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Demystifying Sraffa's Theory of Value in the Light of Arrow and Debreu

DEMYSTIFYING SRAFFA'S THEORY OF VALUE IN THE LIGHT OF ARROW AND DEBREU

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Abstract
This paper compares the models of Arrow and Debreu [1954] and Sraffa [1960], and concludes that (1) the models are informationally distinct conceptions of a capitalist economy, (2) they support radically distinct – though complete and entirely correct – theories of value, (3) the prices in the two theories are different both in terms of definitions and values, (4) in Sraffa's model it is impossible to define constant returns to scale, while in Arrow-Debreu this property is admissible, and (5) in Arrow-Debreu the interpersonal income distribution is determined whereas in Srafa's model the distribution of income between workers and capitalists is undetermined. (100 words)

Keywords: constant returns to scale, theory of value, relations of production, counterfactual information, prices, exchange values, income distribution, general equilibrium, capital, marginal product

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1. Introduction

If two economists, both Nobel Laureates, make very specific but contradictory claims regarding a well-defined issue, it calls for careful scrutiny. In this paper I report on the result of an investigation of one such pair of conflicting claims. And more. Sen [2003] writes (p. 1253),

> Sraffa [1960] … draws exclusively on *observed* information, rather than having to invoke any *counterfactual* presumptions. …. It also relates to other methodological features of Sraffa's analysis, including his strenuous – but entirely correct – insistence that his analysis does *not* need any assumption of constant returns to scale.

On the other hand, jointly with Erkko Etula, one of the greatest economists of our time, Samuelson [2006] claims to provide multiple proofs[1] (p. 183),

> to confirm that Leontief – Sraffa matrix equations for input/output must obey constant returns to scale[.]

There is no question that there is a conflict between the position taken by Sen [2003] and the one taken by Samuelson and Etula [2006] on the role of *constant returns to scale* in Sraffa [1960].[2] This is not new. Samuelson [1962] has held this position for almost half a century. Decades later, Hahn [1982] expresses a similar doubt regarding constant returns to scale in Sraffa.[3] To put this matter to rest, in this paper I conduct an information theoretic examination of two theories of value, one developed by Sraffa and the other developed by Arrow and Debreu [1954] as in Debreu [1959].[4]

First, for clarification, in the next section I define the concept of constant returns to scale. I further argue that the information content is so sparse in Sraffa's model of a capitalist economy that in his model it is *impossible* to define the concept of constant returns to scale. This implies that Sen's claim is true, and Samuelson and Etula's is, in fact, unfounded, as is the doubt expressed in Hahn [1982]. In this paper I develop an argument to demonstrate (i) this impossibility and (ii) elucidate some of the implications of the informational differences between the Sraffa model and the Arrow-Debreu model for the two radically distinct economic theories of values of commodities.

Further, it would be a grave error to treat the arguments that I present here as an issue only in the history of economic thought, important as that is. In fact, this contribution has significant implications for two existing, distinct, *bone fide* economic theories of value. It is not that the death knell has sounded for one and the other has won a resounding victory. Both theories of

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[1] These "proofs" are examined and debunked in Sinha (2007). Also see the Appendix to Naqvi (2007) for two possible interpretations on which the proofs fail.

[2] Every reference to Sraffa in this paper is exclusively to Sraffa [1960].

[3] "Sraffa prices can be found once the rate of profit is known without any appeal to the preferences of households between goods. This of course is also true of a special neoclassical model which … posits constant returns to scale which Sraffa claims not to posit. I have been at a loss to understand him here." Hahn [1982, p. 359]

[4] Every reference to Arrow and Debreu in this paper is exclusively to the rendition in Arrow and Debreu [1954], although Debreu [1959] is a complete and comprehensive classic.

value are alive and kicking. This is precisely because they are based on entirely distinct information sets, so that the validity of one, *by itself*, does not invalidate the other.

Second, *assertions* have continually been made that Sraffa's model is *not* based on the assumption of constant returns to scale, by Sraffa himself, Schefold [1985, 1989, 1996], and more recently, among many others, by Sen [2003] and Sinha [2007]. However, in serious contributions from Samuelson [1962] to Hahn [1982] and Samuelson and Etula [2006], among others, claims to the contrary have also been made, that in Sraffa's model the constant returns to scale *restriction* is either *imposed* or *entailed*. To bring this matter to a final resolution, I do not merely make an assertion, but, in fact, construct an argument to provide an *explanation* as to why one assertion is true, and the other necessarily false. This is also a distinguishing feature of this contribution.

Very many academics in the economics profession are quite familiar with the Arrow-Debreu theory of value, as told by Debreu [1959]. By contrast, as students and teachers of economics, the understanding of Sraffa's theory of value is considerably less widespread. Is it that Sraffa's theory of value fails to qualify as a *bone fide* theory of value? No. On the contrary, one of the greatest economists of our times asserts that "[Sraffa's] pen writes as if a lawyer were at hand to ensure that no vulnerable sentence appears. I honor him for that[.]"[5] With this assertion as the backdrop, therefore, I conduct an information-theoretic comparison of the Arrow-Debreu and the Sraffa models, and find them to be entirely distinct conceptions of a capitalist economy, with each constituting the basis of a distinct, coherent and *bone fide* theory of value. That is how I attempt here to demystify Sraffa (for those who are mystified by him), in relation to the Arrow-Debreu theory of value.

In Section 2, I construct an argument to demonstrate the impossibility of defining constant returns to scale in Sraffa's model, and draw out *six* distinct, though interrelated, implications of the informational difference between this model and the Arrow-Debreu model. Section 3 contains a more detailed description of a particularly simple version of the Sraffa model of a capitalist economy that is drawn exclusively from Part I of his book, which deals with single-product industries without any durable goods.[6] Section 4 outlines the standard Arrow-Debreu model, again in a particularly simple form. Section 5 deals with some rather significant and controversial implications arising from the differences between the Sraffa and the Arrow-Debreu theories of value, in turn based on their distinct conceptions of a capitalist economy. Section 6 contains some concluding remarks.

2. A PRELIMINARY COMPARISON OF TWO THEORIES OF VALUE

Constant Returns to Scale (CRS) is a property of a production activity. An activity that transforms inputs of commodities and labor into outputs may or may not satisfy certain pre-specified requirements or axioms. An activity could also be thought of as occurring by distinct *processes* insofar as different *quantities* of the *same* inputs and outputs are involved in production.

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[5] Samuelson [2000, p. 134, fn. 7.]
[6] As an additional simplification, I assume that labor is employed directly in the production of every commodity.

