the ZLB is high.

3 Preliminary Results

In this section, we first examine how the cost channel affects the equilibrium values in both states n and z. We do so by comparing the equilibrium outcomes both with and without the cost channel. Abstracting from the cost channel amounts to setting $\delta = 0$, such that the model collapses to the textbook model. On the contrary, we set $\delta = 1$ as the baseline value for the interest rate pass-through.

A. *Where the Cost Channel can work*

Before discussing the effect of the cost channel on the equilibrium outcomes of the output gap, inflation and the short-term interest rate, this subsection aims to clarify where and, if so, how the cost channel can make a difference at the ZLB in general and in this framework in particular.[9]

[9] For an interesting overview of monetary policy at the ZLB in a cost channel economy, see (Chattopadhyay and Ghosh, 2016).

From an economic perspective, a cost channel is present when firms' marginal costs depend on the nominal rate of interest. This can be motivated under the assumption that firms need to hold working capital in order to paying factors of production before receiving their revenues. Technically, the cost channel translates into an additional term δi_t on the supply side of the model. That is, contrary to the textbook model, where the short-term interest rate solely appears on the demand side, under the cost channel it also has a direct effect on the supply side. Since the central bank aims to minimize both output and inflation variance in each period, the cost channel also introduces a direct effect on the specific targeting rule, that is the optimal relationship between output and inflation that must hold when the central bank adjusts its interest rate. Note that the targeting rule reads

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right) \frac{1}{[\sigma(1-\delta)+\eta]} x_t.$$

For $\delta = 0$, the targeting rule collapses to the textbook case $\pi_t = -\lambda\,(\tilde{\kappa})^{-1}\, x_t$, where $\tilde{\kappa} \equiv \kappa\,(\sigma + \eta)$. However, under the cost channel, i.e. when $\delta > 0$, the optimal relationship between inflation and the output gap changes. More precisely, for a given increase in the output gap, the central bank adjusts the short-term interest rate such that the inflation rate is higher in absolute terms than it would be in the absence of the cost channel. From a technical point of view, this is because the 'strength' of the cost channel appears in the denominator of the targeting rule. From an economic point of view, since the short-term interest rate appears on both, the demand and the supply side, a positive (negative) demand shock cannot be perfectly stabilized by the central bank and leads to inflation *and* output gap fluctuations under optimal policy. This is contrary to the standard New Keynesian model of Clarida et al. (1999), where an optimal response of the central bank to demand shocks guarantees that neither inflation nor the output gap deviate from their steady-state equilibrium values.

As we have different states in our framework, we will also have equilibrium outcomes for each state. However, it will be easy for us to trace back the source of the diverging results from the models with and without the cost channel, respectively. In the absence of forward guidance, this is because the short-term interest rate is already determined at the ZLB, i.e. in state z. As a result, in state z the equilibrium conditions for the demand and supply side will coincide among both models. In state n however, the cost channel will play an important role because of the direct effect on the supply side as well as the changed relationship between the output gap and inflation in the targeting rule. Although we will discuss the theoretical framework for forward guidance in the next chapter, it is clear that under forward guidance, the short-term interest rate is kept at zero as well. In effect, any equilibrium condition in any state except state n in the model with and without the cost channel will perfectly coincide. To sum up, without forward guidance, i.e. if the central bank implements the targeting rule when the economy exits the ZLB as well as for any forward guidance horizon considered, any deviation in the results from the textbook model must stem from equilibrium outcomes in state n, i.e. when the ZLB constraint is not binding.

B. *Equilibrium outcomes away from the ZLB*

As discussed in the previous subsection, we would expect that without recurring episodes of the ZLB, equilibrium outcomes in state n are different from zero. This is because the cost channel introduces a supply-side effect. As a result, the central bank cannot perfectly stabilize the shock ρ in state n. Contrary to the textbook model without the cost channel, it follows that $\pi_n = x_n = 0$ under the cost channel is no more achievable. Moreover, we would expect that under the cost channel, the equilibrium outcomes in both states n and z are greater in absolute terms, because the equilibrium conditions under the cost channel are the same as under the textbook model and given that the equilibrium values in state z are higher in absolute terms, so should be equilibrium outcomes in state z. This is exactly what we see in Figure (2) and (3). Starting with the equilibrium outcomes for inflation

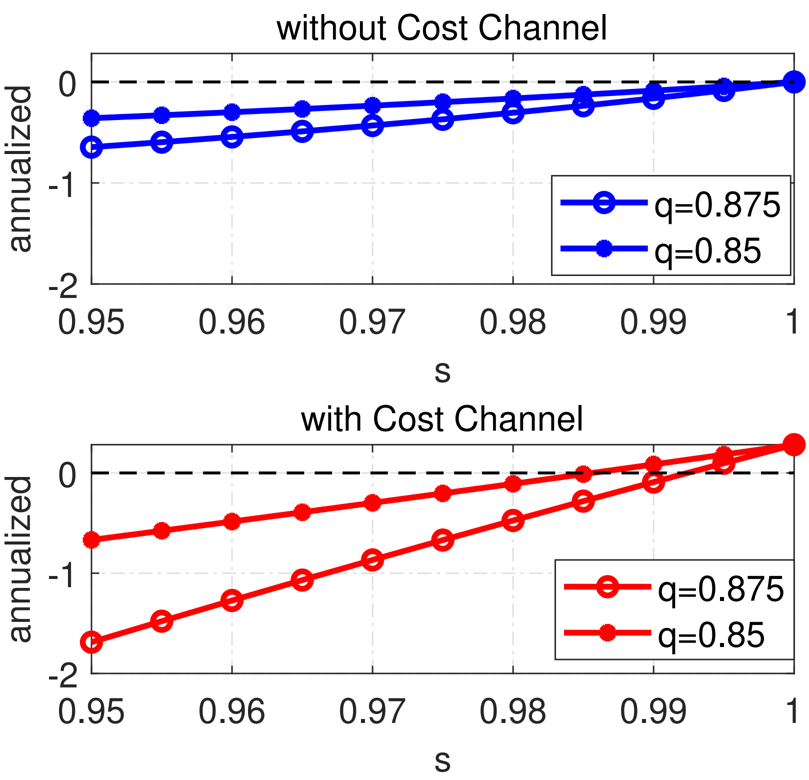

Figure 2: *Outcomes for π_n in equilibrium for different combinations of s and q. Values are annualized and multiplied by 100.*

away from the ZLB, Figure (2) shows that in equilibrium, inflation is always higher in absolute terms under the cost channel, as against the model without the cost channel. In both models, inflation monotonically decreases with the probability of reverting to the ZLB and with the probability of staying at the ZLB. For example, while for $s = 0.95$ and $q = 0.85$, inflation in state n is -0.359 in the textbook model, whereas the counterpart in the model with the cost channel is about -0.668, i.e., almost twice as large. By the same token, a calibration of $s = 0.95$ and $q = 0.875$ delivers $\pi_n = -0.645$ when there is no cost channel while we get a value of -1.688 when the cost channel is included, i.e., almost three times as large as in the model without the cost channel.

The difference between the equilibrium outcomes in the two models becomes apparent, the more likely the economy stays at the ZLB. Finally, note that Figure (2) shows for $s = 1$, inflation in state n under the cost channel delivers a value of 0.146, i.e. a value different from zero. As explained above, this stems from the supply-side effect which always induces a tradeoff between the stabilization of the output gap and inflation, even when ZLB episodes are one-off events. That is, even once the economy leaves the ZLB and never returns back, both zero inflation and an output gap equal to zero cannot be achieved. Figure (3) shows the same results for the output gap in

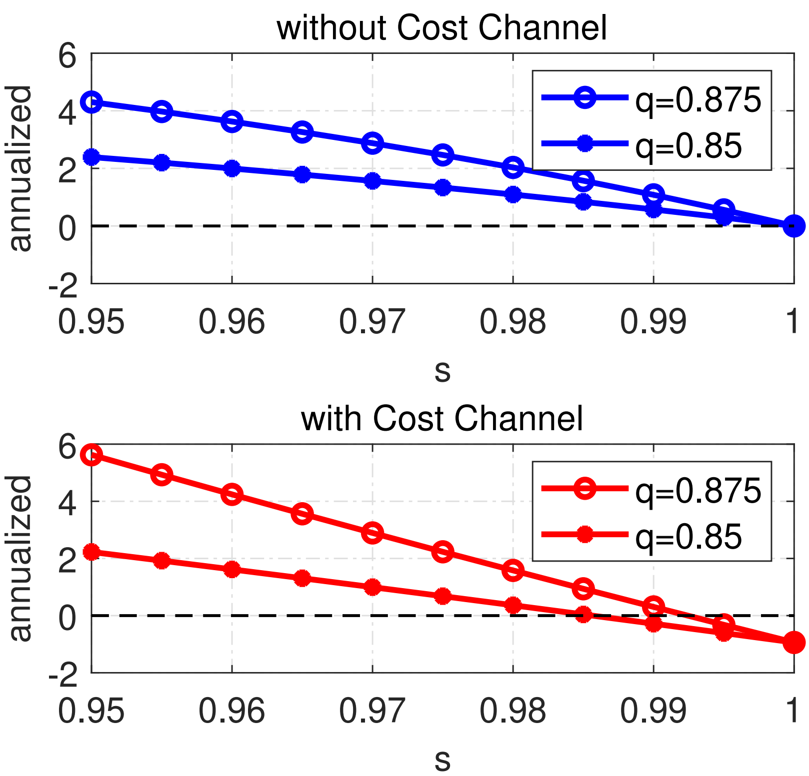

Figure 3: *Outcomes for x_n in equilibrium for different combinations of s and q. Values are annualized and multiplied by 100.*

state n. The results are qualitatively similar, saying that the output gap in equilibrium away from the ZLB increases (in absolute terms), the longer the duration of the ZLB episode and the more likely the economy reverts to the ZLB.[10]

