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Asymmetric capital-tax competition, unemployment and losses from capital market integration

Rüdiger Pethig[a] and Frieder Kolleß[b]

Abstract: In a multi-country general equilibrium economy with mobile capital and rigid-wage unemployment, countries may differ in capital endowments, production technologies and rigid wages. Governments tax capital at the source to maximize national welfare. They account for tax base responses to their tax and take as given the world-market interest rate. We specify conditions under which - in contrast to free trade with undistorted labor markets - welfare declines and unemployment increases in some countries (i) when moving from autarky to trade without taxation and/or (ii) when moving from trade without taxation to tax competition.

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Asymmetric capital-tax competition, unemployment and losses from capital market integration

1 The problem

The international mobility of capital has massively increased over the last decades, governments distort trade by taxing or subsidizing capital, and unemployment is a persistent phenomenon in many countries. The free-trade paradigm promises gains from international trade in a perfectly competitive world in the absence of taxation but leaves unanswered the question what the allocative impact is of trade when capital is mobile, when countries suffer from unemployment, and when their governments engage in capital-tax competition.

The present paper aims at exploring the impact of capital market integration in a multi-country economy with heterogeneous countries, persistent rigid-wage unemployment and capital-tax competition. Each country produces the same consumption good with the help of labor and capital, and unemployment results from excessively high and rigid wage rates. Countries may differ with respect to their rigid wage rates, capital endowments and production technologies.[1] Governments levy capital taxes at the source whose rates are not sign constrained and whose revenues are recycled to the consumers. Governments choose their tax to maximize national income (= welfare) taking account of how the domestic firms' demands for capital respond to the tax. We use that model to investigate the changes in the countries' allocation and welfare, when moving from autarky to trade without taxation and from trade without taxation to tax competition.

There is a large and growing literature on capital tax competition. The classic papers of Zodrow and Mieszkowski (1986) and Wilson (1986) analyze the impact of capital-tax competition on the provision of public goods when labor markets are perfectly competitive. Some of the subsequent literature took up the issue of tax competition in the presence of labor market distortions, e.g. Fuest and Huber (1999), Leite-Monteiro et al. (2003), Eggert and Goerke (2004), Ogawa et al. (2006) and Aronsson and Wehke (2008). However, these studies assume identical countries and therefore yield limited insight only in allocative effects of the transition from autarky to trade and tax competition. Symmetric tax competition means that trade does not take place and that inefficiencies of tax competition, if any, hit all countries alike.

[1] The size of the countries' populations (= labor endowment) may vary as well. However, since all countries are assumed to suffer from unemployment the population size is irrelevant.

Asymmetric tax competition is studied, e.g., by Wilson (1991), Bucovetsky (1991), DePater and Myers (1994), Peralta and van Ypersele (2005) and Sato and Thisse (2007). But none of these contributions deals with labor market distortions and unemployment[2]. Peralta and van Ypersele (2005) address the issue of 'gains from trade' and find for quadratic production functions that "… fiscal competition erodes some, but not all, of the gains from liberalization." (ibidem, p. 259). As we will present cases of trade losses in the present paper, Peralta and van Ypersele's result suggests that it is the *combination* of asymmetric tax competition and labor market distortions that has the potential of rendering capital market integration unfavorable for some countries.

Taking the time-honored free-trade paradigm as a reference, there is, of course, a large literature on trade under various conditions of second best. In the present context the contributions of Kemp and Negishi (1970) and Eaton and Panagariya (1979) are worth mentioning who focus on gains from trade when commodities are taxed and factor markets are distorted. However, they do not model capital-tax competition. More recently, the issue of gains from trade has been linked to capital-tax competition, e.g. by Kessler et al. (2003) and Lockwood and Makris (2006), who analyze capital-tax financed redistribution policies and voting in economies without labor market distortions. Aloi et al. (2009) present a model of two countries which are identical except that the labor market in one country is perfectly competitive and unionized in the other country. They determine conditions under which either country prefers autarky to capital market integration. In their model, no tax competition takes place.

Summing up, to our knowledge the consequences of capital market integration in a multi-country economy with labor market distortions and asymmetric capital-tax competition have not yet been analyzed in the literature. The present paper aims to fill that gap. It considers 'small countries' as in Zodrow and Mieszkowski (1986) and Wilson (1986) rather than modeling governments playing Nash in tax rates. With the concept of rigid-wage unemployment our model relates most closely to Ogawa et al. (2006). We deviate from their approach by dropping the issue of public-good provision[3], by considering heterogeneous countries and by addressing the consequences of moving from autarky to trade and capital-tax competition. Fuest and Huber (1999) and later Ogawa et al. (2006) show that a government's optimal capital tax

[2] A limiting case is Sato and Thisse (2007) who consider a labor market with heterogeneous skills, costly training and the need of matching the firms' skill needs. Yet their analysis relates to full employment except for a hint in the concluding remarks that fiscal competition might well trigger unemployment in a country.

[3] The issue of optimal provision of public goods is dropped in many contributions to the tax competition literature such as Leite-Monteiro et al. (2003), Eggert and Goerke (2004), Peralta and van Ypersele (2005) and Sato and Thisse (2007). Suppressing the fiscal purpose of capital taxation allows isolating the welfare-maximizing government's incentive to stimulate or discourage the use of capital in production.

rate may be positive or negative depending on whether capital and labor are substitutes or complements in production[4]. These properties of the production technology will turn out to have a major impact on the allocative consequences of capital market integration. That is why we will analyze technologies with capital and labor being either substitutes or complements. Ultimately, it is an empirical issue, of course, what the relevant production technology is like. However, as the pertaining empirical evidence is quite complex, if not ambiguous, e.g. Griliches (1969), Bergström and Panas (1992) and Duffy et al. (2004), it appears to be appropriate and necessary clarifying the analytical consequences of alternative assumptions on production technologies.

The paper is organized as follows. After having introduced the model in Section 2, we identify conditions in Section 3 under which countries gain or lose in the transition from autarky to trade if capital is internationally mobile and governments do *not* tax capital. In Section 4 we first characterize an individual government's optimal capital tax policy and show that it increases domestic employment for some given world market rate of interest. Then we investigate the allocative displacement effects that occur when the economy moves from trade without capital taxation to capital-tax competition. Under certain conditions that transition turns out to be welfare decreasing for some countries. Section 5 combines the results from the two previous sections and identifies conditions under which some countries suffer a welfare loss and a rise in unemployment in the transition from autarky to tax competition. Section 7 concludes. Formal proofs of all propositions are delegated to the Appendix.

2 The model: rigid wages, mobile capital and capital taxation

Consider an *n*-country economy in which each country $i = 1, \ldots, n$ produces the amount

$$y_i = Y^i\left(k_i, \ell_i\right) \tag{1}$$

of a consumption good by means of capital input k_i and labor input ℓ_i according to the strictly concave[5] production function Y^i that exhibits positive first derivatives. The consumption good and capital are traded on competitive world markets at price $p_y \equiv 1$ and interest rate

[4] If the output $Y\left(k, \ell\right)$ is produced with capital input *k* and labor input ℓ, capital and labor are said to be substitutes in production, if $Y_{k\ell} < 0$, and complements, if $Y_{k\ell} > 0$.

[5] Assuming strictly concave production functions is indispensible because otherwise we would not obtain well-defined factor demand functions. See equation (2) below. Unfortunately, in their general form strictly concave production functions give limited insights only. Therefore, we will later consider more specific parametric functional forms as well.

r, respectively. Immobile labor is traded on domestic markets at the wage rate w_i, $i = 1, \ldots, n$. The governments of all countries tax capital at the source such that the 'aggregate' producer in country i faces the after-tax rental rate of capital, $\rho_i := r + t_i$, where t_i is the sign-unconstrained[6] capital tax rate. Firm i maximizes profits $\pi_i := y_i - w_i \ell_i - \rho_i k_i$ as a price taker giving rise to the standard factor demand functions

$$k_i = K^i(\rho_i, w_i) \qquad \text{and} \qquad \ell_i = L^i(\rho_i, w_i), \tag{2}$$

where[7] $K^i_\rho = Y^i_{\ell\ell}/D < 0$, $K^i_w = -Y^i_{k\ell}/D$, $L^i_w = Y^i_{kk}/D < 0$, $L^i_\rho = -Y^i_{k\ell}/D$, and $D := Y^i_{kk}Y^i_{\ell\ell} - -\left(Y^i_{k\ell}\right)^2 > 0$; $Y^i_{k\ell}$ may be positive or negative.