Definition 1: A production *activity* is said to satisfy *Axiom CRS* if and only if, in any *pair-wise* comparison of distinct production processes of this activity, if *all* inputs of one process are *proportionate* positive multiples of the respective inputs of the other, then all outputs of the process will also be the *same* multiple of the other.

This definition is general enough to cover joint production in multi-product production correspondences, although such a feature is not entertained in this particular investigation.[7]

In what follows, I take a *production pattern* as a specific distribution of the quantities of all commodity *outputs* actually observed to have been produced in an economy. If this is taken together with the quantities of the various commodities and labor also actually observed as *inputs* in this 'pattern of production', then we have,

Definition 2 (Sraffa): The set of all *actually observed* activities of production of all commodities by means of commodities and labor per period of time is called the set of *relations of production.*

It is noteworthy that, as defined here, the relations of production are based solely on *observed information*. This plays a crucial role in the argument that follows.

Consider the following (actually observed) relations of production of two commodities by means of two commodities and labor. This example is contained in Robinson and Naqvi [1967, p.585] and, as they state, these relations or production represent an "image in miniature of an actual system" [of observed inputs and outputs]. The image "represents a system in which one unit of current labor is employed and the surplus consists of a single commodity."

$$(1) \quad \begin{array}{lllll} \dfrac{14}{9} \, \text{t. wheat} & \& \; \dfrac{20}{9} \, \text{t. iron} & \& \; \dfrac{4}{5} \, \text{labor} & \rightarrow & \dfrac{10}{3} \, \text{t. wheat} \\[2ex] \dfrac{7}{9} \, \text{t. wheat} & \& \; \dfrac{10}{9} \, \text{t. iron} & \& \; \dfrac{1}{5} \, \text{labor} & \rightarrow & \dfrac{10}{3} \, \text{t. iron} \end{array}$$

In particular, the input of iron in both industries equals its gross output. However, the gross output of wheat is one ton more than its aggregate input usage in both sectors. Also, labor input in both sectors taken together is one unit. In (1) '$\rightarrow$' refers to 'is associated with the production of'. Sraffa writes, "each commodity, which initially was distributed between the industries according to their needs, is found at the end of the year to be entirely concentrated in the hands of its producer."[8] And he seeks "a unique set of exchange-values which if adopted by the market restores the original distribution of products and *makes it possible for the process to be repeated; such values spring directly from the relations of production.*"[9]

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[7] What is here called pair-wise comparison of distinct *processes* of a production *activity* is sometimes referred to in some works as proportionate positive 'change' in all inputs (say, all inputs get doubled). The assumption of constant returns to scale entails that, if this is the case, the output 'changes' by the same proportion (gets doubled too).

[8] p. 3.

[9] p. 3, italics added.

However, for Sraffa, $\forall \lambda > 0$,

(2)
$$
\begin{array}{llllll}
\lambda\frac{14}{9}\text{ t. wheat} & \& & \lambda\frac{20}{9}\text{ t. iron} & \& & \lambda\frac{4}{5}\text{ labor} & \rightarrow & \text{? t. wheat} \\
\lambda\frac{7}{9}\text{ t. wheat} & \& & \lambda\frac{10}{9}\text{ t. iron} & \& & \lambda\frac{1}{5}\text{ labor} & \rightarrow & \text{? t. iron}
\end{array}
$$

The question mark (?) in (2) refers to 'Nobody knows for sure, so it is pure *counterfactual speculation*.' This is because Sraffa is willing to consider only observed information as in (1), and simply does not entertain any counterfactual information regarding what would happen to the output of a commodity if all its inputs *were to be*, counterfactually, higher (or lower) by the same proportion, as in (2). Never saw it, did not observe it, and Sraffa is unwilling to speculate as to what the outputs would have been in the case such as (2), in the *unobserved event* that all inputs were to have been higher (or lower) by the same proportion.[10]

Notice that the concept of constant returns to scale is constitutively counterfactual. And merely *to define* this property, information regarding at least *one additional* process of production of a commodity must be available, besides that contained in (1). Since Sraffa's model lacks the information pertaining to this additional process of production, it is impossible in his model to define constant returns to scale. Thus, it is not that the property of constant returns to scale is violated, nor that it is imposed, nor indeed that it is entailed in Sraffa's model. The fact is,

Proposition 1: *Constant returns to scale as a concept cannot be defined in Sraffa, because there is insufficient information in his model of a capitalist economy to define this property.*

In fact, in Sraffa's words,

> No changes in output and … no changes in the proportions in which different means of production are used by an industry are considered, so that no question arises as to the variation or constancy of returns. (p. v)

By contrast, Arrow and Debreu do, in fact, include counterfactual information in addition to observed information (1), so that in their model of a capitalist economy, an *entailment* of their assumptions regarding production is that *there exists* the possibility that, $\forall \lambda > 0$, the economy is described by (3)

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[10] In fact, Sraffa clearly states that "The investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the proportions of 'factors'." (p. v). Notice that in this paper I do not ask *why* Sraffa does what he does, and thus do not deal with any writings of his other than Sraffa [1960]. I therefore take his claims on face value. For a detailed examination that dwells extensively on Sraffa's unpublished writings, see Sinha [2007], among others.

$$\lambda \frac{14}{9} \text{ t. wheat} \quad \& \quad \lambda \frac{20}{9} \text{ t. iron} \quad \& \quad \lambda \frac{4}{5} \text{ labor} \quad \rightarrow \quad \lambda \frac{10}{3} \text{ t. wheat}$$

(3)

$$\lambda \frac{7}{9} \text{ t. wheat} \quad \& \quad \lambda \frac{10}{9} \text{ t. iron} \quad \& \quad \lambda \frac{1}{5} \text{ labor} \quad \rightarrow \quad \lambda \frac{10}{3} \text{ t. iron}$$

Here (3) defines an additional set of infinite *processes* of production activities that are assumed to be possible, though unobserved and thus counterfactual, for all real, positive and finite values of $\lambda \neq 1$ *also*, when compared with the *less*-information-based economy described by (1) and (2).

Clearly, in this particular regard, the information set on which the production part of the Arrow-Debreu model is based is *strictly greater* than the information on which Sraffa's model is based. About that, there should be no doubt. In particular, Sraffa's model assumes information contained in (3) *only* for $\lambda = 1$; by contrast the Arrow-Debreu model contains sufficient information to *admit the possibility* that (3) is true for (i) $\lambda = 1$ *and* for (ii) $\lambda \neq 1, \forall \lambda > 0$. This should settle the controversy regarding constant returns to scale in Sraffa's model.[11]

Once adequate note is taken of this *informational difference* between the two models – the Arrow-Debreu model being based on strictly greater information that the Sraffa model – then any controversy regarding whether constant returns to scale is imposed or entailed in Sraffa can be dismissed altogether. That is why Sen [2003, p. 1253] writes, "Sraffa [1960] … draws exclusively on *observed* information, rather than having to invoke any *counterfactual* presumptions." This is a simple enough point, but it needs to be made, and emphasized, if only because there is much too much confusion surrounding this issue in the literature. Further, there are several significant implications of this fundamental informational difference between the models on which the theories of value are based in Sraffa versus Arrow-Debreu.