C. *Welfare evaluation*

Throughout the paper, we follow Walsh (2018) and solve Bellman equations in order to account for the evolution of states. That is, we evaluate the equilibrium in terms of the present discounted value of losses. Let $l(\tau)$ be the current loss under state τ. Then, considering the whole sequence of events that will follow in the future, the Bellman equation in state τ reads

$$L_\tau = l_\tau + \beta\Big\{p(\tau|\tau) \times L(\tau) + p(\tau'|\tau) \times L(\tau')\Big\},$$

where $p(\tau|\tau)$ is the probability to remain in state τ, given we are in state τ and $p'(\tau'|\tau)$ is the probability of switching to state τ', accordingly. In our case of a two state Markov structure, we need one Bellman equation for each state. That is, our present discounted values L_n^d and L_z^d are given by

$$L_n^d = \frac{1}{2}\Big[(\pi_n^d)^2 + \lambda(x_n^d)^2\Big] + \beta s L_n^d + \beta(1-s)L_z^d \tag{11}$$

$$L_z^d = \frac{1}{2}\Big[(\pi_z^d)^2 + \lambda(x_z^d)^2\Big] + \beta q L_z^d + \beta(1-q)L_n^d, \tag{12}$$

where the expression in squared brackets is the current loss in either state.[11] Figure (4) shows the corresponding present value of future discounted losses in state n, i.e. when the economy is away from the ZLB. As we would expect, for both models, i.e. with and without the cost channel, losses increase with the probability q of staying at the ZLB. Moreover, for both models the losses also increase the more likely the economy reverts to the ZLB, although this effect is much stronger under the cost channel. For instance when $q = 0.85$ and $s = 0.97991$, losses in state n correspond to 0.119 percent of steady-state consumption without the cost channel and 0.122 percent when the cost channel is included, respectively. If we assume transition probabilities of $q = 0.875$ and $s = 0.98326$ instead, losses increase to 0.382 percent and 0.510 percent, respectively. This mirror images our results from before, as we found that the supply-side effect induced by the cost channel

---

[10] The corresponding outcomes for state z, i.e. the equilibrium values at the ZLB, can be found in the appendix.

[11] Common practice is to express the present value of losses in terms of steady-state consumption equivalence (see Billi, 2017, (Billi, 2011)). Therefore, we follow Billi (2017) and Walsh (2018) and express the present value of losses L_τ^d in state $\tau = n, z$ as the share of steady-state consumption $\mu_\tau = 100 \times (1-\beta)\left[\frac{\omega\theta(1+\eta\theta)}{(1-\omega)(1-\omega\beta)}\right] L_\tau^d$ where θ is the price elasticity of demand faced by individual firms. We follow Woodford (2003) and Walsh (2018) and set $\theta = 7.88$.

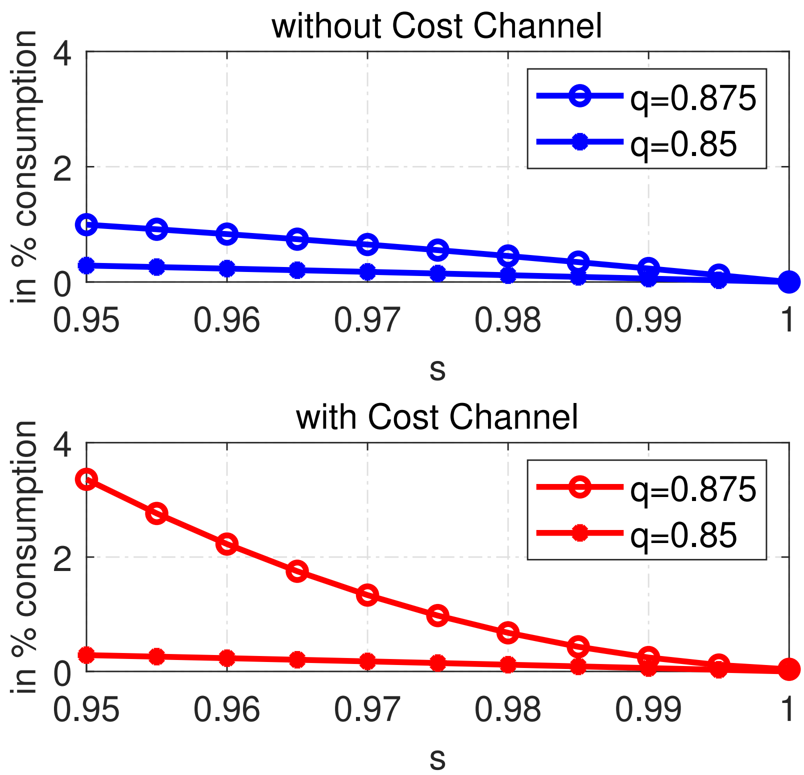

Figure 4: *Loss in state n for different combinations of s and q. Values are expressed in percent of steady-state consumption.*

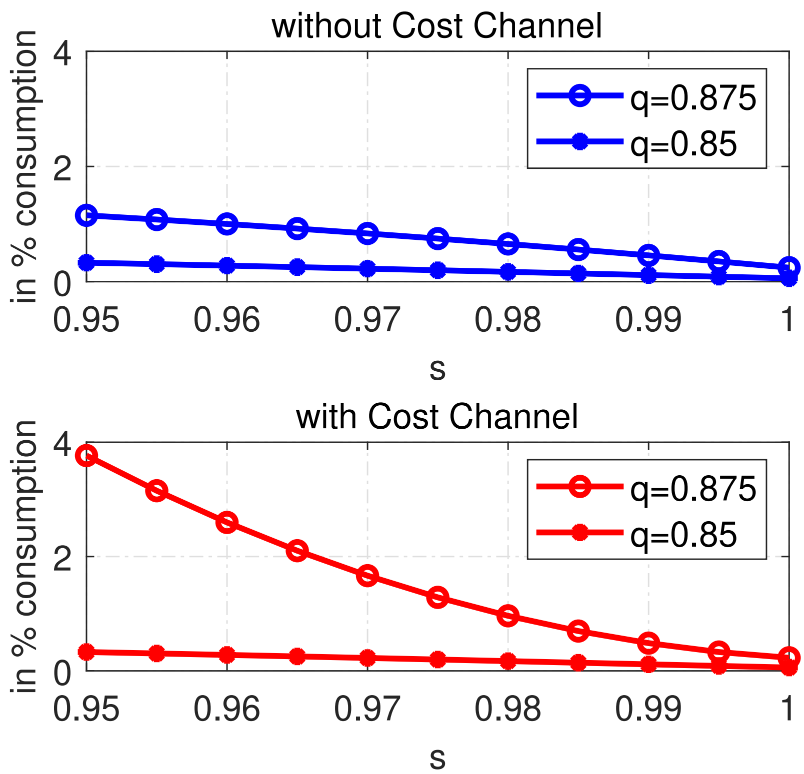

Figure 5: *Loss in state z for different combinations of s and q. Values are expressed in percent of steady-state consumption.*

introduces more volatility. Figure (5) shows the corresponding losses at the ZLB. We see that losses are higher under the cost channel in terms of steady-state consumption. Not surprisingly, the outcomes in state z are in general higher than in state n, although the effect is small. As the sustainability