Except for brief references to the benchmark model with flexible wage rates we focus exclusively on scenarios of persistent rigid wages that are sufficiently high as to make all countries suffer from unemployment. More formally, denote by m_i the number of consumers residing in country i, let each consumer offer one unit of labor and consider situations of excess supply of labor,[8]

$$m_i > L^i(\rho_i, w_i) \qquad \text{for all } i = 1, \ldots, n \tag{3}$$

According to (3), $m_i - L^i(\rho_i, w_i) > 0$ consumers are unemployed and the $L^i(\rho_i, w_i)$ jobs offered by firm i are randomly allocated to consumers. As consumers spend their income on a single consumption good only we can do without utility functions.

The national income of country i is $x_i := r\bar{k}_i + \pi_i + t_i k_i + w_i \ell_i$, where $\bar{k}_i$ is country i's aggregate capital endowment and where profits π_i and the tax revenues $t_i k_i$ are recycled to the consumers[9]. Combining (1), (2) and $\pi_i = y_i - w_i \ell_i - \rho_i k_i$ yields national income (= welfare) x_i as a function of r, t_i and w_i:

$$X^i(t_i, r, w_i) := Y^i\left[K^i(r + t_i, w_i), L^i(r + t_i, w_i)\right] - r\left[K^i(r + t_i, w_i) - \bar{k}_i\right]. \tag{4}$$

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[6] If $t_i < 0$, capital is subsidized. To avoid clumsy wording we will use the term tax irrespective of the sign of t_i.

[7] Capital letters denote functions and subscripts to capital letters denote first derivatives. To simplify notation we write $Y^i_{k\ell}$ instead of $Y^i_{k_i\ell_i}$ etc.

[8] More precisely, (3) is assumed to hold in all equilibria to be specified below.

[9] We need not specify the shares of profits and tax revenues allocated to individual consumers because we refrain from focusing on utility distributions.

The government of country i = 1, …, n is supposed to maximize national income $X^i\left(t_i, r, w_i\right)$ with respect to the capital tax rate t_i. As the definition of $X^i\left(t_i, r, w_i\right)$ in (4) shows, it accounts for the impact of tax variations on its firm's factor demands (2) but takes as given the world price, r, of capital.

For predetermined tax rates $t_1,...,t_n$ the condition for clearing the world capital market is

$$\sum_j \bar{k}_j = \sum_j K^j\left(r + t_j, w_j\right). \tag{5}$$

If (5) is satisfied, the world market for the consumption good is also cleared which follows from summing (4) over all i. The concept of general equilibrium for the n-country economy is straightforward: For given capital endowments $\bar{k}_1,...,\bar{k}_n$ and for persistent rigid wage rates $w_1,...,w_n$ a *tax-competition equilibrium* with unemployment is formally determined by the set $E := \left\{t_1,...,t_n, r, \left(x_i, y_i, k_i, \ell_i\right)_{i=1,...,n}\right\}$ where the allocation $\left(x_i, y_i, k_i, \ell_i\right)_{i=1,...,n}$ and the interest rate r satisfy the equations (1) – (5) for $\left(t_1,...,t_n\right)$, and where government i = 1, …, n chooses its tax rate t_i as to maximize (4).

It will turn out to be useful to consider also equilibria $E := \left\{t_1,...,t_n, r, \left(x_i, y_i, k_i, \ell_i\right)_{i=1,...,n}\right\}$ in which tax rates are exogenously fixed rather than optimally chosen by the governments. We will call such equilibria *constant-tax trade equilibria* in contrast to tax-competition equilibria as defined in the last paragraph. Note that the *no-tax trade equilibrium* $\left(t_1 = ... = t_n \equiv 0\right)$ is a special constant-tax trade equilibrium. When we later investigate the incidence of tax competition, we will exploit an *equivalence* between tax-competition equilibria and constant-tax trade equilibria which arises because due to (5) the capital market equilibrium depends on r and $t_1,...,t_n$ through $\rho_1,...,\rho_n$ only. In formal terms, we state that equivalence in

Proposition 1 (Neutrality of uniform variations in tax rates).
If $E_\tau := \left\{t_{1\tau},...,t_{n\tau}, r_\tau, \left(x_{i\tau}, y_{i\tau}, k_{i\tau}, \ell_{i\tau}\right)_{i=1,...,n}\right\}$ *is a tax-competition equilibrium or a constant-tax trade equilibrium,* $E_\theta := \left\{t_{1\tau} + \theta,...,t_{n\tau} + \theta, r_\tau - \theta, \left(x_{i\tau}, y_{i\tau}, k_{i\tau}, \ell_{i\tau}\right)_{i=1,...,n}\right\}$ *is a constant-tax trade equilibrium for all* $\theta < r$.

Proposition 1 is a standard result in tax incidence theory. Uniform variations in capital tax rates are non-distortionary because the total supply of capital $\left(=\sum_j \bar{k}_j\right)$ is perfectly price *in*elastic. Thus uniform changes in all tax rates can be exactly offset by changes in the interest rate of equal size and opposite sign. Proposition 1 will be used in the proof of our main result in Section 4.2 below.

3 From autarky to trade without taxation

Our first step toward investigating gains or losses from tax competition is to explore the allocative changes that occur when the countries move from autarky to trade in the absence of capital taxation. For the model introduced in the previous section, the reference scenario of autarky is straightforward. All capital markets are national and (5) is replaced by $\bar{k}_i = K^i\left(r_i + t_i, w_i\right)$ for $i = 1, \ldots, n$ with r_i denoting the interest rate in country *i*. Note first that in autarky capital taxation is non-distortionary because the supply of capital is perfectly inelastic in each country. Hence we set $t_i \equiv 0$, for convenience. Since we allow countries to differ in their fundamentals 'capital endowments', 'production technologies' and 'wage rates', the equilibrium interest rate in autarky will generally differ across countries. Using the general functional form (1) of the production function it is hard to specify properties of the mapping from the fundamentals to the autarkic equilibrium interest rate. We therefore resort to CES production functions in

Proposition 2 (Determinants of the size of the equilibrium interest rate in autarky)

Suppose capital is untaxed and country i's production function is CES, i.e. it satisfies

$$Y^i\left(k_i, \ell_i\right) = a_{oi}\left(a_{ki}k_i^{-e_i} + a_{\ell i}\ell_i^{-e_i}\right)^{-\frac{b_i}{e_i}}, \qquad (6)$$

where $a_{oi} > 0$, $a_{ki} > 0$, $a_{\ell i} > 0$, $\sigma_i > 0$, $\sigma_i \neq 1$, $b_i \in \left]0,1\right[$, *and* $e_i := \left(1-\sigma_i\right)/\sigma_i$.

For any given elasticity of substitution, $\sigma_i > 0, \sigma_i \neq 1$, *country i's autarkic equilibrium interest rate,* r_{ia}, *is decreasing in its capital endowment,* $\bar{k}_i$.

Following an increase in the rigid wage rate, r_{ia} *rises / remains unchanged / declines depending on whether* σ_i *is greater than / equal to / smaller than* $c_i = 1/\left(1-b_i\right) > 1$.

Clearly, expanding the capital endowment increases capital abundance and hence reduces the price for capital ($r_{ia} \downarrow$). Increasing the wage rate makes capital scarcer ($r_{ia} \uparrow$) or less scarce ($r_{ia} \downarrow$) depending on whether capital and labor are substitutes ($\sigma_i > c_i$) or complements ($\sigma_i < c_i$). Proposition 2 can be conveniently used to compare the autarkic equilibrium interest rate of different countries whose production functions are CES. To see that suppose some countries *i* and *j* are characterized by the parameters $\left(\bar{k}_i, w_i\right)$ and $\left(\bar{k}_j, w_j\right)$ and observe that implicitly Proposition 2 defines a function, say R^h, such that $r_{ha} = R^h\left(\bar{k}_h, w_h\right)$ for *h* = *i*, *j*. If both countries use the same production function ($R^i = R^j = R$), they have the same autarkic interest rate ($r_{ia} = r_{ja}$), if and only if $R\left(\bar{k}_i, w_i\right) = R\left(\bar{k}_j, w_j\right)$. Moreover, the inequality $r_{ia} > r_{ja}$ holds, if, ceteris paribus, either { $\bar{k}_i < \bar{k}_j$ } or { $w_i > w_j$ and $\sigma > c$ } or { $w_i < w_j$ and $\sigma < c$ }.