First, based on very different information sets, the two theories of value are profoundly different: there are strictly positive *Sraffan prices* and there are *Arrow-Debreu prices*, and these two sets are completely distinct – both by definition and in their values.[12] Sraffa clearly states that the prices that are inherent in the relations of production are "a unique set of exchange-values *which if adopted by the market* restores the original distribution of products and makes it possible for the process to be repeated."[13] It is important to note that Sraffa does *not* say that these prices are

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[11] Notice that in the Arrow-Debreu model, the returns to scale are required to be non-increasing, which permits the possibility that they could be constant.

[12] Without using this terminology, Sen [2003] has attempted to bring this matter out by making a distinction between a *mathematical* determination of prices in Sraffa versus a *causal* determination of prices, (as, for instance, in Arrow-Debreu). In the context of "Prices and Two Senses of Determination" [2003, p.1247], Sen [p.1253] writes, "The sense of "determination" invoked by Sraffa concerns the mathematical determination of one set of facts from another set. To illustrate the point … a sundial may allow us to "determine" what time it is by looking at the shadow of the indicator (gnomon), but it is not the case that the shadow of the indicator "causally determines" what time it is. The value of the clock does not lie in its ability to "fix" – rather than "tell" – the time of day."

[13] Italics added.

exclusively market-clearing prices, but only that *if* they were adopted by the market, *then* a replication of the production activities would become possible. By contrast, as we shall see in Section 4, for Arrow and Debreu the prices are, in fact, solely market-clearing prices in the sense that excess demands for all commodities are zero for strictly positive prices and non-positive for non-negative prices. However, Arrow and Debreu do not explicitly raise any issue of replication of the production activities. I return to this issue in Section 5.

Second, neither theory has so far been *rigorously* shown to be false, in terms of making a claim *A*, and reaching the conclusion *not A*. Third, to ascertain the truth or falsity of either theory of values of commodities, the investigation must be conducted within the respective model, not from outside it. It is, in this sense, *not* a legitimate exercise to use the Arrow-Debreu model to criticize Sraffa's theory of value, nor is it legitimate to use Sraffa's model to criticize the Arrow-Debreu theory of value. Fourth, due to this third implication, the entire Cambridge-Cambridge debate, with hindsight, ends up being utterly futile, because it led to no definitive conclusion. The issues raised in that debate remain unresolved to date. Indeed, as Sen [2005, p.6] writes in a different context, "We need to take note not only of the opinions that won – or allegedly won – in debates, but also of other points of view that were presented and are recorded or remembered. A defeated argument that refuses to be obliterated can remain very alive."

Fifth, additional issues that simply do not arise in Sraffa's model, as, for example pertaining to consumers' demand functions for commodities, simply must not be raised in the context of his theory of value. Sraffa has *nothing* to say about such matters, *one way or the other*, so that asserting anything regarding them is tantamount to putting words in his mouth, to which he has gone on record to object quite strenuously.[14] Similarly, it is not legitimate to raise issues pertaining to matters for which the Arrow-Debreu model was not designed, as, for instance, of replication of production activities, because they also have nothing to say on the matter, one way or the other. Of course, replication cannot occur in the Arrow-Debreu formulation, because every last bit of chocolate is eaten up by the consumers, thanks to the Strong Monotonicity axiom as we shall see in Section 4, so that everybody's endowment of every commodity but labor is wiped out by the end of the production cycle. Further discussion of this matter has to be postponed until after a fuller discussion of the Arrow-Debreu model in Section 4.

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[14] Witness the Arun Bose [1965] case. In a letter to Bose in 1964, Sraffa wrote:

"I am sorry to have kept your MS so long—and with so little result.

The fact is that your opening sentence is for me an obstacle which I am unable to get over. You write: 'It is a basic proposition of the Sraffa theory that prices are determined exclusively by the physical requirements of production and the social wage-profit division with consumers demand playing a purely passive role.'

Never have I said this: certainly not in the two places to which you refer in your note 2. Nothing, in my view, could be more suicidal than to make such a statement. You are asking me to put my head on the block so that the first fool who comes along can cut it off neatly.

Whatever you do, *please* do not represent me as saying such a thing."

Sixth, 'capital', in the sense of the value of intermediate goods, does not arise as a concept in the Arrow-Debreu model, because Arrow and Debreu do not engage in the aggregation of the intermediate goods (which are used as inputs rather than net final output) in the production of a commodity, so that the issue of distribution of income between capital owners and workers is completely absent in their theory of value.[15]

Regarding the two theories of value, there are additional implications that can be inferred, over and above the six mentioned so far, but their discussion requires a more complete description of the full-blown models of both Sraffa and Arrow and Debreu. To these, I turn next.

3. Sraffa's Theory of Value
As noted, the theory of value I describe here is taken exclusively from Part I of Sraffa's book, which deals only with the simplest case in which each industry produces only one commodity and there is only one process of production of each commodity, i.e., there is neither any joint production nor any issue of choice of technique, and all commodities have a life of one period.

The observed relations of production from which Sraffa starts are a matrix of inputs, A, and a matrix of outputs, C, both assumed to be non-singular. The entry in row i, column j in matrix $A = [a_{ij}]$, $\forall i, j = 1, \dots m$, represents the amount of commodity j actually observed to have been used as input in the production of c_i quantity of commodity i. Since each industry produces only one product, C is a diagonal matrix with the amounts of outputs produced $[c_1, \dots c_m]$ along the main diagonal.