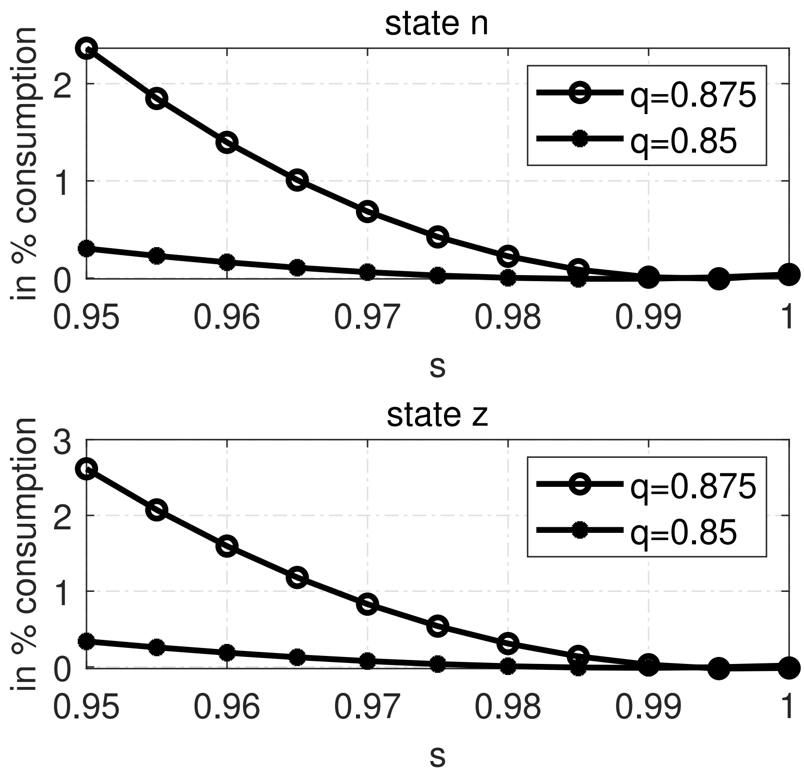

Figure 6: *Difference between loss in state n and z for different combinations of s and q, with the cost channel ($\delta = 1$) and without the cost channel ($\delta = 0$). Values are expressed in percent of steady-state consumption.*

of forward guidance crucially depends on the benchmark under discretion, Figure (6) plots the difference between the results with and without the cost channel, respectively. Both the upper and bottom panels show the difference between $L_n^d(\delta = 1) - L_n^d(\delta = 0)$ for the same values of s and q as in Figures (4) and (5). Most strikingly, the difference in both states is always positive and shows the same patterns regarding the sensitivity to s and q as before.[12] More specifically, the difference between losses at and away from the ZLB between the model with and without the cost channel very much depend on the transition probabilities. While for both $q = [0.85; 0.875]$ the difference between losses is very high, the more likely the economy reverts to the ZLB, this effect almost disappears the more s gets to one. To sum up, these results show that the supply-side effect introduced by the cost channel changes the dynamics both at and away from the ZLB.

[12]This result is qualitatively similar to (Pathberiya, 2016), who studies optimal monetary policy under the cost channel at the ZLB. However, the results therein rely on a different modeling of the shock as well as on perfect foresight.

4 Forward Guidance

Now that we have analyzed how the economy both with and without the cost channel works under optimal discretionary monetary policy, in this section we introduce forward guidance. Consider that the ZLB constraint no longer holds. Forward guidance amounts to the promise of the central bank to keep the nominal interest rate at zero for another k periods even when the ZLB is no longer binding in order to stimulate the economy. Having fulfilled its promise, after k periods, the central bank implements the optimal discretionary policy, given that the economy still has not returned to the ZLB.

For forward guidance to be sustainable, two conditions must be satisfied. First, losses at the ZLB under forward guidance must be less or equal to losses under optimal discretionary policy. Hence, it must hold that $L_z^{fg} \leqslant L_z^d$, where the superscript fg denotes losses under forward guidance and d denotes losses under optimal discretionary policy (i.e. $k = 0$), respectively. Second, given that the central bank keeps the nominal interest rate at zero when the ZLB is not a binding constraint, it must have no incentive to defect and implement the optimal policy instead. This incentive arises because when the ZLB is not a binding constraint, the central bank can implement $x = \pi = 0$ in the model without the cost channel and values close to zero under the cost channel. By keeping the nominal interest rate at zero instead, the central bank allows values that are potentially significantly different from zero. However, since equilibrium values in either state jointly depend on each other, it is still possible that under forward guidance, the present value of losses can be higher than under optimal discretionary policy. To sum up, it must hold that $L_n^d < L_e^{fg}$. When this condition is satisfied, the central bank has no temptation to defect from promises made in the past such that fulfilling past promises is the central bank's best strategy in the exit period.[13]

In the case of one-period forward guidance, we need one additional state (state e, the exit period) consisting of two additional equations such that the

[13]More generally, we adopt the definition of sustainability as in Walsh (2018) who labels a policy plan as sustainable if the present value of losses obtained by implementing the policy (i.e., forward guidance in this case) is, in every state, less than or equal to the present value of losses under discretionary policy.

joint set of equilibrium conditions reads

$$
\begin{aligned}
x_z^{fg} &= \left[q x_z^{fg} + (1-q) x_e^{fg}\right] + \sigma^{-1}\left(\left[q\pi_z^{fg} + (1-q)\pi_e^{fg}\right] + r_z\right) \\
\pi_z^{fg} &= \beta\left[q\pi_z^{fg} + (1-q)\pi_e^{fg}\right] + \kappa\,(\sigma+\eta)\, x_z^{fg} \\
x_e^{fg} &= \left[s x_n^{fg} + (1-s) x_z^{fg}\right] + \sigma^{-1}\left(\left[s\pi_n^{fg} + (1-s)\pi_z^{fg}\right] + \rho\right) \\
\pi_e^{fg} &= \beta\left[s\pi_n^{fg} + (1-s)\pi_z^{fg}\right] + \kappa(\sigma+\eta) x_e^{fg} \\
x_n^{fg} &= \left[s x_n^{fg} + (1-s) x_z^{fg}\right] - \sigma^{-1}\left(i_n^{fg} - \left[s\pi_n^{fg} + (1-s)\pi_z^{fg}\right] - \rho\right) \\
\pi_n^{fg} &= \beta\left[s\pi_n^{fg} + (1-s)\pi_z^{fg}\right] + \kappa\left[(\sigma+\eta) x_n^{fg} + \delta i_n^{fg}\right] \\
\lambda x_n^{fg} &+ \kappa\left[\sigma(1-\delta)+\eta\right]\pi_n^{fg} = 0.
\end{aligned}
$$

Notice that model-consistent expectations in state n now imply that households expect the central bank to keep the nominal interest rate at zero for one additional period, i.e. in the period the economy exits the ZLB. Hence, households place a positive weight for equilibrium values in state e which is the exit period in the case of one-period forward guidance ($k = 1$). By the same token, households realize that after keeping the nominal interest rate at zero for one period, the central bank will implement optimal discretionary policy thereafter.

The corresponding valuation functions read

$$
\begin{aligned}
L_e^{fg} &= \frac{1}{2}\left[(\pi_e^{fg})^2 + \lambda (x_e^{fg})^2\right] + \beta s L_n^{fg} + \beta(1-s) L_z^{fg} \\
L_n^{fg} &= \frac{1}{2}\left[(\pi_n^{fg})^2 + \lambda (x_n^{fg})^2\right] + \beta s L_n^{fg} + \beta(1-s) L_z^{fg} \\
L_z^{fg} &= \frac{1}{2}\left[(\pi_z^{fg})^2 + \lambda (x_z^{fg})^2\right] + \beta q L_z^{fg} + \beta(1-q) L_e^{fg},
\end{aligned}
$$

and can be jointly solved as in the case of pure discretionary policy.

A. *Results for one-period forward guidance*

In this subsection, we analyze whether forward guidance is credible based on the underlying conditions stated above, i.e. we separately look at (i) the gain of forward guidance and (ii) the temptation to renege on the commitment. Recall that the former case corresponds to the difference of losses at the ZLB, i.e. in state z when implementing the promised path of interest rates. The latter case corresponds to the difference of losses in the exit period which controls whether fulfilling past promises is the central bank's best strategy. Figure (7) plots the gain $G = L_z^d - L_z^{fg}$ for different combinations of s and q, both for the model without the cost channel (upper panel) and with the cost

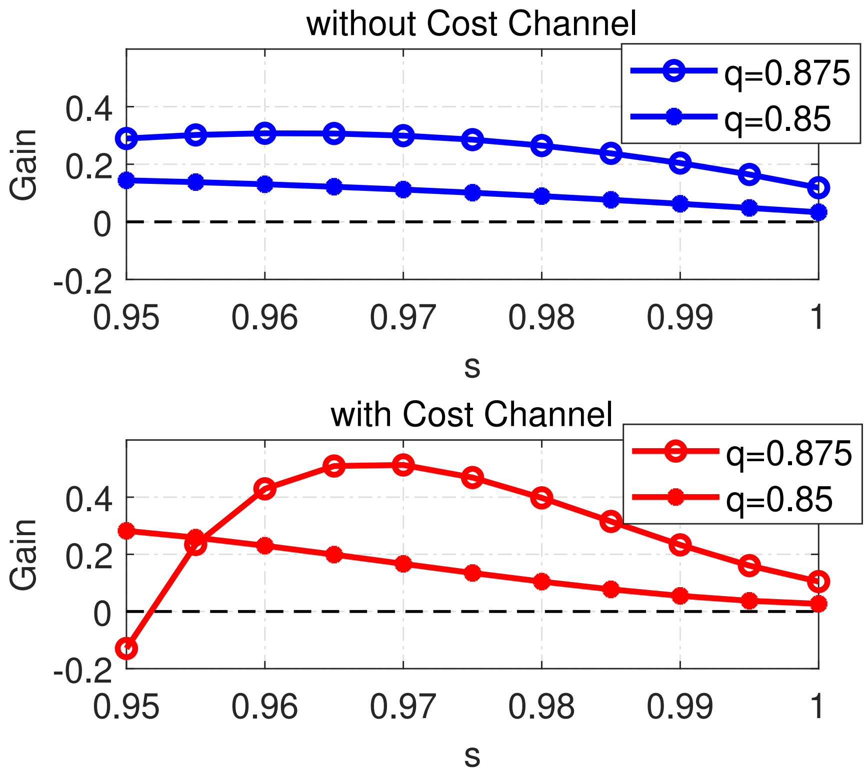

Figure 7: *Gain of one-period forward guidance as* $G = L_z^d - L_z^{fg}$ *for different combinations of s and q. Values are expressed in percent of steady-state consumption.*

channel (bottom panel).