Suppose now that all countries have attained their autarkic equilibrium and the borders are subsequently opened for trade in capital and the consumption good while all governments refrain from taxation. It is straightforward from (5) (with $t_i \equiv 0$ for all *i*) that the no-tax trade equilibrium interest rate, denoted r_o, satisfies $r_o \in \left]r_a^{\min}, r_a^{\max}\right[$, when $r_a^{\min}$ is the smallest and $r_a^{\max}$ is the largest autarkic equilibrium interest rate of all countries and $r_a^{\min} < r_a^{\max}$. For any country *i* with autarkic interest rate r_{ia} the allocative consequences of the transition from autarky to the no-tax trade equilibrium clearly depend on the sign of the difference $r_o - r_{ia}$. We take this difference as our point of departure for analyzing the impact of moving from autarky to the no-tax trade equilibrium and explicitly allow for different production technologies. The results are summarized in

Proposition 3 (Transition from autarky to no-tax trade)

Suppose all governments refrain from taxation ($t_i \equiv 0$ for all i) and consider the transition of the n-country economy from its autarky equilibrium (subscript a) to its zero-tax trade equilibrium (subscript o).

(i) The allocative impacts of that transition are summarized in Table 1.

(ii) Suppose the cases 1 or 7 in Table 1 apply and the production functions are Cobb-Douglas, defined by

$$Y^i\left(k_i,\ell_i\right)=k_i^{\alpha_i}\ell_i^{\beta_i} \quad \textit{for all i with } \alpha_i>0,\ \beta_i>0 \textit{ and } \alpha_i+\beta_i=:b_i<1. \tag{7}$$

If the production functions are the same across countries ($\alpha_i=\alpha$ *and* $\beta_i=\beta$ *, all i) then the following equivalences hold:*

$$\bar{k}_i \begin{Bmatrix}>\\=\\<\end{Bmatrix} \omega_i\bar{k}_\varnothing \Leftrightarrow r_o \begin{Bmatrix}>\\=\\<\end{Bmatrix} r_{ia} \Leftrightarrow \ell_{io} \begin{Bmatrix}<\\=\\>\end{Bmatrix} \ell_{ia} \Leftrightarrow x_{io} \begin{Bmatrix}<\\=\\>\end{Bmatrix} x_{ia}, \tag{8}$$

$$\textit{where } \bar{k}_\varnothing := \frac{\sum_j \bar{k}_j}{n} \textit{ and } \omega_i := \frac{nw_i^{-\beta c}}{\sum_j w_j^{-\beta c}}.$$

(iii) If Case 6 in Table 1 applies and the production function is CES, (6), country i loses from trade if the elasticity of substitution in production, σ_i *, satisfies*

$$\sigma_i > c_i + \frac{\left(k_{io}-\bar{k}_i\right)\left(w_i+r_oq_{io}\right)}{w_ik_{io}} > c_i, \textit{ where } q_i := \left(\frac{a_{ki}}{a_{\ell i}}\right)^{\sigma_i}\left(\frac{w_i}{r}\right)^{\sigma_i} \textit{ and } c_i := \frac{1}{1-b_i} > 1.$$

	$r_o > r_{ia}$			$r_o = r_{ia}$	$r_o < r_{ia}$		
	$L_r^i < 0$	$L_r^i > 0$	$L_r^i = 0$	$L_r^i \gtreqless 0$	$L_r^i = 0$	$L_r^i > 0$	$L_r^i < 0$
Case No.	1	2	3	4	5	6	7
$\ell_{io}-\ell_{ia}$	-	+	0	0	0	-	+
$k_{io}-k_{ia}$	-	-	-	0	+	+	+
$y_{io}-y_{ia}$	-	?	-	0	+	?	+
$x_{io}-x_{ia}$	? (-)*)	+	+	0	-	? (-)*)	+

*) For details see the Propositions 2ii and 2iii

Table 1: Allocative impacts of the transition from autarky to no-tax trade

A few remarks on Table 1 are in order. The top row distinguishes the cases in which country *i*'s autarkic equilibrium interest rate, r_{ia}, is lower than, equal to or higher than the world market interest rate in the no-tax trade equilibrium, r_o. Note that one can combine that information with the results established in Proposition 2 to trace the difference $r_o - r_{ia}$ to differences in capital endowments and rigid wages[10]. The second row in Table 1 relates to properties of

[10] Recall that Proposition 2 allows identifying the capital endowments and the wage rates as determinants of the differences $r_o - r_{ia}$ for the case of identical CES production functions.

the labor demand of country *i*'s firm, which in turn are determined by properties of the production function as shown in equation (2). To be more specific, if production functions are Cobb-Douglas, (7), we have $Y^i_{k\ell} > 0$. Hence for Cobb-Douglas the Cases 1, 4 and 7 apply. CES production functions (6) exhibit $Y^i_{k\ell} \lesseqgtr 0$, if and only if $\sigma_i \gtreqless c_i$. Therefore, such functions are examples for the Cases 2, 4 and 6, if $\sigma_i > c_i$, for the Cases 1, 4 and 7, if $\sigma_i < c_i$, and for the knife-edge Cases 3 and 5, if $\sigma_i = c_i$. Case 4 is trivial but not entirely uninteresting for some conclusions in the next section.

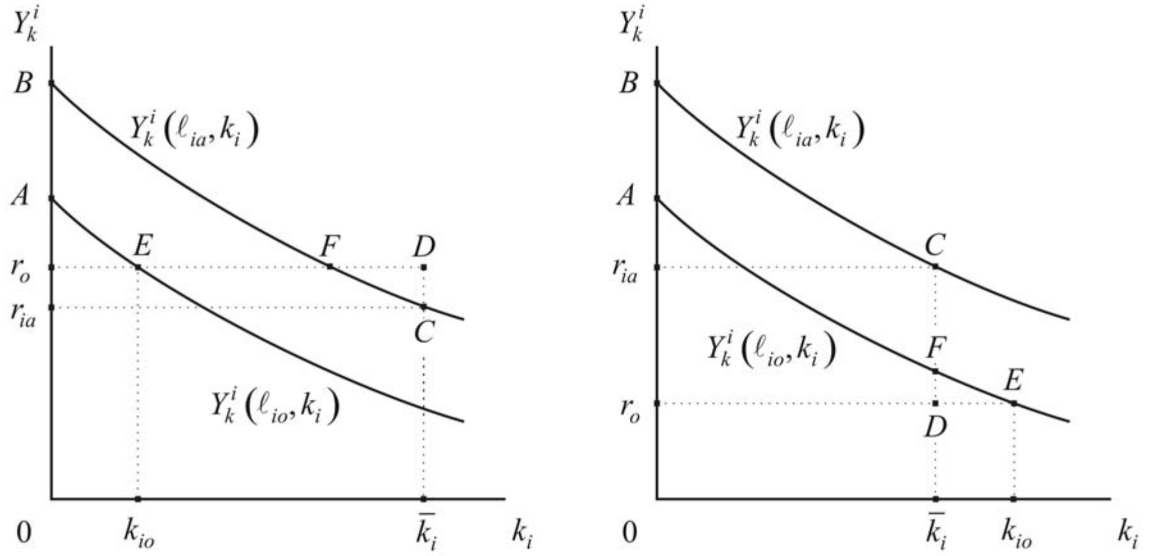

Figure 1: Illustration of the cases 1 and 6 in Table 1

The results listed in Table 1 can be easily illustrated. In Figure 1 we take up the Cases 1 and 6 of Table 1 that are unclear in the sign of $x_{io} - x_{ia}$ and leave the illustration of the other cases to the reader. In both panels of Figure 1, y_{ia} is given by the area $0BC\bar{k}_i$, and y_{io} is given by $0AEk_{io}$. In Case 1 capital is exported, and the value of these exports is equal to the area $\bar{k}_i DEk_{io}$ in the left panel of Figure 1. In Case 6 capital is imported, and the value of these imports is equal to the area $\bar{k}_i DEk_{io}$ in the right panel of Figure 1. It follows that in Case 1 [Case 6] we have $x_{io} = 0AED\bar{k}_i$ such that x_{io} is smaller than / equal to / greater than x_{ia}, if and only if the area *ABFE* [*ABCF*] is greater than / equal to / smaller than *DFC* [*DFE*].