Suppose further that at least one industry produces a surplus over and above the total input requirement of that commodity in all industries. Then the value of the surplus is $\boldsymbol{p}.(\boldsymbol{u}.C - \boldsymbol{u}.A)$, where $C = \text{diag } [c_1 \dots c_m]$ such that $(\forall j: c_j \geq \sum_{i=1}^{m} a_{ij}) \& (\exists j: c_j > \sum_{i=1}^{m} a_{ij})$, $\boldsymbol{u}$ is a m-vector of ones, and $\boldsymbol{p}$ is an m-vector of prices of the m commodities. For such an information-set-based economy, Sraffa asserts the following m independent relationships in $m + 2$ unknowns

(4) $$(1 + r)A.\boldsymbol{p} + w\boldsymbol{l} = C.\boldsymbol{p}.$$

This is on the assumption that wages are paid *post factum*. In (4), $\boldsymbol{l}$ is a strictly positive m-vector of (current or direct) labor actually employed in each of the m industries.[16] Thus the only information – factual information – that Sraffa considers as available for ascertaining the "unique set of exchange values" embedded in the relations of production is that which is contained in A, C, and $\boldsymbol{l}$, and nothing else whatsoever.

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[15] The concept of capital does appear in other renditions of economic theory, especially the "neoclassical" rendition identified by Hahn [1982], which is not the concern of this paper. Instead I deal only with Arrow-Debreu and Sraffa.
[16] Sraffa does *not* require that labor be employed directly in the production of every commodity, but that is another simplification I adopt, to do away with the distinction between what he calls basic and non-basic commodities, simply because this distinction is not of significance for the purpose of a comparison with the Arrow-Debreu model.

Since every element of $l = (l_1, l_2, \dots l_m)$ is a given number in the relations of production, so is its sum, which is never considered to change in any examination of the model. It is harmless, therefore, to set this sum equal to unity, which is what Sraffa does, $\sum_{j=1}^{m} l_i = 1$, with the understanding that $\forall i: l_i > 0$. In (4), among the $m + 2$ unknowns, (i) w is the wage rate that is assumed to be the same in all industries and is assumed to be non-negative, (ii) r is the rate of profit on the value of capital that is assumed to be the same in every industry and also assumed to be non-negative, and (iii) the remaining m unknowns are the Sraffan prices $(p_1, \dots p_m)$, such that $\forall i: p_i > 0$. To ensure that all Sraffan prices are strictly positive, some conditions have to be imposed on A and C.

Conditions 1: Define $B = C^{-1}A$. Note that B is a square matrix, and is non-singular because, by assumption, A and C are non-singular. Then it follows from a theorem of Perron and Frobenius that the price vector p is (a) strictly positive and (b) unique up to multiplication by a positive real number, if (1) B is indecomposable, (2) all elements of B are non-negative and at least one element is strictly positive, and (3) p is a non-negative characteristic vector associated with the maximal real-valued characteristic root of B.[17]

As already noted, the object of Sraffa's exercise is to ascertain the values of w, r and the m Sraffan prices. Of course, it is not possible to obtain unique values of these $m + 2$ unknowns from the m independent relationships in (4). Arrow and Debreu face a similar indeterminacy problem. To get around this, as we shall see in Section 4, they assume that the m-vector of prices in *their* model belongs to the unit simplex, which solves the problem of indeterminacy of the m prices. It is also common in many fields such as international trade theory to take one of the commodities as the *numéraire*, so that its price is set at unity, and all other prices and the wage rate are expressed in terms of, say, p_i units of that *numéraire* commodity per unit of commodity i.[18] On the other hand, the rate of profit on the value of capital is, of course, a unit-free number such as 0.25 that refers to a 25% rate.

Srafa uses a different, though equally legitimate normalization rule. He takes the national income of the economy as equal to one. This is the value of net output of the economy. All Sraffan prices and the wage rate are then expressed in units of net national product. Formally, the normalization rule that national income equals one is

(5) $$p.(u.C - u.A) = 1.$$

Equations (4) and (5) consist of $m + 1$ independent equations in $m + 2$ unknowns, thereby rendering the system still underdetermined, unlike the case of the Arrow-Debreu model with the normalization that their price vector belongs to the unit simplex.

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[17] For details, see Kurz and Salvadori [1995, p. 517].
[18] If Commodity m is taken to be the *numéraire*, then $(p_1, \dots p_{m-1})$ would be the relative prices of the $(m − 1)$ commodities that are expressed in units of Commodity m, and the wage rate w would also be expressed in the quantity of Commodity m per year of labor.

To gain additional insight, it is helpful to return to the example of a Sraffan economy utilized in Section 2. Consider the counterpart of (4) in the example referred to in Robinson and Naqvi [1967, p. 585-86] as Technique A. This takes the form

$$\left(\frac{14}{9} + \frac{20}{9}\, p\right)(1 + r) \quad + \frac{4}{5}w \quad = \quad \frac{10}{3}$$

(4a)

$$\left(\frac{7}{9} + \frac{10}{9}\, p\right)(1 + r) \quad + \frac{1}{5}w \quad = \quad \frac{10}{3}p$$

Noting that the net output of this economy is one ton of wheat in the Robinson-Naqvi case, using Sraffa's normalization rule of setting the value of net output of the economy equal to one, with the price of wheat multiplied by 1 ton of wheat equal to one, it follows that the price of wheat is one unit of national income per ton of wheat. Further, using (4a), p, the price of iron in terms of national income (and also in terms of wheat, in this example) can be eliminated to solve for the wage rate as a function of the profit rate. This yields the equation of the w-r curve. This relatioship for the specific Robinson-Naqvi example is contained in Figure 1. Figure 1 shows the inverse, though non-linear, relationship between the rate of profit on the value of capital on the horizontal axis, and the *total wages* accruing to all the workers in the economy on the vertical axis, and this relationship is embedded entirely in the observed relations of production.

With total labor employment equal to one, in general the wage rate, w, equals the total income of workers, which, in turn, equals the fraction of national income accruing to workers; the remainder of the value of net output of the economy, $(1 - w)$, accrues to capital owners, who have property rights over the commodities that constitute the means of production. For the Robinson-Naqvi example, Figure 1 displays such a tradeoff involved in the distribution of national income between workers and capitalists.
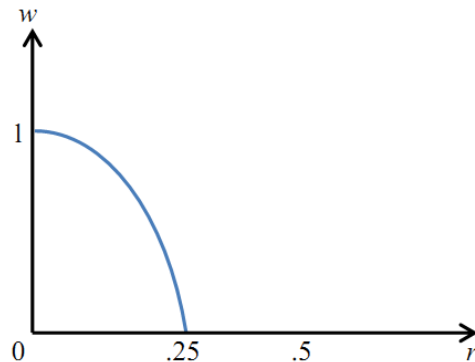

Figure 1

At the one extreme, $w = 0$, so that workers get nothing out of national income and capitalists get the entire national income, which corresponds in the Robinson-Naqvi example to the maximum

rate of profit on the value of capital of 25% in each industry, and thus in the economy, and also reveals as embedded in these relations of production the Sraffan price of iron equal to one-half of the national income per ton of iron, which happens to equal to ½ ton of wheat per ton of iron.[19] At the other extreme, however, the entire national income accrues to workers, with $w = 1$, so that $r = 0$, and the Sraffan price is 0.44 tons of wheat per ton of iron. Thus the price of iron varies from 0.44 to 0.5 tons of wheat per ton of iron as (i) r varies from 0 to 0.25, or equivalently, as (ii) w varies from 1 to 0. From the foregoing discussion, the following proposition can be asserted.