Two things stand out. First, for $q = 0.85$, the gain of forward guidance is always positive. For our two baseline calibrations of $q = [0.85; 0.875]$ and $s = [0.97991; 0.98326]$, the gains are equal to $[0.090; 0.248]$ in the textbook model and $[0.105; 0.344]$ under the cost channel, respectively. Note that for $q = 0.85$, the gain of forward guidance increases, the more likely a reversion to the ZLB is. That is, in both models the gain monotonically increases in $1 - s$.[14] Moreover, under $q = 0.85$, the gain is always higher under the cost channel than in the model without the cost channel. Intuitively, this means that the improvement of outcomes at the ZLB is larger when the cost channel is included as opposed to the case when the cost channel is absent. Second, when we assume a higher probability of staying at the ZLB, forward guidance can be unsustainable under the cost channel if, at the same time, the probability of reverting to the ZLB is high. This can be seen as the gain in the bottom panel is negative for $q = 0.875$ and when s is close to 0.95. In this case, a promise to keep the nominal interest rate at zero after the economy has exited the ZLB would not be sustainable. However, this is not the case in the textbook model, where also under $q = 0.875$, the gain is always positive

[14] Note that Figures (7) and (8) mirror image the results typically found in the literature. Specifically, for $s = 1$, we find that the improvement at the ZLB delivers a positive gain for both values of q in both models. Nevertheless, the temptation is slightly positive. It follows that, in this case, the central bank is better off reneging on past promises and switch to the discretionary policy instead.

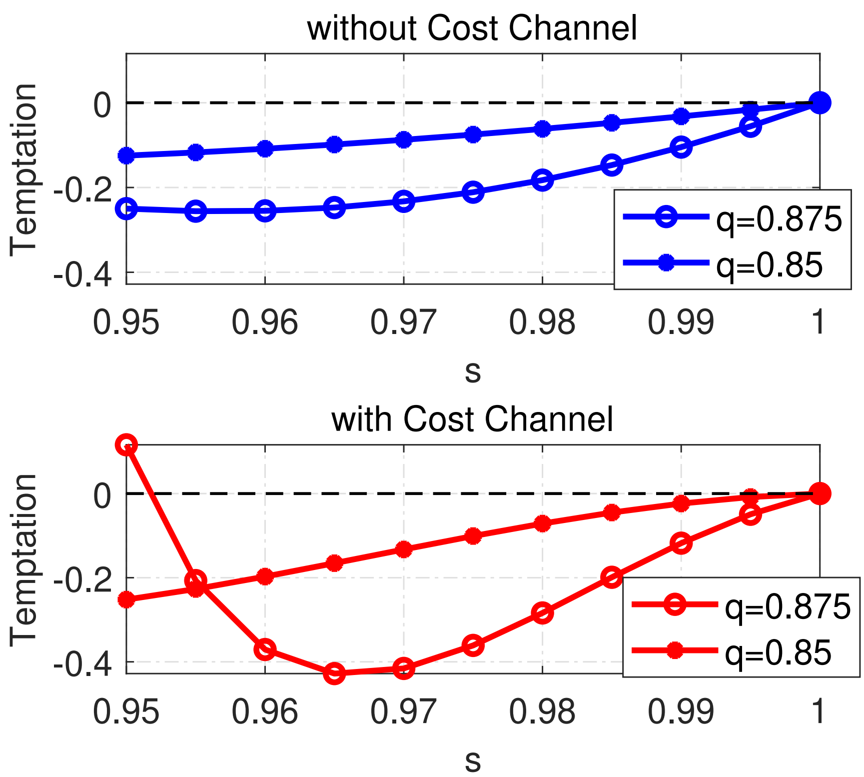

Figure 8: *Temptation to defect for one-period forward guidance as* $T = L_e^{fg} - L_n^d$ *for different combinations of s and q. Values are expressed in percent of steady-state consumption.*

for all values of s between 0.95 and 1 and looks qualitatively very much like in the case of $q = 0.85$.

Figure (8) plots the corresponding temptation to defect. Roughly speaking, the results look qualitatively similar to the case where one would flip the paths from (7) upside-down. As a result, we find that for one-period forward guidance, there is no temptation to defect from past promises when losses at the ZLB under forward guidance are improved over outcomes under optimal discretionary policy.

Table (2) reports equilibrium outcomes for the output gap, inflation, and the nominal interest rate in all three states (except for the nominal interest rate as it is already determined in states e and z) under both, optimal discretionary policy and forward guidance. While the upper part of the table reports outcomes for $q = 0.875$ and $s = 0.98326$, the bottom part of the table assumes $q = 0.85$ and the corresponding value of $s = 0.97991$.

Not surprisingly, in equilibrium, all outcomes for inflation and the output gap are higher in absolute value, the longer the expected duration of the ZLB, both under optimal discretionary policy and under forward guidance. This observation holds both for the textbook model as well as under the cost channel. For example, when $q = 0.85$, deflation at the ZLB is equal to -1.409 (annualized) with a corresponding output gap of -10.346. Away

$s = 0.98326$, $q = 0.875$

without Cost Channel							
k	x_n	x_e	x_z	π_n	π_e	π_z	i_n
0	1.735	1.735	-15.185	-0.260	-0.260	-2.511	0.794
1	1.320	3.008	-11.762	-0.198	-0.164	-1.911	0.844
with Cost Channel							
k	x_n	x_e	x_z	π_n	π_e	π_z	i_n
0	1.161	1.161	-17.080	-0.348	-0.348	-2.876	0.760
1	0.690	2.356	-13.336	-0.207	-0.190	-2.170	0.833

$s = 0.97991$, $q = 0.85$

without Cost Channel							
k	x_n	x_e	x_z	π_n	π_e	π_z	i_n
0	1.097	1.097	-9.477	-0.164	-0.164	-1.350	0.857
1	0.753	2.557	-6.773	-0.113	-0.077	-0.927	0.902
with Cost Channel							
k	x_n	x_e	x_z	π_n	π_e	π_z	i_n
0	0.368	0.368	-10.346	-0.110	-0.110	-1.409	0.868
1	-0.021	1.845	-7.382	0.006	0.025	-0.908	0.933

Table 2: *Results for one-period forward guidance. Outcomes for i are multiplied by 100, values for the π and x are annualized and multiplied by 100.*

from the ZLB, the equilibrium inflation rate is −0.110. The central bank adjusts the nominal interest rate in order to satisfy the targeting rule, which results in an output gap in state n of 0.368. When the expected duration of the ZLB episode is higher with $q = 0.875$, outcomes at the ZLB are almost twice as large with $\pi_z = -2.876$ and $x_z = -17.080$. Also, away from the ZLB, i.e. in state n, both the output gap and inflation are remarkably higher than before. However, when the central bank announces to keep the nominal interest rate at zero, once the economy has left the ZLB, much of the variance at and away from the ZLB can be reduced. This is where the gain of forward guidance comes from in both models, i.e. the improvement of outcomes of the inflation rate as well as the output gap at the ZLB. As we would expect, the variance of the output gap in the exit period is larger under forward guidance than under optimal discretionary policy. However, this effect is overcompensated through the improvement in inflation over discretionary policy because the output gap only has a relative weight of $\lambda = 0.003$ in the central banks' loss function. These results are qualitatively similar under the standard textbook model, although the relative differences in state n are higher under the cost channel.

Table (3) summarizes our findings and reports the corresponding losses in each states as well as the gain and temptation under the same calibrations

$s = 0.98326$, $q = 0.875$					
without Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.382	0.382	0.592	0.000	0.000
1	0.222	0.222	0.344	0.248	-0.160
with Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.510	0.510	0.783	0.000	0.000
1	0.281	0.281	0.439	0.344	-0.229

$s = 0.97991$, $q = 0.85$					
without Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.119	0.119	0.171	0.000	0.000
1	0.056	0.057	0.082	0.090	-0.062
with Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.122	0.122	0.181	0.000	0.000
1	0.050	0.050	0.075	0.105	-0.072

Table 3: *Results for one-period forward guidance. Losses are expressed in percent of steady-state consumption.*

as in table (2). As it can be seen, the reported values for both, the gain and temptation imply that one-period forward guidance is sustainable and credible across both models, as the gain for $k = 1$ is positive in both cases and the temptation to defect is negative.[15] Altogether, it stands out that the gain is higher under the cost channel because of the induced tradeoff between the stabilization of the output gap and inflation in state n. Overall, our results are comparable to Walsh (2018), who finds that even in a discretionary environment, forward guidance can be credible as long as the ZLB is occasionally binding. In the next section, we extend our analysis to the multiperiod case for $k > 1$.