According to Table 1 country *i* unambiguously gains from trade in the Cases 2, 3 and 7, while the welfare change in the Cases 1 and 6 remains unclear for general production functions. To gain additional insights in changes of national income in the unclear cases, we have specified the production technology in Proposition 3ii by Cobb-Douglas functions thus restricting the focus of Proposition 3ii to the Cases 1 or 7 of Table 1, as argued above.

The striking result of Proposition 3ii is that with Cobb-Douglas functions countries lose from trade – and suffer from higher unemployment - in Case 1 of Table 1 and they gain from trade – and enjoy higher employment - in Case 7 of Table 1.[11] Proposition 3iii demonstrates that countries with CES production functions may also lose from trade if the elasticity of substitution is large enough.

As mentioned above, for CES production functions we can use Proposition 2 to specify conditions on fundamentals under which a country will export or import capital after the borders are opened. For Cobb-Douglas functions Proposition 3ii establishes an even more informative clear relationship between the difference in interest rates, $r_{ia} - r_o$, on the one hand and capital endowments and wages *of all countries*, on the other hand. It is therefore worthwhile analyzing and interpreting (8) in some more detail. Suppose first the countries differ in their capital endowments only. If the level of rigid wages is the same in all countries, we have $\omega_i = 1$ for all *i* such that (8) is turned into

$$\bar{k}_i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \bar{k}_ø \Leftrightarrow r_o \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} r_{ia} \Leftrightarrow x_{io} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} x_{ia} .$$

Under these conditions country *i* loses [gains] from trade, if and only if its capital endowment exceeds [falls short of] the countries' average capital endowment, $\bar{k}_ø$. If capital is relatively abundant ($\bar{k}_i > \bar{k}_ø$), country *i*'s equilibrium interest rate in autarky is low ($r_{ia} < r_o$) and is bound to rise when capital is internationally traded. Recall that $K_r^i < 0$ holds for all strictly concave production functions, and that $L_r^i < 0$ holds because of (7). Therefore country *i* will use less capital and labor and will consequently produce less output, so much less, that the (new) revenues from exporting capital do not compensate for the reduction in output.[12] Conversely, countries with a relatively small capital endowment ($\bar{k}_i < \bar{k}_ø$) will face a lower interest rate in trade equilibrium ($r_{ia} > r_o$) which, in turn, boosts the input of both capital and labor such that the extra value of output is greater than the expenditure on capital imports. This re-

[11] Case 7 applies for more general production functions as well.

[12] Note that if the wage rate were flexible, w_i would shrink to restore the full employment equilibrium in the labor market which would then tend to boost domestic production.

sult confirms the gains from trade stated already under more general conditions in the Case 7 of Table 1.

To focus on the role played by rigid wages, suppose all countries are endowed with the same amount of capital, $\bar{k}_1 = ... = \bar{k}_n = \bar{k}_ø$, but differ with respect to their wage rates. Invoking (8), we establish the following equivalences:

$$1 \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \omega_i \;\Leftrightarrow\; w_i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \tilde{w} \;\Leftrightarrow\; r_o \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} r_{ia} \;\Leftrightarrow\; x_{io} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} x_{ia}, \text{ where } \tilde{w} := \left(\sum_j \frac{w_j^{-\beta c}}{n} \right)^{-\frac{1}{\beta c}}.$$

Hence in that case, there is a positive number $\tilde{w}$, the same for all countries, such that country i loses / is equally well off / gains from trade, if and only if its wage w_i is above / equal to / below $\tilde{w}$. The magnitude of the threshold value $\tilde{w}$ is unclear. We would like to know, in particular, how $\tilde{w}$ relates to $w_Ø := \sum_j w_j / n$, the average wage. One can show[13] that

$$\tilde{w} > w_Ø, \text{ if and only if } n > \left[\left(\sum_j w_j \right)^{\beta c} / \sum_j w_j^{\beta c} \right]^{\frac{1}{1+\beta c}} > 0.$$

In other words, in an economy with a sufficiently large number of countries only those countries lose from trade whose wage rate is well above the average wage rate. Note that the threshold value which needs to be exceeded for the number of countries to be large enough depends on the level of wage rates and on parameters of the Cobb-Douglas technology.

To highlight the consequences of rigid wages in the transition from autarky to trade from another perspective, we establish

Proposition 4 (Transition from autarky to no-tax trade with *flexible* wages)

Suppose that all governments refrain from taxation ($t_i \equiv 0$ for all i), that wages are flexible, and that production functions are Cobb-Douglas, (7), the same across countries. The transition of the n-country economy from autarky (subscript a) to its no-tax trade equilibrium (subscript o) is characterized by

$$\bar{k}_i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \bar{m}_i \bar{k}_ø \;\Leftrightarrow\; r_o \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} r_{ia} \;\Leftrightarrow\; w_{io} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} w_{ia} \;\Leftrightarrow\; x_{io} \begin{Bmatrix} > \\ = \\ > \end{Bmatrix} x_{ia}, \text{ where } \bar{m}_i = \frac{n m_i^{\beta\gamma}}{\sum_j m_j^{\beta\gamma}} \qquad (9)$$

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[13] The proof is provided at the end of the Appendix.

As expected, Proposition 4 confirms for the model at hand the general gains-from-trade result for economies with a full set of perfectly competitive markets. In the present context the purpose of Proposition 4 is to use (9) as a reference for further interpretation of the equivalences (8) from Proposition 3ii. To keep the exposition simple, we restrict that comparison to wage rates and labor endowments satisfying the condition $\bar{\omega}_i = \bar{m}_i = 1$, which holds, e.g., if we set $w_1 = w_2 = ... = w_n$ in Proposition 3ii and $m_1 = m_2 = ... = m_n$ in Propositions 4. Obviously, in that case the first equivalences in (8) and (9) are the same. However, (9) shows that maintaining full employment of labor after opening the borders results in a lower wage rate in capital-rich countries and a higher wage rate in capital-poor countries. In the capital-rich country we find that $w_{io} < w_{ia} < w_i$, where the wage rates w_{io} and w_{ia}, respectively, are equilibrium rates in autarky and trade in the flexible-wage scenario of Proposition 4, while w_i is the rigid wage rate of Proposition 3ii. Hence when the wage rate is rigid, allowing for trade widens the difference between the rigid wage rate and the respective equilibrium wage rates in case of flexible wages $\left[\left(w_i - w_{io}\right) > \left(w_i - w_{ia}\right)\right]$. As a consequence, the labor market disequilibrium is aggravated and unemployment rises, which in turn reduces national income (last equivalence in (8)). In contrast, for the capital-poor country we find the inequalities $w_{ia} < w_{io} < w_i$ which imply that opening the borders for trade reduces the difference between the rigid wage rate and the respective equilibrium wage rates in case of flexible wages $\left[\left(w_i - w_{io}\right) < \left(w_i - w_{ia}\right)\right]$. Therefore trade diminishes the labor market disequilibrium and thus raises employment as well as national income.

4 From trade without taxation to capital tax competition

Having clarified the allocative consequences of the transition from autarky to trade without taxation in the previous section we now take as our point of departure the trade equilibrium without taxation and analyze the allocative impact of tax competition. As a first step toward that end it is necessary and useful to take a closer look at the government's optimization calculus.

4.1 Properties and implications of an individual government's optimal capital tax

By assumption, governments choose their capital tax as to secure the maximum national income (= welfare), (4), for any given interest rate. The resultant optimal tax rates are characterized in

Proposition 5 (Properties of the optimal capital tax).

Government i's optimal tax rate satisfies the condition[14, 15]

$$t_i = -\frac{w_i L^i_\rho}{K^i_\rho} = \frac{w_i Y^i_{\ell k}}{Y^i_{\ell\ell}}. \qquad (10)$$

If the production functions are CES , (6), or Cobb-Douglas, (7), respectively, (10) is turned into

$$t_i = \frac{\sigma_i - c_i}{\left(a_{ki}/a_{\ell i}\right)^{\sigma_i} c_i w_i^{\sigma_i - 1}\rho_i^{-\sigma_i} + \sigma_i \rho_i^{-1}} \qquad \text{and} \qquad t_i = -\beta_i r < 0. \qquad (11)$$

where $a_{ki} > 0$, $a_{\ell i} > 0$, $\sigma_i \neq 1$, $c_i := 1/\left(1-b_i\right)$ *and* $\rho_i := r + t_i$.