Proposition 2: *In general, the actual distribution of national income between workers, on the one hand, and capitalists, on the other, is not uniquely revealed as inherently embedded in the relations of production that are based exclusively on the factual information contained in Sraffa's conception of a capitalist economy.*

This is can be seen as the one of the central messages of Sraffa. There is nothing counterfactual in Proposition 2. Further, as noted above, with w representing the share of national income that accrues to workers, it follows that $(1 - w)$ is the share that accrues to capitalists, $\forall w \in [0, 1]$. Let r_{max} refer to the maximum rate of profit embedded in the relations of production. The factual information contained in the observed relations of production implies that as w varies between 0 and 1, r varies between r_{max} and 0, and conversely. Formally, for a given set of factually observed relations of production (4) and the normalization (5),

$$\{\forall w \in [0, 1] \to \exists [\boldsymbol{p}(w) > 0 \ \& \ r(w) \geq 0]\} \ \& \ \{\forall r \in [0, r_{max}] \to \exists [\boldsymbol{p}(r) > 0 \ \& \ w(r) \geq 0]\},$$

where $\boldsymbol{p}(w), r(w)$, $\boldsymbol{p}(r)$ and $w(r)$ are unique if Conditions 1 hold. Thus, for a given set of actual relations of production (4) and normalization (5)

Proposition 3: *There is a factual inverse relationship between the share of national income that accrues to workers and the share that accrues to capitalists that is inherently embedded in the relations of production that are based exclusively on factual information.*

This can be seen as the second central message of Sraffa. The rest of his book contains implications of these two fundamental *facts* that are contained in Propositions 2 and 3.[20]

Moreover, at the Sraffan prices, measured in terms of the quantity of national income per unit of a commodity,

(6a) $$K_i(A, C, r) \equiv \sum_{j=1}^{n} p_j(A, C, r) a_{ij}, \forall r \in [0, r_{max}]$$

Or

(6b) $$K_i(A, C, w) \equiv \sum_{j=1}^{n} p_j(A, C, w) a_{ij}, \forall w \in [0, 1]$$

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[19] The example, one might recall, was designed with a one ton surplus output of wheat and no surplus output of iron.
[20] The Robinson and Naqvi [1967] results, and those contained in Schefold [1989], among many others, are direct implications of these two fundamental factual assertions.

are, by definition, the values of capital in industry i, and $K = \sum_{i=1}^{n} K_i(A, C, x)$ is the value of aggregate capital in the economy, for $x = w, r$. Naturally, since Sraffan commodity prices vary depending on the share of national income that accrues to workers contained in the value of the parameter r (or w), the value of capital is not unique, nor is the concept of ranking industries, or anything else, in terms of "capital intensity" construed as the ratio of capital to labor. While this argument follows from Sraffa's analysis, and is thoroughly devastating for explaining the rate of profit as determined by the marginal productivity of capital, it is still not the fundamental issue that Sraffa is after, despite considerable interest in the matter in the subsequent literature. I return to this issue in Section 5.

In the Robinson-Naqvi example, the relations of production imply that there is a 25% maximum rate of profit on the value of capital. Equation (4a) and Figure 1 reveal how the 1 t. wheat surplus is distributed across the two industries in proportion to their respective values of capital, and that *the value of capital itself depends on the value of the distribution parameter $r \in [0, 0.25]$* in (6a), which is *external* to the factual relations of production in Sraffa's model of a capitalist economy. Of course, *the value of capital itself can also be seen as parametrically dependent on the parameter $w \in [0, 1]$* in (6b), which is a distributional issue that is also external to Sraffa's model. In particular, in the two polar cases,

Table 1

Units = t. wheat	Value of Capital at $r = 0.25$	Value of Capital at $r = 0$
Wheat industry	120/45	114/45
Iron industry	60/45	57/45
Total	4.0	3.8

Some economists have tended to view the inverse relationship between the share of national income going to workers and the rate of profit on the value of capital as displaying a class conflict between workers and capitalists, and hence have seen it as an illustration of the antagonistic relation between the two classes. Such an interpretation, however, misses the point. I return to this issue in Section 5, after briefly describing the Arrow-Debreu model in the next section.[21]

4. Arrow-Debreu Theory of Value
Just as I have taken the model from Part I of Sraffa's book, I shall also take up the Arrow-Debreu model in a particularly simple form, while retaining its essential features. There is a society of finite n persons. Each person i is characterized by:

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[21] See Hahn [1982] and Sen [2003], among others.

(i)   a finite m-vector of vector of commodities, called the personal endowment, $\boldsymbol{\omega_i} = (\omega_{i1} \dots \omega_{im}), \omega_{ij} \geq 0, \forall j = 1, \dots, m, \forall i = 1, \dots, n$, with at least one commodity in a strictly positive quantity that a person is endowed with. This commodity could be labor – say 40 hours per week over 52 weeks – that the person is endowed with. Each commodity has a life of one period.

(ii)   by a scalar $\theta_{is} \geq 0$, a non-negative fraction that represents the *share* of a producing-unit s called a *firm* $\forall s = 1, \dots, l$, and there exist a finite l number of firms, each with a one-period production cycle. Also, $\forall s, \exists i = 1, \dots, n, \theta_{is} > 0$ & $\sum_{i=1}^{n} \theta_{is} = 1$, indicating that any given firm is entirely owned privately by some persons.

(iii) R_i , which is a *binary* relation of weak preference that stands for "is at least as good as" defined on a subset S_i of the m-dimensional real commodity space, with the convention that if a person ends up being a net buyer (seller) of a commodity then its quantity is denoted by a strictly positive (negative) real number, zero otherwise.

Each firm s produces exactly one commodity in the period under consideration.[22] A firm s buys commodities, including labor, from persons and transforms them into a single commodity, so that it is characterized by an m-vector called its *net* output vector $\boldsymbol{y_s} = (y_{s1}, \dots, y_{sm})$, with exactly one positive element, at least one negative element and the rest non-positive, with $\boldsymbol{y_s} \in Y_s \subset \mathfrak{R}^m$, which is the m-dimensional real space, where Y_s is called the production set of firm $s, \forall s = 1, \dots l$. Further, there are s_i number of firms that produce commodity i, where s_i is a large positive, though finite, number. Thus the economy ends up with $l = \sum_{i=1}^{m} s_i$ firms.