B. *The multiperiod case*

Suppose the central bank promises to keep the nominal interest rate at zero for the general case of $k > 0$ periods after exiting the ZLB. Given that the economy has not reverted to the ZLB for the entire k periods, the central bank implements optimal discretionary policy in period $k + 1$ by means of the targeting rule (6). If the economy instead reverts to the ZLB after $\underline{k} < k$

[15] Notice that for all calibrations, we also ensured that the non-negative constraint on the short-term interest rate is satisfied.

periods, the process starts over.

calibration	s	q
A	0.98326	0.875
B	0.97991	0.85

Table 4: *Baseline values for s and q.*

In order to jointly solve the model with multiperiod promises, we need k additional states, that is one state for each period that belongs to the promise made by the central bank. In particular, for each additional state we need two more equations because we have two additional unknowns.[16] For $k > 0$ periods, we have two equations in state z, i.e. at the ZLB, three equations in state n, i.e. when the economy is staying away from the ZLB under optimal discretionary policy and $2 \times k$ for each period (i.e. state) under forward guidance.

In this section, we focus on the credibility and sustainability of forward guidance for values of $k = 0$ up to $k = 5$ under the baseline calibration for the structural parameters and the history-consistent calibrations for s and q as summarized in table (4). However, it is worth noting that the analysis requires to be cautious about the level of the short-term interest rate in state n. More precisely, for each k under consideration we have to ensure that the interest rate in state n does not violate the non-negative constraint (3).

Let us start with our results for $s = 0.98326$ and $q = 0.875$ (calibration A). Table (5) reports the present value of losses in states e, n and z as well as the gain and the temptation to defect from the promised forward guidance policy for $k = 0$ (i.e. optimal discretion) up to $k = 5$. In the absence of the cost channel, promises to keep the short-term interest rate at zero for up to four periods after exiting the ZLB deliver significantly better outcomes over discretion. As equilibrium outcomes in state n also depend on outcomes in state z, losses away from the ZLB are also being improved for promises to keeping the interest rate at zero for up to four periods. However, the present value of losses at the ZLB is minimized for $k = 3$ with a gain of 0.585. Also, note that such a promise is sustainable, as the central bank has no temptation to defect from past promises made when the economy was stuck at the ZLB.

Under the cost channel, the qualitative results are very similar. Losses are lower under forward guidance as against discretion for up to $k = 4$. The gain is maximized for $k = 3$. In this case, the present value of losses at the ZLB is 0.030 as compared to 0.783 under discretion, which results in a gain of 0.753. The corresponding value of -0.478 in the last column states that this promise is also credible.

[16]The interest rate is obviously not unknown under forward guidance and determined as zero.

A: $s = 0.98326$, $q = 0.875$

k	L_n	L_e	L_z	Gain	Temptation
without Cost Channel					
0	0.382	0.382	0.592	0.000	0.000
1	0.222	0.222	0.344	0.248	-0.160
2	0.082	0.083	0.128	0.464	-0.300
3	**0.004**	**0.005**	**0.007**	**0.585**	**-0.377**
4	0.072	0.076	0.112	0.480	-0.307
5	*0.469*	*0.480*	*0.726*	*-0.134*	*0.097*
with Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.510	0.510	0.783	0.000	0.000
1	0.281	0.281	0.439	0.344	-0.229
2	0.085	0.085	0.136	0.647	-0.424
3	**0.030**	**0.031**	**0.030**	**0.753**	**-0.478**
4	0.426	0.433	0.563	0.220	-0.077
5	*2.292*	*2.317*	*3.194*	*-2.411*	*1.807*

Table 5: *Results for k-period forward guidance. Losses are expressed in percent of steady-state consumption. Rows with values in bold font indicate that under this horizon k, the gain is maximized and forward guidance is sustainable. Rows with values in italics correspond to horizons where forward guidance is not sustainable, either because (1) the gain is negative, (2) temptation is positive or (3) both at the same time.*

What if the expected duration of ZLB episodes is shorter and reversions to the ZLB episodes are more likely? Table (6) reports the same results for $q = 0.85$ and $s = 0.97991$ (calibration B). Across both models losses at the ZLB are in general lower than in the former case, because the expected duration of ZLB episodes is lower. Without the cost channel, forward guidance still yields improved outcomes over discretion at the ZLB for up to four periods. Also, just as in the former case, losses at the ZLB are minimized for $k = 3$, which is also a credible promise as such a promise delivers no positive temptation to defect.

However, under the cost channel, the results are qualitatively different compared to the former case. We find that, when the expected duration of ZLB episodes is lower, forward guidance delivers improved outcomes only for up to $k = 3$ as opposed to $k = 4$ for the case without the cost channel. Also, losses at the ZLB are minimized for $k = 2$ as opposed to $k = 3$ when the cost channel is absent. Recall that the inclusion of the cost channel only makes a difference in state n, as at the ZLB and under forward guidance, the nominal interest rate is equal to zero. Hence, the difference must stem from the supply-side effect of the nominal interest rate in the Phillips Curve as

B: $s = 0.97991$, $q = 0.85$

without Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.119	0.119	0.171	0.000	0.000
1	0.056	0.057	0.082	0.090	-0.062
2	0.012	0.012	0.017	0.154	-0.107
3	**0.006**	**0.007**	**0.009**	**0.162**	**-0.111**
4	0.081	0.084	0.116	0.055	-0.034
5	*0.313*	*0.323*	*0.452*	*-0.281*	*0.205*
with Cost Channel					
k	L_n	L_e	L_z	Gain	Temptation
0	0.122	0.122	0.181	0.000	0.000
1	0.050	0.050	0.075	0.105	-0.072
2	**0.016**	**0.017**	**0.019**	**0.162**	**-0.105**
3	0.078	0.080	0.086	0.094	-0.042
4	0.363	0.369	0.447	-0.267	0.247
5	1.179	1.199	1.514	-1.334	1.077

Table 6: *Results for k-period forward guidance. Losses are expressed in percent of steady-state consumption. Rows with values in bold font indicate that under this horizon k, the gain is maximized and forward guidance is sustainable. Rows with values in italics correspond to horizons where forward guidance is not sustainable.*

well as the targeting rule. Overall, it stands out that while forward guidance is less credible under the cost channel, we find that at the same time, the improvement at the ZLB can be greater under the cost channel than in its absence.

Figure (9) summarizes our results and plots the gain of forward guidance based on the two calibrations of table (4) for $k = 0$ (discretionary policy) up to $k = 5$. Not surprisingly, across both models (i.e., with and without the cost channel), the gain is higher when the expected duration of the ZLB period is longer. This is because improved outcomes for both, inflation and the output gap at the ZLB, imply expected inflation to be closer to zero upon exiting than it is under optimal discretion (see also Walsh (2018) for this point). It stands out that in general, both models show a similar qualitative picture, saying that the gain shows an inverse U-shape, also turning from positive to negative for pretty much the same values of $k = 5$. However, with a reversion to the ZLB more likely (calibration B), the gain under the cost channel is also negative for $k = 4$ but positive in the textbook model.

Figure (10) shows the temptation to defect from past promises for the same calibrations as in the former case. In all cases, the temptation shows a hump-shaped pattern and, most importantly, implies that temptation is always negative when the gain is positive at the same time. Hence, under

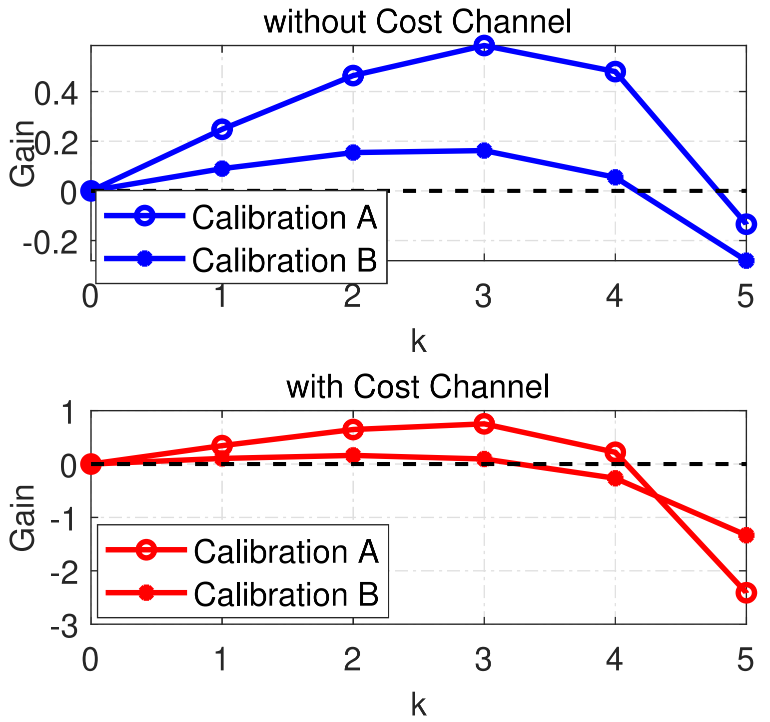

Figure 9: *Gain of forward guidance as* $G = L_z^d - L_z^{fg}$ *for different combinations of* s *and* q*. Values are expressed in percent of steady-state consumption.*