Since $L^i_r \gtreqless 0 \Leftrightarrow Y^i_{k\ell} \lesseqgtr 0$, the optimal tax rate is negative if the production function is Cobb-Douglas. In case of CES production functions we have $Y^i_{k\ell} \lesseqgtr 0$, if and only if $\sigma_i \gtreqless c_i$, and therefore $t_i \lesseqgtr 0$ if and only if $\sigma_i \lesseqgtr c_i$. When production functions are Cobb-Douglas, the optimal tax rates are uniform across countries, if and only if the technology is the same in all countries. Interestingly, differences with respect to their capital endowments and wage rates do not translate into differences in optimal tax rates. When production functions are CES, the optimal tax rates are the same across countries, if and only if all countries have identical production functions *and* identical wage rates. Differences in capital endowments do not matter. Cases of uniform identical tax rates will be of some interest in Section 4.2 below.

By presupposition, if the government sets its tax rate according to (10), the country's income is maximized. But it is not clear how income maximization changes the level of employment, in particular, if we allow for different signs of $Y^i_{\ell k}$. We provide the answer in

Proposition 6 (Income maximization always promotes employment)

[14] Note that in (10) the terms L^i_ρ and K^i_ρ are functions of t_i.

[15] The equation (10) has been derived by Fuest and Huber (1999) and later by Ogawa et al. (2006). Fuest and Huber (1999) employ a right-to-manage model of wage bargaining for the labor market. When they model governments maximizing utility for a given wage rate; they derive an optimal tax rate in their equation (26) equal to (10) for $e' = 0$. Ogawa et al. (2006) derive (10) in a setup where governments levy a head tax *and* a capital tax and finance a public good. Irrespective of these differences, the rationale of levying the capital tax in the present model is the same as in Fuest and Huber (1999) and Ogawa et al. (2006).

Let t_i^* be the optimal tax rate of government i (satisfying (10)), if some interest rate r_o prevails. Irrespective of the sign of t_i^* (i.e. the sign of $Y_{\ell k}^i$) it is true that the government's optimal tax rate increases employment along with national income as compared to the no-tax strategy $\left[L^i\left(r_o + t_i^*, w_i\right) > L^i\left(r_o, w_i\right)\right]$.

To see the rationale of that synergism observe that the change in national income resulting from a small variation in the tax rate is given by

$$X_t^i = Y_k^i K_\rho^i + Y_l^i L_\rho^i - rK_\rho^i = \left(\rho_i - r_o\right) K_\rho^i + w_i L_\rho^i = t_i K_\rho^i + w_i L_\rho^i ,$$

if the wage rate is rigid. If the labor market were perfectly competitive, the term $Y_\ell^i L_\rho^i$ with $Y_\ell^i = w_i$ would be absent. As an immediate implication optimality would require to abstain from taxation ($X_t^i = 0 \Leftrightarrow t_i = 0$). However, since the wage rate is rigid, the derivative of national income with respect to the tax rate is equal to the sum of the components $\left(\rho_i - r_o\right) K_\rho^i$ and $w_i L_\rho^i$. To interpret these terms suppose capital is taxed ($t_i = \rho_i - r_o > 0$) in the initial situation and the tax rate is increased, $dt_i = d\rho_i > 0$. We then observe the partial marginal benefit effect $-rK_\rho^i > 0$ of increased capital export revenues or reduced expenditures on capital imports, and $\rho_i K_\rho^i < 0$ is the marginal cost of reduced output. Hence $\left(\rho_i - r_o\right) K_\rho^i < 0$. The sign of $w_i L_\rho^i$ depends on whether $Y_{\ell k}^i > 0$ or $Y_{\ell k}^i < 0$. If $Y_{\ell k}^i > 0$, the drop in capital input (following $dt_i = d\rho_i > 0$) diminishes the marginal productivity of labor, Y_ℓ^i, ceteris paribus. However, the first-order condition for profit maximization, $Y_\ell^i = w_i$, and the rigid wage rate w_i induce the firm to 'restore' the former level of Y_ℓ^i. It does so by reducing its labor input, $L_\rho^i < 0$ such that $w_i L_\rho^i < 0$ is another marginal cost term yielding $X_t^i = \left(\rho_i - r_o\right) K_\rho^i + w_i L_\rho^i < 0$, if $t_i > 0$. To attain the first-order condition for a maximum of x_i it is therefore necessary to choose $\rho_i < r$ and hence a capital subsidy, $t_i < 0$, in this scenario. Analogous arguments apply to the case $Y_{\ell k}^i < 0$.

Recall the qualification of Proposition 6 that government *i*'s promotion of employment is subject to the condition that it takes the interest rate as given. To fix our ideas take as the baseline the no-tax trade equilibrium with its equilibrium interest rate r_o and suppose that all governments introduce their optimal capital tax taking the interest rate r_o as given. The clear implication of Proposition 6 is that all countries will raise their income as well as their level of employment. The bad news is, however, that the world capital market will not be in equilibrium anymore. Hence to restore equilibrium the world interest rate will have to change, in general, giving rise to the possibility that tax competition may partly offset or perhaps even defeat the government's effort to promote both welfare and employment. We will study that issue in the following subsection.

4.2 The impact of tax competition as compared to trade without taxation

Starting from the scenario of trade without taxation we wish to determine how tax competition changes the allocation attained in the no-tax trade equilibrium. In other words, we now take as a benchmark the no-tax trade equilibrium and compare the pertaining allocation with the allocation attained in tax-competition equilibrium. Analogous to the role of the difference $r_{ia} - r_o$ in case of the transition from autarky to trade (Proposition 3), the change in the interest rate from r_o (no-tax trade equilibrium) to r_τ (tax-competition equilibrium) will now turn out to play an important role. It is therefore important to know what determines the sign of the difference $r_\tau - r_o$. A clear-cut answer is possible, if $\operatorname{sign} t_{i\tau}$ is the same for all *i*. If $t_{i\tau} < 0$ for all *i*, then $r_\tau > r_o$ because $\sum_j \bar{k}_j < \sum_j K^j\left(r_o + t_{j\tau}, w_j\right)$. In that case *r* must rise to restore equilibrium on the capital market, (5). Likewise, if $t_{i\tau} > 0$ for all *i*, then $r_\tau < r_o$ because $\sum_j \bar{k}_j > \sum_j K^j\left(r_o + t_{j\tau}, w_j\right)$. Now *r* must decline to restore equilibrium on the capital market. If technologies are mixed such that $t_{i\tau} < 0$ for some countries and $t_{i\tau} > 0$ for others, the difference $r_\tau - r_o$ may take on either sign.

Keeping these preliminaries in mind we are now ready to compare the allocations of the no-tax trade equilibrium and the tax-competition equilibrium in

Proposition 7 (Transition from trade without taxation to tax competition)

Consider the no-tax trade equilibrium $E_o := \left\{t_{1o} = 0, ..., t_{no} = 0, r_o, \left(x_{io}, y_{io}, k_{io}, \ell_{io}\right)_{i=1,...,n}\right\}$ *and the associated tax-competition equilibrium* $E_\tau := \left\{t_{1\tau}, ..., t_{n\tau}, r_\tau, \left(x_{i\tau}, y_{i\tau}, k_{i\tau}, \ell_{i\tau}\right)_{i=1,...,n}\right\}$, *where* $t_{i\tau}$ *satisfies (10) for all i. The allocation* $\left(x_{i\tau}, y_{i\tau}, k_{i\tau}, \ell_{i\tau}\right)_{i=1,...,n}$ *of the tax-competition equilibrium deviates from the no-tax trade allocation* $\left(x_{io}, y_{io}, k_{io}, \ell_{io}\right)_{i=1,...,n}$ *as shown in Table 2.*

sign $(r_\tau - r_o)$	sign L^i_ρ	sign $(t_{i\tau} + \triangle r)$	No.	$\ell_{i\tau} - \ell_{io}$	$k_{i\tau} - k_{io}$	$y_{i\tau} - y_{io}$	$x_{i\tau} - x_{io}$
$r_\tau > r_o$	$L^i_\rho < 0$	$t_{i\tau} + \triangle r < 0$ *)	1	+	+	+	+
		$t_{i\tau} + \triangle r > 0$	2	-	-	-	-
	$L^i_\rho > 0$	$t_{i\tau} + \triangle r > 0$	3	+	-	?	-
$r_\tau \neq r_o$	$L^i_\rho \neq 0$	$t_{i\tau} + \triangle r = 0$	4	0	0	0	0
$r_\tau = r_o$	$L^i_\rho = 0$	$t_{i\tau} = 0$	5	0	0	0	0
$r_\tau < r_o$	$L^i_\rho < 0$	$t_{i\tau} + \triangle r < 0$	6	+	+	+	-
	$L^i_\rho > 0$	$t_{i\tau} + \triangle r > 0$	7	+	-	?	+
		$t_{i\tau} + \triangle r < 0$	8	-	+	?	-