A firm is a legal entity owned entirely by some or all of the n persons. A commodity that is produced is identified with an industry that produces that commodity, so that there are m industries, each populated by a large number of firms that produce that commodity. Each firm buys commodities, including labor, from the persons who sell these commodities to the firms from their respective endowments. A firm *chooses* the quantities of commodities it buys as inputs and the quantity of its net output that it produces and sells at a *parametrically given*, strictly positive, m-vector of prices $\boldsymbol{p} = (p_1 \dots p_m)$ of the m commodities.[23] The sole motive of

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[22] In the original Arrow and Debreu [1954] contribution, joint production and multi-product correspondences are admissible. The case of joint production in Sraffa's formulation of the Standard Commodity is an extension by Schefold [1989]. The Arrow-Debreu model has also been extended in a number of important ways, including by Chipman [1970] to include parametric (or external to a firm but internal to an industry) increasing returns to scale, and to the case of asymmetric information between buyers and sellers by Akerlof [1970], among other significant extensions. However, we do not deal with these issues here.

[23] In the general model, contained in Arrow and Debreu [1954], some prices can be zero if they end up with negative excess demands in the aggregate, but that is not a generalization to our purpose, because our goal is to make a comparison of the Arrow-Debreu model with the Sraffa model. The Sraffa model, Part III onwards, is also more general than the one presented here. The purpose here is to consider such versions of the two models that capture the salient features of the two models, without aiming at the greatest possible generality, so as to isolate the precise nature of the differences between the two conceptions of the same economic reality, and *inter alia*, to quarantine both the *sources* of the differences and the *entailments* of the differences in the two theories of values of commodities that they respectively support.

a firm in choosing these quantities of its net output vector $\boldsymbol{y}_s$ is the maximization of $\pi_s = \boldsymbol{p}.\boldsymbol{y}_s : \boldsymbol{y}_s \in Y_s$.

Each person chooses the quantities of each of the m commodities $\boldsymbol{x}_i = (x_{i1}, \dots, x_{im})$, including labor, to buy from firms, at the *parametrically given* strictly positive price vector $\boldsymbol{p}$. Thus a person can play a double role, one as a buyer and seller of commodities out of the endowment $\boldsymbol{\omega}_i$, including selling labor (as a negative quantity purchased), and the second role as a possible fractional owner of a firm or firms. Thus the income of person i is $v_i = \boldsymbol{p}.\boldsymbol{\omega}_i + \sum_{s=1}^{l} \theta_{is}\pi_{is} > 0$. The sole motivation of every person i is to *choose* such quantities $\boldsymbol{x}_i = (x_{i1}, \dots, x_{im})$ as to maximize personal preference R_i subject to the budget constraint $\boldsymbol{p}.\boldsymbol{x}_i \leq v_i$. Further, by assumption, $\sum_{i=1}^{n} \boldsymbol{\omega}_i = \boldsymbol{\omega}$, the economy's endowment vector of the m commodities, all elements of which are strictly positive.

Moreover, $\boldsymbol{x} = \sum_{i=1}^{n} \boldsymbol{x}_i$ is the aggregate m-vector of commodities chosen to be consumed by all the n persons in the economy. Also, $\boldsymbol{y} = \sum_{s=1}^{l} \boldsymbol{y}_s \in Y$, where $\boldsymbol{y}$ is the net output vector of the economy and Y is the production set of the economy. An entailment of the closure of the economy is $\boldsymbol{p}.\boldsymbol{x} = \boldsymbol{p}.\boldsymbol{y}$.[24]

To obtain the existence of a strictly positive Arrow-Debreu price vector $\boldsymbol{p}$, some restrictions are placed on each firm's production set Y_s, the production set of the economy Y, and on the personal preference relation R_i defined on S_i, to be discussed presently. Labor is a commodity like any other, except that a person may have an endowment of labor, but no firm produces labor. That is, labor is a primary factor of production, and it may be considered the only non-produced commodity.[25] A sufficient set of restrictions are

Conditions 2a: In the capacity of a buyer of commodities, a person's preference relation R_i $\forall 1 = 1, \dots, n$ defined on S_i is: (i) Binary, (ii) Reflexive, Transitive and Complete, (iii) Strongly monotonic, (iv) Convex, and (v) Continuous. Also, S_i is (a) closed, and (b) bounded below.

Conditions 2b: The production set of a firm, (i) Y_s is closed, $\forall s = 1, \dots, l$. The economy's production set, Y is (ii) Convex, (iii) admits of the possibility of Inaction ($0 \in Y$), (iv) satisfies Irreversibility of production, in so far as $[\forall s: \boldsymbol{y}_s \in Y \ \& -\boldsymbol{y}_s \in Y \rightarrow \boldsymbol{y}_s = 0]$, and the property that (v) nothing can be produced out of thin air, in that $[\forall s: \boldsymbol{y}_s \geq 0 \ \& \ \boldsymbol{y}_s \in Y \rightarrow \boldsymbol{y}_s = 0]$.

The following existence result can be proved, and has been proven by Arrow and Debreu [1954]:

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[24] It is noteworthy that this condition holds as a weak inequality. However, it turns out that with the requirement of (i) strong monotonicity of personal preferences, and (ii) finite markets, this relationship holds as a strict equality, and thus entails a strictly positive price vector under the full set of conditions. It may also be noted that if there are infinite markets, as in the typical *Overlapping Generations* model, then this condition holds as a strict inequality, which renders the stationary outcome Pareto suboptimal, so that the introduction of financial intermediation, among other possibilities, raises it to a Pareto optimal level.

[25] Of course, the general model does not require that there be only one primary factor of production, but that, again, is not a generalization to our purpose.

Proposition 4: *There exists a price vector $\boldsymbol{p}$, with the normalization that $\boldsymbol{p}$ belongs to the unit simplex, such that quantities demanded and supplied are equal for every commodity, rendering all excess demands $\boldsymbol{z}(\boldsymbol{p})$ zero at strictly positive prices, $\boldsymbol{z}(\boldsymbol{p}) = \boldsymbol{x}(\boldsymbol{p}) - \boldsymbol{y}(\boldsymbol{p}) = \boldsymbol{0}$, if Conditions 2a and 2b are satisfied.*[26]

Arrow and Debreu seek a set of (strictly positive) commodity prices $\boldsymbol{p}$ such that quantity demanded equals quantity supplied in the market for every commodity, and the price vector belongs to the unit simplex. And, they accomplish this goal with the greatest of rigor possible. Notice that Arrow and Debreu do *not* ask the now-familiar question Sraffa does of the set of commodity prices, as to whether they would permit a replication of the production process in the economy.[27] Nor, indeed, does Sraffa ask the well-known question Arrow and Debreu do of the set of commodity prices as to whether they would result in zero excess demand for every commodity. They ask different questions, come up with different answers, both based on their distinct − though entirely complete and entirely correct − characterizations of a capitalist economy, based on informationally distinct models.