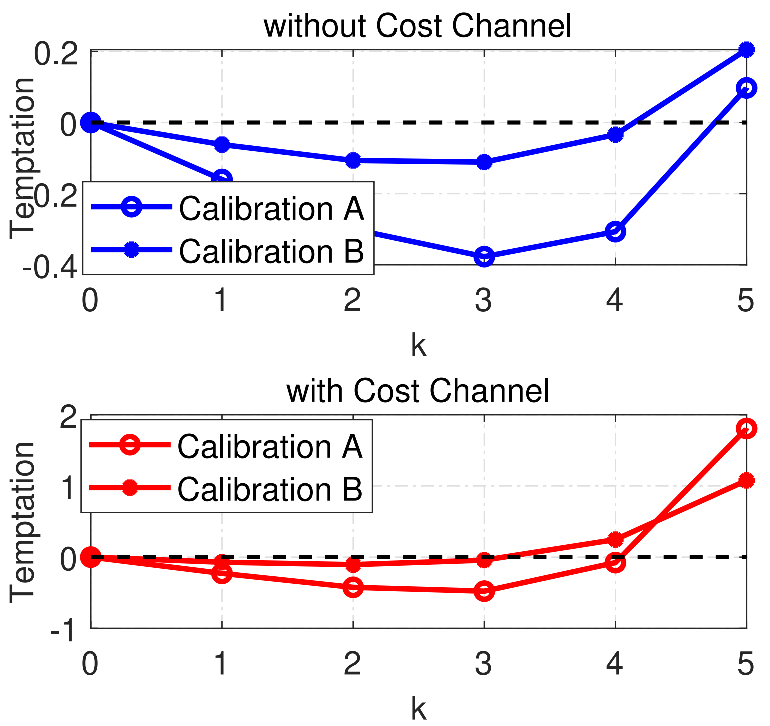

Figure 10: *Temptation of forward guidance as* $G = L_z^d - L_z^{fg}$ *for different combinations of* s *and* q*. Values are expressed in percent of steady-state consumption.*

the calibration considered, we do not have the case that the central bank has a positive temptation to defect while still being able to improve outcomes at the ZLB.

5 Sensitivity Analysis

A. *The effect of the real interest rate*

In this subsection, we examine how our results depend on the effect of the real interest rate. Our benchmark calibration assigns a value of $\sigma = 2$ to the interest rate elasticity of output. Even though this value is widely accepted in the literature, we check how our results change if we try a different value of σ, namely $\underline{\sigma} = 0.16$. This value is used for example in Woodford (2003), Adam and Billi (2006) as well as Adam and Billi (2007).[17] From an economic perspective, σ measures the intertemporal elasticity of substitution of aggregate spending. That is, lower values of σ imply that households respond stronger to changes in the real interest rate. Table (7) reports the results for the gain and temptation, both for our calibration of $\sigma = 2$ as well as the alternative value of $\underline{\sigma} = 0.16$. Results are reported for both calibrations for s and q as in the main text.

A few things stand out. First, under calibration A, we find that a promise to keep the interest rate at zero after the economy has left the ZLB improves outcomes at the zero lower bound over discretion for a horizon up to $k = 3$. Interestingly, however, a promise to keep the interest rate at zero for three quarters yields a positive temptation, both under the cost channel as well as in the textbook model. Hence, such a promise is not sustainable because the central bank has an incentive to renege on past promises and switch to discretionary policy instead. Second, the results for both the gain and temptation for all horizons are much closer across both models for $\underline{\sigma}$. Intuitively, this is because with a smaller value for σ, we increased the effect of the real interest rate on aggregate expenditure. That is, we technically shrink down the relative importance of the cost channel. For both models, the highest gain is achieved when $k = 2$. This promise is also sustainable.

Under calibration B, the striking qualitative difference compared to the main part of the paper here is that in the previous section, promises were sustainable for up to $k = 4$. With $\underline{\sigma}$ increasing the effect of the real interest rate, this is no longer the case. Now, forward guidance is sustainable for up to two periods after the economy has left the ZLB. However, contrary to the previous case, all of these promises are also sustainable, and the central

[17] In these papers (or book in the case of Woodford, 2003), the demand side is modeled as $x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t)$. A value of $\sigma = 0.16$ in our model therefore refers to $\sigma = 6.25$ in the papers cited above.

A: $s = 0.98326$, $q = 0.875$

	without Cost Channel			
	$\sigma = 2$		$\overline{\sigma} = 0.16$	
k	Gain	Temptation	Gain	Temptation
0	0.000	0.000	0.000	0.000
1	0.248	−0.160	0.358	−0.220
2	0.464	−0.300	**0.478**	**−0.250**
3	**0.585**	**−0.377**	*0.074*	*0.135*
4	0.480	−0.307	*−1.293*	*1.275*
5	*−0.134*	*0.097*	*−4.195*	*3.613*
	with Cost Channel			
	$\sigma = 2$		$\overline{\sigma} = 0.16$	
k	Gain	Temptation	Gain	Temptation
0	0.000	0.000	0.000	0.000
1	0.344	−0.229	0.380	−0.247
2	0.647	−0.424	**0.491**	**−0.280**
3	**0.753**	**−0.478**	*0.036*	*0.134*
4	0.220	−0.077	*−1.432*	*1.342*
5	*−2.411*	*1.807*	*−4.465*	*3.774*

B: $s = 0.97991$, $q = 0.85$

	without Cost Channel			
	$\sigma = 2$		$\underline{\sigma} = 0.16$	
k	Gain	Temptation	Gain	Temptation
0	0.000	0.000	0.000	0.000
1	0.090	−0.062	0.512	−0.350
2	0.154	−0.107	**0.635**	**−0.385**
3	**0.162**	**−0.111**	*−0.008*	*0.164*
4	0.055	−0.034	*−1.826*	*1.615*
5	*−0.281*	*0.205*	*−5.200*	*4.260*
	with Cost Channel			
	$\sigma = 2$		$\underline{\sigma} = 0.16$	
k	Gain	Temptation	Gain	Temptation
0	0.000	0.000	0.000	0.000
1	0.105	−0.072	0.546	−0.385
2	**0.162**	**−0.105**	**0.642**	**−0.423**
3	0.094	−0.042	*−0.061*	*0.158*
4	*−0.267*	*0.247*	*−1.965*	*1.665*
5	*−1.334*	*1.077*	*−5.403*	*4.352*

Table 7: *Results for k-period forward guidance. Losses are expressed in percent of steady-state consumption. Rows with values in bold font indicate that under this horizon k, the gain is maximized and forward guidance is sustainable. Rows with values in italics correspond to horizons where forward guidance is not sustainable.*

bank's best strategy is to fulfill its promises. Note that also under calibration, we find that the difference between the textbook model as well as the model featuring the cost channel became much smaller. Since, also under the benchmark calibration, the most significant improvement over discretion could be achieved for $k = 2$, this makes no qualitative difference under the cost channel.

Summing up, we find that our qualitative results do not change considerably. While the optimal forward guidance horizons are exactly the same under both models, results only change for the textbook model. Still, we find that forward guidance in both models can be sustainable as long as the forward guidance horizon is not too long. Most importantly, however, we find again that for the optimal horizon k, forward guidance is more powerful with the cost channel than without the cost channel.

B. *Results without recurring ZLB episodes*

Throughout the paper, we mainly focused on calibrated values for s and q that match the historical data for the US. In this section, we relax this assumption and try different combinations for s and q, i.e. calibrations that do not match with movements of the federal funds rate. More precisely, we now try the combinations $(s, q) = (1, 0.9)$ as well as $(s, q) = (0.9999, 0.9)$. For the former case, our model collapses to the model of Eggertsson and Woodford (2003), who abstract from reoccuring ZLB episodes and assume that the binding constraint on the nominal interest rate is a one-off event instead. The second calibration is chosen because Walsh (2018) shows that forward guidance in a discretionary framework can be sustainable when there is even a marginal probability of reoccuring ZLB episodes. This experiment is interesting for two reasons. First, while we would expect that for $s = 1$, forward guidance can never be sustainable in the textbook model, we do not know whether this is also the case under the cost channel. From an economic perspective, this is because under the cost channel, any demand shock always creates a tradeoff between the stabilization of the output gap and inflation and leads to fluctuations even under an optimal policy, because shocks on the demand side cannot be perfectly stabilized any more. Therefore, under the cost channel, $\pi_n = x_n = 0$ cannot be part of a feasible solution. Second, we found that forward guidance under the cost channel is at best as sustainable as under the textbook model. Therefore, we assess whether forward guidance can also be sustainable when the probability of reverting to the ZLB is only minuscule.