*) $\triangle r := r_\tau - r_o$

Table 2: Transition from trade without taxation to tax competition

Table 2 calls for some comments. The very first column lists the possible constellations of the respective equilibrium interest rates analogous to the top row in Table 1. As in Table 1, each of the Cases 1 - 8 of Table 2 is characterized by the sign of L^i_ρ (second column) but a new attribute is the sign of $t_{i\tau} + \triangle r$ (third column). The meaning and role of the term $t_{i\tau} + \triangle r$ is made precise in the proof of Proposition 7 in the Appendix. There we show that if the interest rate r_o of the no-tax trade equilibrium prevails (rather than the interest rate r_τ of the tax-competition equilibrium) and if all governments *j* choose $t_{j\tau} + \triangle r$, then the allocation of the tax-competition equilibrium is attained.[16]

It is informative to combine the results of Table 2 with the information provided in Proposition 5. Recall from (11) that $L^i_\rho < 0$ holds, in particular but not only, for Cobb-Douglas functions and CES functions satisfying $\sigma_i < c_i$. Yet if the technology is Cobb-Douglas, the Cases 1 and 2 apply only if the Cobb-Douglas functions differ across countries. Otherwise Case 4 applies. Case 5 represents a knife-edge case corresponding to the Cases 3 and 5 of Table 1 which happens to occur when all production functions are when all production functions are

[16] Consider for example the constellation $r_\tau > r_o$ (or $\triangle r = r_\tau - r_o > 0$) and $L^i_\rho < 0$ in Table 2. Owing to $L^i_\rho < 0$ we have $t_{i\tau} < 0$ such that $t_{j\tau} + \triangle r$ may take on either sign generating one of the Cases 1, 2 or 4.

CES with $\sigma_i = c_i$. The properties of production functions leading to the Cases 6, 7 and 8 in Table 2 can be indentified in analogy to the properties on the Cases 1, 2 and 3.

Note finally that if $r_\tau > r_o$ and $L^i_\rho > 0$ (Case 3), $t_{i\tau} + \vartriangle r < 0$ is not a feasible outcome. If $r_\tau < r_o$ and $L^i_\rho < 0$ (Case 6), $t_{i\tau} + \vartriangle r > 0$ is not a feasible outcome either. A necessary condition for the Cases 3 and 6 to occur is that there are technologies exhibiting $Y_{k\ell} > 0$ for some countries and $Y_{k\ell} < 0$ for others.

In those special tax-competition equilibria in which the tax rates $t_{1\tau}, ..., t_{n\tau}$ are uniform across countries (Case 4 and trivially Case 5 of Table 2)[17] the allocations of the tax-competition equilibrium E_τ and the no-tax trade equilibrium E_o coincide as a consequence of Proposition 1. Tax competition then has no allocative impact at all.

Consider next the tax incidence in the plausible case of tax competition where tax rates differ across countries. Focusing on employment we observe, rather unexpectedly, that variations in employment are clear in sign under all conditions: Unless tax rates are uniform (see above) employment either improves or shrinks. For either sign of the difference $r_\tau - r_o$ employment declines [increases] if $t_{i\tau} \cdot (t_{i\tau} + \vartriangle r) < 0$ [$t_{i\tau} \cdot (t_{i\tau} + \vartriangle r) > 0$]. The constellation $t_{i\tau} < 0$ and $(t_{i\tau} + \vartriangle r) > 0$ or vice versa occurs under two conditions: (i) $t_{i\tau}$ and $r_\tau - r_o$ must exhibit opposite signs and (ii) $t_{i\tau}$ must be sufficiently close to zero.

While all signs of changes in factor inputs and income are clear, the changes in the level of output are ambiguous in the Cases 3, 7 and 8 where sign $(\ell_{i\tau} - \ell_{io}) \cdot (k_{i\tau} - k_{io})$ is negative.

The striking result of Proposition 7 is that countries may lose from tax competition if the scenario of trade without taxation is taken as the baseline. Under the conditions specified in Proposition 7 country *i* may suffer a welfare loss and such a loss can occur under various assumptions. Country *i*'s technology may satisfy $Y^i_{k\ell} > 0$ (Cases 2 and 6) or $Y^i_{k\ell} < 0$ (Cases 3 and 8) or the constellation $r_\tau > r_o$ (Cases 2 and 3) or $r_\tau < r_o$ (Cases 6 and 8) may be given. To see the driving force for the welfare loss, consider Case 2 in Table 2 as an example. $t_{i\tau} + \vartriangle r$ will be positive, if country *i*'s tax rate $t_{i\tau}$ is negative but relatively small in absolute value (see

[17] Case 4 applies, e.g., when all countries produce with identical Cobb-Douglas functions or when all countries produce with identical CES functions *and* their rigid wage rates are identical. See our comments on Proposition 5.

the proof of Proposition 7)[18]. Hence in tax-competition equilibrium we find that $L^i\left(r_\tau + t_{i\tau}, w_i\right) = L^i\left(r_o + t_{i\tau} + \vartriangle r, w_i\right) < L^i\left(r_o, w_i\right)$ because $L^i_\rho < 0$ is presupposed in Case 2, and $K^i\left(r_\tau + t_{i\tau}, w_i\right) = K^i\left(r_o + t_{i\tau} + \vartriangle r, w_i\right) < K^i\left(r_o, w_i\right)$ because $K^i_\rho < 0$. The straightforward implication is that the output shrinks ($y_{i\tau} < y_{io}$). On the other hand, since $k_{i\tau} < k_{io}$, country i's value of exports rises or the cost of its imports declines. However that partial income increase is smaller than the loss from reduced production because the per unit cost of capital as an input, $r_o + t_{i\tau} + \vartriangle r$, is higher than r_o, the per unit revenue from increased capital exports or from reduced capital imports.

5 From autarky to tax competition

Proposition 3 scrutinized the shift from autarky to the no-tax trade equilibrium and Proposition 7 analyzed the allocative impact of moving from the no-tax trade equilibrium to the tax-competition equilibrium. It is therefore necessary combining both steps in an effort to answer the question what the allocative consequences are for individual countries of moving from autarky to tax competition. As for income changes, closer inspection of the Tables 1 and 2 reveals that the sign of those changes is unclear in several cases. Obviously, since the sign of the difference $x_{io} - x_{ia}$ is unclear in the Cases 1 and 6 of Table 1, the net welfare change from autarky to tax competition is bound to be ambiguous. However, even if the partial welfare effects in the Tables 1 and 2 are clear in sign, the net effect is also ambiguous, whenever the partial effects exhibit opposite signs. Although it is not possible to fully exploit the complex information presented in the Propositions 3 and 7, we restrict our attention to changes in unemployment and welfare and select some specific cases in

Proposition 8 (Losses from autarky to tax competition).

Consider the transition of the n-country economy from autarky to tax competition.

(i) Country i suffers a welfare loss, if all countries use identical Cobb-Douglas production functions , (7), and if $\bar{k}_i > \omega_i \bar{k}_\varnothing$ *holds.*

[18] For example, if all production functions are Cobb-Douglas and differ across countries with respect to their parameter β, we conclude from $\vartriangle r > 0$ (which is presupposed in Case 2) and $t_{i\tau} = -\beta_i r_\tau < 0$ from equation (11) that $\tilde{t}_{i\tau} = t_{i\tau} + \vartriangle r = -\beta_i r_\tau + \vartriangle r > 0$, if β_i is a sufficiently small component in $\left(\beta_1, \beta_2, ..., \beta_n\right)$.