5. Comparison of the Two Theories of Value
It is noteworthy that throughout his book Sraffa never refers to the concept of equilibrium. Arrow and Debreu, by contrast, prove the existence of equilibrium by specifying a sufficient set of conditions. In the light of Propositions 2 and 3, in Sraffa's conception of a capitalist economy, unique values of Sraffan prices and the distribution of income between workers and owners of commodities that constitute inputs are *undetermined* based exclusively on observed facts pertaining to an economy. By contrast, based on facts and counterfactual presumptions about a capitalist economy, it is clear from Proposition 4 that a set of Arrow-Debreu commodity prices and the interpersonal income distribution are *completely determined* in equilibrium. These conclusions pertain solely to the descriptive characteristics of the Sraffan economy and the

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[26] For a proof, see Arrow and Debreu [1954], where the function $\boldsymbol{z}(\boldsymbol{p})$ is allowed to be multi valued, so that it can be called an excess demand correspondence.

[27] Notice that $\boldsymbol{x}(\boldsymbol{p}) = \boldsymbol{y}(\boldsymbol{p})$ implies, as noted in Section 2, that replication of production activities in the next period in the manner of Sraffa cannot occur in the Arrow-Debreu formulation, because every last bit of every commodity ends up getting consumed, thereby wiping out everybody's endowment vector $\boldsymbol{\omega}_i$ but for their respective primary factor endowments. There simply aren't any commodities left to produce commodities in the next period, in the Arrow-Debreu model.

Moreover, if intertemporal considerations are entertained, there are serious problems in assigning dates to commodities in the Arrow-Debreu model. Since consumers are maximizers of personal preferences subject to their respective budget constraints, it is well known that they will engage in *inconsistent planning* unless their discount rates are identical across all dates, which is not a restriction imposed either in Arrow and Debreu [1954] or in Debreu [1959]. Thus, once a person has calculated a personal consumption plan for a finite T number of periods (and both Arrow and Debreu [1954] and Debreu [1959] formulations are finite dimensional) starting at time $t = 1$, then, upon arriving at a later date, such as $t = 2, 3$, and so on, this person will recalculate and *not* follow the "original" optimal consumption plan.

However, as pointed out in Section 2, it is not legitimate to ask of the Arrow-Debreu theory of value if production can be replicated in as much as it is not legitimate to ask of Sraffa's theory of value as to what happens to demand for commodities as their prices change − each of the two models is designed for its respective purpose, not for answering questions meant to be answered by the other model.

distinct Arrow-Debreu economy. Some far-reaching implications can be inferred from these observations that have been the source of both needless confusion and unwarranted controversy. Let me explain.

While Sraffa sets out to find a unique set of exchange values that would redistribute the commodities concentrated at the end of the production cycle in the hand of their respective producers back to the industries so as to permit a replication of the production activities, he finds that such exchange-values do *not* uniquely exist if the sole basis of ascertaining them is the factual information about inputs of commodities and labor and commodity outputs, contained in A, C, and l, and nothing else whatsoever. Instead, he finds that the factual information regarding production in an economy reveals that there is an inverse relationship between the share of national income that accrues to workers, w, and the rate of profit on the value of capital, r. And, this is a fact. Further, for every value of w between 0 and 1 (or for every value of r between 0 and r_{max}) that is exogenously specified from outside the economic system, there is a unique set of Sraffan prices. Therefore, the following claim can be made.

Proposition 5: *There is nothing factual about an economy that endogenously determines what the actual distribution of income between workers and capitalists will be. The explication of this distribution is to be found in society outside the economic system.*

This claim is completely at odds with the conclusion reached by Arrow and Debreu, based in turn on both factual and counterfactual information. For, once the prices of all commodities at zero excess demands are causally determined, then given the values of ω_i and θ_{is} the following proposition holds:

Proposition 6: *The interpersonal income distribution is completely determined endogenously in the Arrow-Debreu economy based on the information regarding personal endowments and personal corporate-ownership shares.*

Proposition 6 asserts that in the Arrow-Debreu economy, that is characterized by both factual and counterfactual information, the values of commodities and the distribution of income are completely determined endogenously, whereas Proposition 5 says that this is simply not true in the Sraffan economy that is based solely on facts.

The question is not whether Propositions 5 is true or Proposition 6 is true. Indeed both are true, in their respective models of a capitalist economy. The question, then, turns on which characterization of a capitalist economy is a more accurate description of reality – Sraffa's based only on facts or the Arrow-Debreu explanation based on facts and counterfactual information.

This issue can be examined at three levels. The first is purely information theoretic. If the information set on which Sraffa's model is based is *S*, and the information set on which the Arrow-Debreu model is based is *D*, then *S* is a proper subset of *D*. Moreover, while *S* contains only factual information, *D* contains both the factual information in *S and* additional counterfactual information. Since Sraffa's theory of value is based on weaker assumptions – in the sense of depending on less information – than the assumptions on which the Arrow-Debreu theory of value is based, it necessarily follows that Sraffa's is a more general theory – no ifs, ands or buts about it – regardless of what the conclusions of the two might be. This is a simple

point of logic. Since the information set on which Sraffa's model is based is a proper subset of the information set on which the Arrow-Debreu model is based, challenging the accuracy of Sraffa's model as the basis of the description of capitalism would prove fatal for the Arrow-Debreu model also.

Second, at a philosophical level, there is a well-known, long-standing debate in epistemology regarding the use of counterfactual information. It has been argued that there is an element of unreliability in propositions that are predicated on counterfactuals that is absent in purely observational propositions that are based exclusively on facts.[28] In this regard, Sraffa's approach of eschewing counterfactual information bypasses such difficulties in his purely descriptive theory of value. The Arrow-Debreu descriptive theory of value, however, remains open to this epistemological critique. On the other hand, it is also evident that the sole concern of economics is not with *description*. The concern with *prescription* is also inescapable and significant. Therefore, it is not altogether clear why one would want to eschew counterfactuals in descriptive economics, only to take them on board in the normative exercise of social evaluation aimed at policy prescription. This qualification notwithstanding, many more philosophers are liable to 'buy' Sraffa's story than will be prone to 'buying' the story of Arrow and Debreu; this, of course, may or may not be a great trophy. Also, just because counterfactual presumptions are necessary for normative social evaluation, it does not follow that there is any need to swallow counterfactuals hook, line and sinker in descriptive economics as well.