Table (8) reports the results for both models. For $s = 1$, we find exactly

without Cost Channel				
	$s = 0.9999$		$s = 1$	
k	Gain	Temptation	Gain	Temptation
0	0.000	0.000	*0.000*	*0.000*
1	2.010	-0.020	*2.003*	*0.000*
2	3.649	-0.035	*3.623*	*0.000*
3	4.742	-0.045	*4.709*	*0.001*
4	**5.060**	**-0.047**	*5.025*	*0.003*
5	4.327	-0.036	*4.305*	*0.006*
with Cost Channel				
	$s = 0.9999$		$s = 1$	
k	Gain	Temptation	Gain	Temptation
0	0.000	0.000	*0.000*	*0.000*
1	2.010	-0.018	*1.901*	*0.000*
2	3.440	-0.031	*3.396*	*0.000*
3	4.364	-0.038	*4.302*	*0.001*
4	**4.456**	**-0.036**	*3.382*	*0.008*
5	3.412	-0.020	*0.915*	*0.015*

Table 8: *Results for k-period forward guidance for different values of s. In all cases, $q = 0.9$. All values are expressed in percent of steady-state consumption. Rows with values in bold font indicate that under this horizon k, the gain is maximized and forward guidance is sustainable. Rows with values in italics correspond to horizons where forward guidance is not sustainable.*

what we would expect for the textbook model.[18] That is, although the gain is positive and there seems to be an opportunity to improve outcomes at the ZLB, such promises are never sustainable because the central bank would always renege on past promises and implement discretionary policy in the exit period instead. However, also under the cost channel, the gain of forward guidance is positive, even when future ZLB episodes never reoccur. Again, we find that the central bank is better off if it defects from past promises. As a result, forward guidance is not sustainable in the Eggertsson and Woodford (2003) framework. Intuitively, the reason is that in the exit period, either inflation or the output gap or both are closer to zero under discretionary policy than under forward guidance, although demand shocks cannot be perfectly stabilized.

When $s = 0.9999$, promises are credible for the entire horizon from $k = 1$ up to $k = 5$. In both models, the largest improvement over discretionary policy at the ZLB can be achieved for $k = 4$.

Summing up our results, we find that forward guidance can be sustainable under the cost channel, even if the probability of reoccurring ZLB

[18]In fact, these are one-by-one the results as in Walsh (2018).

episodes is very small (0.01%). However, we also find our usual qualitative result that forward guidance is less sustainable under the cost channel. This is because if we extend the horizon to $k = 7$, we find that forward guidance is sustainable for up to $k = 6$ in the textbook model and up to $k = 6$ under the cost channel.

C. *The strength of the cost channel*

In this subsection, we assess how our results behave when we change the strength of the cost channel, which is controlled by the coefficient of δ. Values of δ that are smaller than one indicate that changes in the central bank's policy rate do not translate into a one-to-one increase on the supply side. This can be motivated by the assumption that firms only partly rely on financial intermediaries in order to pay factors of production in advance. As a result, a value of $\delta > 1$ would imply that firms have to pay a markup when borrowing from financial intermediaries. This could be motivated by the assumption that the pass-through from the central bank's policy rate to the actual landing rate is not complete. A value of $\delta = 1$ implicitly assumes that no friction disciplines an incomplete pass-through from the central bank's policy rate to the lending rate.

Our calibration of $\delta = 1$ as our benchmark value is taken from Ravenna and Walsh (2006), who use a broad set of instruments and show that the coefficient of δ is not significantly different from one. However, the uncertainty around their estimate also covers values of δ that are far above and below one. Chowdhury et al. (2006) estimate a hybrid Phillips curve featuring a cost channel and find a value for $\delta = 1.3$ for the US. Hence, we work with two different values of δ, namely $\underline{\delta} = 0.5$ and $\overline{\delta} = 1.5$.

The results for the gain and temptation for values of $k = 0$ to $k = 5$ under our benchmark calibrations A and B are presented in table (9) for the set of $\boldsymbol{\delta} = \left[\underline{\delta}, \delta, \overline{\delta}\right]$. Overall, we find that the calibration of δ makes a difference. Broadly speaking, we find that a lower value of δ moves the results in the direction of the results under the textbook model. While for $\underline{\delta}$, the optimal horizon for k is equal to the benchmark value of $\delta = 1$, we find that under calibration B, a promise to keep the interest rate at zero for four periods after the ZLB episode is now sustainable, as there is an improvement of outcomes at the ZLB and because the central bank has no temptation to defect when it comes to fulfill its promise in the exit period. In this sense, the results are qualitative equal to the textbook model regarding the overall sustainability as well as the optimal horizon for k. However, for $\overline{\delta}$ we find a different picture. Now, the optimal horizon is one quarter lower compared to the results under the baseline calibration. While for calibration A, $k = 3$ periods

A: $s = 0.98326$, $q = 0.875$

	$\underline{\delta} = 0.5$		$\delta = 1$		$\bar{\delta} = 1.5$	
k	Gain	Temptation	Gain	Temptation	Gain	Temptation
0	0.000	0.000	0.000	0.000	0.000	0.000
1	0.277	−0.180	0.344	−0.229	0.718	−0.508
2	0.523	−0.340	0.647	−0.424	**1.147**	**−0.749**
3	**0.657**	**−0.423**	**0.753**	**−0.478**	*−2.063*	*1.972*
4	0.497	−0.309	0.220	−0.077	*−88.358*	*70.324*
5	*−0.400*	*0.299*	*−2.411*	*1.807*	*−369.642*	*286.471*

B: $s = 0.97991$, $q = 0.85$

	$\underline{\delta} = 0.5$		$\delta = 1$		$\bar{\delta} = 1.5$	
k	Gain	Temptation	Gain	Temptation	Gain	Temptation
0	0.000	0.000	0.000	0.000	0.000	0.000
1	0.096	−0.067	0.105	−0.072	**0.077**	**−0.033**
2	**0.163**	**−0.112**	**0.162**	**−0.105**	*−0.156*	*0.208*
3	0.161	−0.107	0.094	−0.042	*−1.443*	*1.357*
4	0.010	−0.007	*−0.267*	*0.247*	*−6.953*	*6.098*
5	*−0.453*	*0.349*	*−1.334*	*1.077*	*−39.043*	*33.256*

Table 9: *Results for different values of δ. All values are expressed in percent of steady-state consumption. Rows with values in bold font indicate that under this horizon k, the gain is maximized and forward guidance is sustainable. Rows with values in italics correspond to horizons where forward guidance is not sustainable.*

were optimal in the benchmark case, the largest improvement can now be achieved for $k = 2$. By the same token, the optimal horizon in calibration B changed from $k = 2$ to $k = 1$.

Another striking result is that under an incomplete interest rate pass-through, forward guidance is now far less sustainable than in the benchmark case as well as for a value of δ that is lower than one. We find that under calibration A, promises are only sustainable for promises up to $k = 2$, whereas, under calibration B, only one-period forward guidance can improve losses at the ZLB. Interestingly, for $k = 2$ an improvement could be achieved, but the central bank would defect from past promises and implement the time-consistent optimal policy plan instead.

Summing up our results, we find that the strength of the cost channel significantly changes our results, both quantitatively and qualitatively. A strong cost channel makes forward guidance less sustainable, whereas a weak cost channel pushes results in the direction of our benchmark results.

D. *Ignoring the cost channel*

So far, we compared the sustainability of forward guidance between a standard New Keynesian model and an otherwise standard model that includes a cost channel. While the assumption that firms need to pay factors in advance and rely on financial intermediaries to do so before receiving revenues is convincing per se, this is not a standard assumption in New Keynesian models nowadays. Most researchers typically ignore the cost channel and rely on a standard textbook model, or add other frictions instead. But what happens if, instead, a cost channel is present, but mistakenly ignored by the central bank? To tackle this question, we apply a poor man's approach and assume that the central bank optimizes (4) subject to (1), (3) and a Phillips curve that reads

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\sigma + \eta)x_t,$$

where the term δi_t is now missing. However, the *true* Phillips curve is still given by (2), i.e. the cost channel actually exists but is overseen or ignored by the central bank. As a result, the corresponding targeting rule in state n is now given by

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)\frac{1}{\sigma + \eta}x_t.$$

If a present cost channel is not taken into account by the central bank, forward guidance can be less sustainable, because the central bank does not respond optimally in state n anymore. Because the equilibrium outcomes in the other states also depend on state n, the equilibrium values in these states will also change. We will therefore focus on the missing benefit that would have been achieved when the true model had been considered. Thus, it is straightforward to report the difference of the gain that would have been achieved, given that the central bank had behaved optimally subject to the model it perceives to be the true one. To put it differently, we actually derive the foregone benefit (improvement) of lower losses at the ZLB that would have been achieved, had the central bank taken the correct model into account.