(ii) *Country i suffers a welfare loss, if the production functions satisfy $Y^i_{k\ell} > 0$ for all i (as e.g. in case of Cobb-Douglas or CES with $\sigma_i < c_i$), if $r_{ia} = r_o$ and if $t_{i\tau}$ is small enough in absolute value relative to the other countries' optimal tax rates.*

(iii) *Country i suffers a welfare loss, if $r_{ia} > r_o$ (implying $k_{io} > \bar{k}_i$), if its production function is CES satisfying $\sigma_i > c_i + \left(k_{io} - \bar{k}_i\right)\left(w_i + r_o q_{io}\right) / w_i k_{io}$, and*

- ***either** if $Y^i_{k\ell} < 0$ for all i and $t_{i\tau}$ is small enough relative to the other countries' optimal tax rates,*
- ***or** if all countries use the same CES production function as country i and wage rates are the same across countries.*

(iv) *In all scenarios of the Propositions 8i, 8ii and 8iii country i suffers from increasing unemployment in the transition from autarky to tax competition.*

The principal message of Proposition 8 is that capital market liberalization with tax competition can lead to rising unemployment and welfare losses under various conditions. Welfare losses can be derived by combining in various ways the countries' fundamentals, i.e. their wage rates, capital endowments and production technologies. It is conceded that all cases presented in Proposition 8 make use of conditions that are more or less restrictive. Yet all these conditions are sufficient but not necessary. In our view, it is therefore safe to conjecture that losses from tax competition are not an elusive phenomenon. Take for example Proposition 8ii. Its range – and relevance - is certainly limited because the condition $r_{ia} = r_o$ is very special, if it is fulfilled at all for any country. However, a welfare loss will also occur if the condition $r_{ia} = r_o$ of Proposition 8ii is replaced by $r_{ia} \neq r_o$ as long as if the difference $\left| r_{ia} - r_o \right|$ is small enough. When the condition $r_{ia} = r_o$ (Case 4 of Table 1) is weakened in this non-rigorous way, welfare losses can also be identified in the Cases 3, 6 and 8 of Table 2. Moreover, in all cases of uniform optimal tax rates which we identified in our remarks on equation (11) some countries lose when moving from autarky to tax competition, if and only if they lose in the transition from autarky to trade without taxation.

7 Concluding remarks

We have shown that unemployment markedly changes the impact of capital market liberalization and capital-tax competition among heterogeneous countries as compared to the case of perfectly competitive labor markets. With autarky as the reference scenario, the introduction of international capital mobility and tax competition turned out to have the potential of reducing the welfare and/or of exacerbating unemployment in some countries. Hence such countries will not be in favor of capital market liberalization unless they succeed in removing wage rigidity. Since we allowed countries to differ with respect to capital endowments, rigid wage rates and production technologies, there is a great variety of outcomes and welfare changes which can hardly be characterized completely. Nonetheless, we identified a number of specific cases where countries suffer higher unemployment and a welfare loss during the transition from autarky to tax competition and traced the reasons for that outcome. As could be expected, less general assumptions on production functions yielded more informative results. For example, in case of Cobb-Douglas technology we were able to fully characterize the allocative displacement effects. We showed that in the transition from autarky to tax competition countries fare the better, ceteris paribus, the greater is their capital endowment or the lower is their rigid wage rate.

The rigid-wage assumption is a very simple and coarse way to model unemployment given the great variety of sophisticated and complex theories of non-competitive wage formation developed in labor economics (Nickel 1990). One small step in relaxing that assumption in future research work would be to retain downward rigidity but allow for upward flexible wages. More complex and arguably more realistic labor market theories have already been employed in some studies of capital-tax competition with labor-markets imperfections some of which we have referenced in the Introduction. However, as we pointed out, none of these studies tackles unemployment *and* heterogeneous countries. The trade-off between realistic complexity in modeling, tractability and informative insights appears to necessitate and warrant compromises.

References

Aloi, M., Leite-Monteiro, M., and Lloyd-Braga, T. (2009), "Unionized labor markets and globalized capital markets", *Journal of International Economics* 78, 149-153

Aronsson, T., and Wehke, S. (2008), "Public goods, unemployment and policy coordination", *Regional Science and Urban Economics* 38, 285-298

Bergström, V., and Panas, E. E. (1992), "How robust is the capital-skill complementarity hypothesis?", *The Review of Economics and Statistics 74, 540-546*

Bucovetsky, S. (1991), "Asymmetric tax competition", *Journal of Urban Economics* 30, 171-181

DePater, J. A., and Myers, G. M. (1994), "Strategic capital tax competition: A pecuniary externality and a corrective device", *Journal of Urban economics* 36, 66-78

Duffy, J., Papageorgiou, C. and Perez-Sebastian, F. (2004), "Capital-skill complementarity? Evidence from a panel of countries", *The Review of Economics and Statistics 86, 327-344*

Eaton, J., and Panagariya, A. (1979), "Gains from trade under variable returns to scale, commodity taxation, tariffs and factor market distortions", *Journal of International Economics* 9, 481-501

Eggert, W., and Goerke, L. (2004), "Fiscal policy, economic integration and unemployment", *Journal of Economics* 82, 137–167

Fuest, C., and Huber, B. (1999), "Tax coordination and unemployment", *International Tax and Public Finance* 6, 7-26

Griliches, Z. (1969), "Capital-skill complementarity", *The Review of Economics and Statistics 51*, 465-468

Kemp, M., and Negishi, T. (1970), "Variable returns to scale, commodity taxes, factor market distortions and their implications for trade gains", *The Swedish Journal of Economics* 72, 1-11

Kessler, A.S., Lülfesmann, C., and Myers, G. M. (2003), "Economic versus political symmetry and the welfare concern with market integration and tax competition", *Journal of Public Economics* 87, 847-865

Leite-Monteiro, M., Marchand, M., and Pestieau, P. (2003), "Employment subsidy with capital mobility", *Journal of Public Economic Theory* 5, 327-344

Lockwood, B., and Makris, M. (2006), "Tax incidence, majority voting and capital market integration", *Journal of Public Economics* 90, 1007-1025

Nickell, S. (1990), "Unemployment: A survey", *Economic Journal* 100, 391-439

Ogawa, H., Sato, Y., and Tamai, T. (2006), "A note on unemployment and capital tax competition", *Journal of Urban Economics* 60, 350–356

Peralta, S., and van Ypersele, T. (2005), "Factor endowments and welfare levels in an asymmetric tax competition game", *Journal of Urban Economics* 57, 258-274

Sato, Y., and Thisse, J.-F. (2007), "Competing for capital when labor is heterogeneous", *European Economic Review* 51, 2054-2079

Wilson, J. D. (1986), "A theory of interregional tax competition", *Journal of Urban Economics* 19, 296–315

Wilson, J. D. (1991), "Tax competition with interregional differences in factor endowments", *Regional Science and Urban Economics* 21, 423-451

Zodrow, G. R., and Mieszkowski, P. (1986), "Pigou, Tiebout, property taxation, and the underprovision of local public goods", *Journal of Urban Economics* 19, 356-370

Appendix

Proof of Proposition 1. In the tax-competition equilibrium $E_\tau := \left\{t_{1\tau},...,t_{n\tau}, r_\tau, \left(x_{i\tau}, y_{i\tau}, k_{i\tau}, \ell_{i\tau}\right)_{i=1,...,n}\right\}$ the capital market is in equilibrium, by presupposition: $\sum_j \bar{k}_j = \sum_j K^j\left(r_\tau + t_{j\tau}, w_j\right)$. Consider $\theta \in \left]-\infty, r\right[$ and define $\tilde{t}_{i\tau} := t_{i\tau} + \theta$ for all i and $\tilde{r}_\tau := r_\tau - \theta$. Then we obviously have $\sum_j \bar{k}_j = \sum_j K^j\left(r_\tau + t_{j\tau}, w_j\right) = \sum_j K^j\left(\tilde{r}_\tau + \tilde{t}_{j\tau},\ w_j\right)$. □

Proof of Proposition 2. Under conditions of perfect competition the elasticity of substitution reads $\sigma_i = \frac{dq_i}{q_i} \cdot \frac{w_i / r}{d\left(w_i / r\right)}$, where $q_i := \frac{k_i}{\ell_i}$. Rearrange this equation to obtain $\sigma_i \hat{w}_i - \sigma_i \hat{r}_i = \hat{q}_i$.