Third, and perhaps the most significant, is a political issue. In the Arrow-Debreu conception, the interpersonal distribution of income is endogenously determined. It is what it is. Not so, in Sraffa's conception of the economy, however. It is not merely that the value of aggregate capital and its marginal productivity in explaining the rate of profit are thrown in doubt, but, far more significantly, there is nothing in the economic system of factual relationships that pins down the distribution of income. This income-distributional matter rests outside the purely economic sphere of society, and thus falls in the political domain. The distribution of income between workers and capitalists is determined on the basis of the relative bargaining power of the two classes – a position taken by classical political economists for centuries – which finally receives formalization in Sraffa's work. This is the most fundamental contribution of Sraffa.[29]

Sen [2003, p.1247] writes, "I must confess that I find it altogether difficult to be convinced that one's skepticism of unrestrained capitalism must turn on such matters as the usefulness of aggregate capital as a factor or production and the productivity attributed to it, rather than on the mean streets and strained lives that capitalism can generate[.]" Actually, Sraffa achieves considerably more than that. The fact is, Sraffa provides an *explanation* of the makings of "the mean streets and strained lives that capitalism can generate" by identifying the underlying inequality of income distribution that falls in the domain of political negotiation and balance of power between workers and capitalists. That is the true power of Sraffa's contribution – to

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[28] There is, for instance, a distinction made between *Prima Facie* and *Ultima Facie* justification in epistemology by Senor [1996].

[29] As it happens, the German constitution actually embodies this feature of collective bargaining by trade unions with corporate management. It is somewhat odd that the country that once expelled Karl Marx, the most forceful proponent of this position on the determination of income distribution, has, in a significant way, embraced his ideas on this matter.

provide an answer to a question as old as economics itself – that the factual relations of production, by themselves, fail to determine the actual income distribution.

In some sense, Sraffa was the last of the greats among classical political economists. He saw the disassociation between economics and politics, inherent in the trend of neoclassical economics, as a diminution of the study of society. It is in this precise sense that, in his words, "It is, however, a peculiar feature of the set of propositions now published that, although they do not enter into any discussion of the marginal theory of value and distribution, they have nevertheless been designed to serve as the basis for the critique of that theory."[30] This politico-economic approach of Sraffa may be contrasted with the politics-free neoclassical approach to rehabilitation of the *status quo* income distribution in the Arrow-Debreu formulation, which, along with the entire allocation, can be, and has also been shown to be optimal in a specific sense, viz., that of Pareto.[31]

6. Concluding Remarks

This paper covers much ground in dealing with two very comprehensive theories of value. Both theories of value provide answers to the "determination" of commodity prices and income distribution in a society, in their own distinctive ways. Both are based on perspicaciously articulated characterizations of a capitalist economy. Yet, they offer conclusions that are quite different due to the fact that the characterizations are distinct, philosophically, informationally, and as a consequence, politically. Sraffa's theory of value falls squarely in the domain of classical political economy – a theory that sees society as consisting constitutively of classes, with individuals acting in the interest of the class to which they belong. Arrow and Debreu's theory of value, by contrast, is the culmination of the tradition of neoclassical economics that sees society as constitutively made up of individuals who act in their personal self-interest.

Notice that one could be tempted to make more than is warranted of the economic indeterminacy of income distribution revealed by the work of Sraffa. Merely because of fact (a) that there is an inverse relationship between the share of national income that accrues to workers versus capitalists, and fact (b) that the economic relations fail to resolve the income distribution matter, thereby leaving it to the political process to determine, it does not follow that a violent (or non-violent) revolution is called for. As in several countries, including Germany, both facts can be embraced with tranquility, and a state-aided political resolution to the income distribution issue can be reached under constitutional rule of law, without any call to arms.

There is another momentously significant perspective that emerges from a comparison of Propositions 5 and 6. Proposition 6, arising from the Arrow-Debreu theory of value, lulls one into thinking that the reality of "mean streets and strained lives that capitalism can generate" is what it is, to the extent that the distribution of income is completely determined in the economic sphere, and thus gives room for a conservative orthodoxy to argue that "all is for the best," in the sense of Pareto, "in the best of all possible worlds," even if misery is pervasive. This claim puts a conscientious objector on the defensive in having to justify state intervention for ameliorating

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[30] p. vi. Notice that "marginal" is *not* the operative word here. It is the existing, alternative "theory of value and distribution" to which Sraffa makes reference as aiming to critique. And that is the theory of Arrow and Debreu.
[31] The reference to the "critique" in Sraffa is to a "theory of value and distribution" that is bereft of any political consideration, rather than to one that relies on differential calculus.

widespread misery. However, Proposition 5, based on Sraffa's theory of value, constitutes a basis for a critique of the Arrow-Debreu theory of value by establishing a completely divergent position (that is based solely on factual information), viz., that the economic system does not uniquely dictate any specific income distribution, and that if the income distribution that emerges is fraught with deprivation for the many, it is due solely to a political failure in empowering *les misérables*. Public action in the political domain is the answer to such discontent, not a calm, tranquil and passive acceptance of the *status quo* that arises from reliance on counterfactuals.

Oddly enough, Sraffa writes only about commodities, and there are no people visible in his model of a capitalist economy or in his theory of value, even though his overriding concern is – as that of classical political economists – with the uncovering of the lack of economic determinism of the distribution of income. By contrasts, the Arrow-Debreu model is visibly populated by persons, and based on a larger quantity of information it ends up with the conclusion of economic determinism of interpersonal income distribution. It must be recognized, though, that some doubt has been cast on the concept of persons in Arrow and Debreu. In the words of Sen and Williams [1982, p.4], "Persons do not count as individuals in this any more than petrol tanks do in the analysis of the national consumption of petroleum." Actually, Arrow and Debreu refer to them as "consumption units."

Thus, one finds an absence of people in Sraffa, and persons with highly circumcised, inorganic individuality in Arrow and Debreu. This leaves room for a theory of value that gives greater play to persons as individuals in society, with the full set of cultural identities, political affiliations, familial associations, and personal predicaments and preferences. A way out appears to be Sen's [1985] theory of *Commodities and Capabilities* of persons that aims to capture the freedoms of individuals to achieve and be what they have reason to value constitutively and instrumentally, and of which Kuklys [2005] has done a phenomenal job of operationalizing.

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