This is done in table (10) for our benchmark calibrations for s and q. Starting with $s = 0.98326$ and $q = 0.875$, we find that forward guidance can be sustainable even if the central bank responds to shocks based on a wrong perceived model. However, while the optimal horizon is still given by $k = 3$ which results in a gain of 0.564, the same promise would result in a gain of 0.753 in the true model. Consequently, the foregone benefit at the ZLB in terms of steady-state consumption is equal to 0.189 percent. Given that

A: $s = 0.98326$, $q = 0.875$

	true model		*wrong model*		
k	Gain	Temptation	Gain	Temptation	Δ Gain
0	0.000	0.000	0.000	0.000	0.000
1	0.344	−0.229	0.244	−0.152	0.100
2	0.647	−0.424	0.446	−0.283	0.201
3	**0.753**	**−0.478**	**0.564**	**−0.354**	**0.189**
4	0.220	−0.077	0.467	−0.282	−0.244
5	*-2.411*	*1.807*	*-0.110*	*0.109*	*-2.301*

B: $s = 0.97991$, $q = 0.85$

	true model		*wrong model*		
k	Gain	Temptation	Gain	Temptation	Δ Gain
0	0.000	0.000	0.000	0.000	0.000
1	0.105	−0.072	0.081	−0.055	0.024
2	**0.162**	**−0.105**	0.138	−0.091	0.024
3	0.094	−0.042	**0.140**	**−0.088**	−0.046
4	*-0.267*	*0.247*	0.028	−0.003	−0.295
5	*-1.334*	*1.077*	*-0.304*	*0.240*	−1.030

Table 10: *Results for different values of s. In all cases, $q = 0.9$. All values are expressed in percent of steady-state consumption. Rows with values in bold font indicate that under this horizon k, the gain is maximized and forward guidance is sustainable. Rows with values in italics correspond to horizons where forward guidance is not sustainable.*

a cost channel is indeed present, this amounts to 25.1% of the gain at the ZLB. That is, while there is still an improvement over discretion through forward guidance, this benefit could have been even higher had the central bank taken the true model into account. Interestingly, we find that for all values of k that imply a credible promise in the true model, the gain at the ZLB is always lower if the central bank considers the wrong model as the true one.

Under calibration B, we find a slightly different result. With reversion to the ZLB being more likely, the wrong model yields an optimal horizon of $k = 3$, as opposed to the true model where $k = 2$ is optimal. In this case, however, the relevant foregone benefit has to be calculated as Δ Gain=0.162−0.140=0.022, which is equal to 13.58% of the gain at the ZLB under the true model. However, overall, it stands out that given the central bank behaves optimally according to the model it believes to be the true one, we find that ignoring the cost channel might be costly in terms of aggregate steady-state consumption. This effect is, however, is much stronger, the higher the expected duration of the ZLB episode.[19]

6 Conclusion

A common finding in the literature is that the zero lower bound does not have serious consequences, given that a central bank can commit to future actions. If instead a discretionary policy environment is assumed, the zero lower bound has serious consequences. Any promise of the central bank to keep the interest rate at zero once the ZLB episode is neither credible nor sustainable in this case because the central bank will renege on promises made in the past. This result changes if the ZLB is occasionally binding, i.e., if future ZLB episodes are possible. In this paper, we compare the sustainability of forward guidance from a standard New Keynesian textbook model with the extension of a cost channel. Since under a cost channel, any shock on the demand side always creates a tradeoff between the stabilization of the output gap and inflation, we find that the supply-side effect introduced by the cost channel makes forward guidance, at best, as sustainable as in the absence of the cost channel. At the same time, however, we show that this tradeoff makes forward guidance more powerful under the optimal horizon. Ignoring the cost channel can be costly and results in foregone steady-state consumption. If firms only partially rely on financial intermediaries when paying factors of production in advance, the cost channel does not make a difference concerning the optimal horizon of forward guidance. If, instead, the interest rate pass-through from the central bank's policy rate to the lending rate is instead incomplete, forward guidance lacks credibility when the promised length of zero interest rates is too long.

Digging deeper, one could use a more sophisticated modeling of financial frictions, in which the size of the cost channel is pinned down to agency costs (see, De Fiore and Tristani, 2012). Another interesting topic for future research would be to endogenize the relation between forward guidance to the financial system. In the current model, there is no such feedback. If this assumption would be relaxed instead and forward guidance changes the willingness of banks to lend, this should feed back onto the supply side.

[19] We did the same experiment with values of $\delta > 1$, i.e., under the assumption of an imperfect interest rate pass-through from the central bank's policy rate to the lending rate. In this case, the effect found above is much higher, resulting in sizable foregone improvements at the ZLB.

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Appendix

A. *Solving the model.*

Throughout the paper, we solve the model numerically. In the absence of forward guidance, the system of equations reads

$$
\begin{aligned}
x_n^d &= \left[s x_n^d + (1-s) x_z^d\right] - \sigma^{-1}\left(i_n^d - \left[s\pi_n^d + (1-s)\pi_z^d\right] - \rho\right)\\
\pi_n^d &= \beta\left[s\pi_n + (1-s)\pi_z\right] + \kappa\left[(\sigma+\eta)\, x_n^d + \delta i_n^d\right]\\
x_z^d &= \left[q x_z^d + (1-q) x_n^d\right] + \sigma^{-1}\left(\left[q\pi_z^d + (1-q)\pi_n^d\right] + r_z\right)\\
\pi_z^d &= \beta\left[q\pi_z^d + (1-q)\pi_n^d\right] + \kappa\,(\sigma+\eta)\, x_z^d\\
&\lambda x_n^d + \kappa\left[\sigma(1-\delta)+\eta\right]\pi_n^d.
\end{aligned}
$$

Define $\mathbf{y} = [i_n,\ x_n,\ \pi_n,\ x_z,\ \pi_z]$ and $\mathbf{c} = [\rho\sigma^{-1},\ 0,\ r_z\sigma^{-1},\ 0,\ 0]$. The appropriate matrix (5×5) matrix with reduced coefficients then reads

$$
\mathbf{A} = \begin{bmatrix}
\sigma^{-1} & 1-s & -s\sigma^{-1} & -(1-s) & -(1-s)\sigma^{-1}\\
-\kappa\delta & -\kappa(\sigma+\eta) & 1-\beta s & 0 & -\beta(1-s)\\
0 & -(1-q) & -(1-q)\sigma^{-1} & 1-q & -q\sigma^{-1}\\
0 & 0 & -\beta(1-q) & -\kappa(\sigma+\eta) & 1-\beta q\\
0 & -\lambda & -\kappa\left[\eta+\sigma(1-\delta)\right] & 0 & 0
\end{bmatrix},
$$

such that the model in companion form reads

$$\mathbf{A}\mathbf{y} = \mathbf{c}.$$

The solution of the model can therefore be easily obtained as

$$\mathbf{y} = \mathbf{A}^{-1}\mathbf{c}.$$

Solving the model under forward guidance works exactly as above, except that we need two additional equations for each k, i.e. for each additional state.

B. *Equilibrium outcomes in state z without forward guidance.*

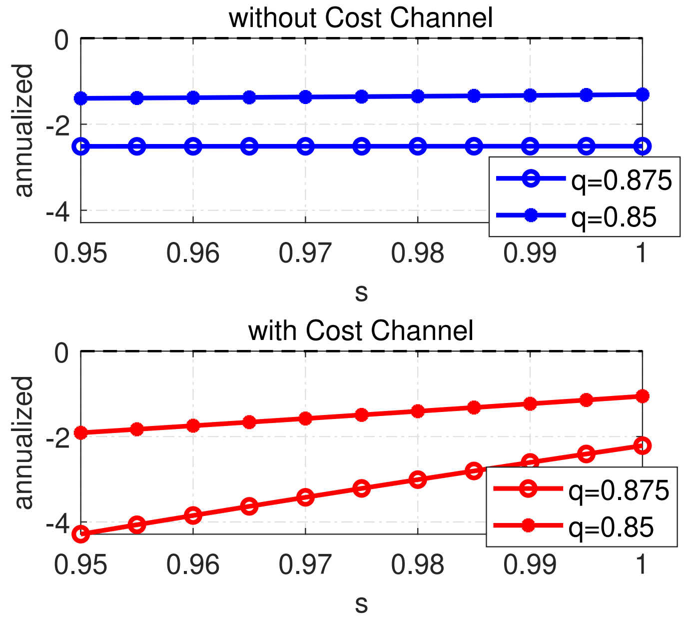

Figure 11: *Outcomes for π_z in equilibrium for different combinations of s and q.*

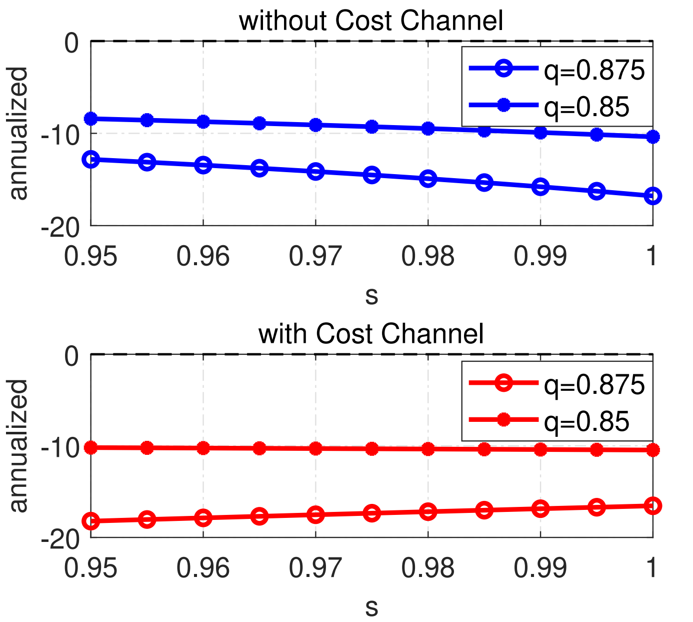

Figure 12: *Outcomes for x_z in equilibrium for different combinations of s and q.*