Furthermore, consider $\hat{q}_i = \hat{k}_i - \hat{\ell}_i$ as well as $\hat{\ell}_i = -\frac{c_i w_i + q_i r_i \sigma_i}{w_i + q_i r_i} \hat{w}_i - \frac{\left(c_i - \sigma_i\right) q_i r_i}{w_i + q_i r_i} \hat{r}_i$ in $\sigma_i \hat{w}_i - \sigma_i \hat{r}_i = \hat{q}_i$ to obtain after some rearrangements of terms

$$\hat{r}_i = -\frac{w_i + q_i r_i}{\sigma_i w_i + c_i q_i r_i} \hat{k}_i + \frac{\left(\sigma_i - c_i\right) w_i}{\sigma_i w_i + c_i q_i r_i} \hat{w}_i,$$

where $c_i := 1/\left(1 - b_i\right) > 1$. If we set $k_i = \bar{k}_i$ and hence $\hat{k}_i = \hat{\bar{k}}_i$ in that equation, $r_i = r_{ia}$ and $\hat{r}_i = \hat{r}_{ia}$ follows. That proves Proposition 2. □

Proof of Proposition 3. Proposition 3i: Denote by x_i and x_{ia} the national incomes when the interest rate is r and r_{ia}, respectively. Obviously, that implies $x_i = x_{ia}$ for $r = r_{ia}$, and if dx_i / dr is monotone in r we are able to determine the sign of the difference $x_{io} - x_{ia}$. Differentiation of x_i with respect to r yields

$$\frac{dx_i}{dr} = w_i L_r^i - \left(k_i - \bar{k}_i\right) \tag{A1}$$

and $\left.\frac{dx_i}{dr}\right|_{r=r_{ia}} = w_i L_r^i$. Suppose that $r > r_{ia}$ and $L_r^i > 0$. Then $k_i = K^i\left(r, w_i\right) < K^i\left(r_{ia}, w_i\right) = \bar{k}_i$ and therefore $\frac{dx_i}{dr} > 0$, proving $x_{io} > x_{ia}$ for the case 2 in Table 1. Along the same lines we show that $x_{io} > x_{ia}$ in the cases 3 and 7 and $x_{io} < x_{ia}$ in case 5 in Table 1 . In the cases 1 and 6 the sign of dx_i / dr is unclear.

Proposition 3ii: (a) We first observe that the term $y_i - r_i k_i$ turns into $y_i - r_i k_i$ = $\ell_i^b q_i^\alpha - r_i k_i$ = $k_i\left(\ell_i^b k_i^{-1} q_i^\alpha - r_i\right)$ when the production function is Cobb-Douglas. Interest rates r_i may differ across countries in autarky but they are uniform in the associated zero-tax equilibrium. We also know that $\ell_i^b k_i^{-1} q_i^\alpha = \ell_i^b k_i^{-1} \frac{k_i}{\ell_i} q_i^{\alpha-1} = \ell_i^{b-1} q_i^{\alpha-1} = \frac{r_i}{\alpha}$, because $\alpha \ell_i^{b-1} q_i^{\alpha-1} = r_i$ is the first-order condition for profit maximization. Hence $y_i - r_i k_i = k_i\left(\frac{r_i}{\alpha} - r_i\right) = \frac{r_i k_i}{\alpha\gamma}$, where $\gamma := \frac{1}{1-\alpha}$.

From this information the equations

$$\left[Y^i\left(k_{io}, \ell_{io}\right) - r_o k_{io}\right] = \frac{r_o k_{io}}{\alpha\gamma} \qquad \text{and} \qquad \left[Y^i\left(\bar{k}, \ell_{ia}\right) - r_{ia}\bar{k}_i\right] = \frac{r_{ia}\bar{k}_i}{\alpha\gamma} \tag{A2}$$

follow, where the indexes *a* and *o* refer to the autarky equilibrium and the zero-tax equilibrium, respectively. We write the difference in income of country i following a switch from autarky to free-trade as $\Delta x_i = x_{io} - x_{ia} = \left[Y^i\left(k_{io}, \ell_{io}\right) - r_o k_{io}\right] - \left[Y^i\left(\bar{k}, \ell_{ia}\right) - r_{ia}\bar{k}_i\right] - \left(r_{ia} - r_o\right)\bar{k}_i$ and consider (A2) to obtain $\alpha\gamma\Delta x_i = r_o k_{io} - r_{ia}\left(1 + \alpha\gamma\right)\bar{k}_i + \alpha\gamma r_o \bar{k}_i = r_o\left(k_{io} - \bar{k}_i\right) + r_o\left(1 + \alpha\gamma\right)\bar{k}_i$ - $r_{ia}\left(1 + \alpha\gamma\right)\bar{k}_i$ or

$$\alpha\gamma\Delta x_i = r_o\left(k_{io} - \bar{k}_i\right) + \left(1 + \alpha\gamma\right)\bar{k}_i\left(r_o - r_{ia}\right). \tag{A3}$$

(b) Next we determine the variables r_{ia}, r_o and k_{io} for the Cobb-Douglas production function. The first-order conditions for profit maximization $r = \alpha k_i^{\alpha-1} l_i^{\beta}$ and $w_i = \beta k_i^{\alpha} l_i^{\beta-1}$ imply $\ell_i = \frac{\beta r_i}{\alpha w_i} k_i$ yielding

$$r_i^{1-\beta} - \alpha^{1-\beta}\beta^{\beta} w_i^{-\beta} k_i^{-\frac{1}{c}} = 0, \text{ where } c := \frac{1}{1-b}. \tag{A4}$$

In autarky, we have $k_{ia} = \bar{k}_i$ such that (A4) is turned into

$$r_{ia} = \alpha\beta^{\frac{\beta}{1-\beta}} w_i^{-\frac{\beta}{1-\beta}} \bar{k}_i^{-\frac{1}{(1-\beta)c}}. \tag{A5}$$

In case of free trade we convert (A4) into $k_{io} = \alpha^{(1-\beta)c}\beta^{\beta c} r_o^{-(1-\beta)c} w_i^{-\beta c}$. Invoke the equilibrium condition on the world capital market, $\sum_j k_{jo} = \sum_j \bar{k}_j$, to obtain $\sum_j k_{jo} = \sum_j \bar{k}_j$ = $\alpha^{(1-\beta)c}\beta^{\beta c} r_o^{-(1-\beta)c} \sum_j w_j^{-\beta c}$ or, equivalently, $\bar{k}_{ø} := \frac{\sum_j \bar{k}_j}{n} = \alpha^{(1-\beta)c}\beta^{\beta c} r_o^{-(1-\beta)c} \sum_j \left(\frac{w_j^{-\beta c}}{n}\right)$ and

$$r_o^{1-\beta} = \alpha^{1-\beta}\beta^{\beta}\left(\sum_j \frac{w_j^{-\beta c}}{n}\right)^{\frac{1}{c}} \cdot \bar{k}_{ø}^{-\frac{1}{c}} \quad \text{or} \quad r_o = \alpha\beta^{\frac{\beta}{1-\beta}}\left(\sum_j \frac{w_j^{-\beta c}}{n}\right)^{\frac{1}{(1-\beta)c}} \cdot \bar{k}_{ø}^{-\frac{1}{(1-\beta)c}} \tag{A6}$$

To determine k_{io} we combine (A4) and (A6) which yields, after some rearrangement of terms,

$$k_{io} = \omega_i \bar{k}_{ø}, \qquad \text{where } \omega_i := \frac{n w_i^{-\beta c}}{\sum_j w_j^{-\beta c}}. \tag{A7}$$

(c) Invoke r_{ia} from (A5) and r_o from (A6) combined with (A7) and observe that

$$r_o \gtreqless r_{ia} \quad \Leftrightarrow \quad \alpha\beta^{\frac{\beta}{1-\beta}} w_i^{-\frac{\beta}{1-\beta}} \cdot \left(\omega_i \bar{k}_{ø}\right)^{-\frac{1}{(1-\beta)c}} \gtreqless \alpha\beta^{\frac{\beta}{1-\beta}} w_i^{-\frac{\beta}{1-\beta}} \bar{k}_i^{-\frac{1}{(1-\beta)c}} \quad \Leftrightarrow \quad \omega_i \bar{k}_{ø} \lesseqgtr \bar{k}_i. \tag{A8}$$

(d) We now insert (A5), (A6) and (A7) in (A3):

$$\alpha\gamma\Delta x_i \quad = \quad r_o \bar{k}_{ø}\left[\omega_i - \kappa_i\right] + \frac{(1+\alpha\gamma)\bar{k}_i \alpha\beta^{\frac{\beta}{1-\beta}}}{\bar{k}_{ø}^{\frac{1}{(1-\beta)c}} w_i^{\frac{\beta}{1-\beta}}}\left[\omega_i^{-\frac{1}{(1-\beta)c}} - \kappa_i^{-\frac{1}{(1-\beta)c}}\right]$$

$$= \delta_i \left[\omega_i \bar{k}_\varnothing^{1-\frac{1}{(1-\beta)c}} + \alpha\gamma \bar{k}_i \bar{k}_\varnothing^{-\frac{1}{(1-\beta)c}} \right] - (1+\alpha\gamma)\bar{k}_i r_{ia}, \tag{A9}$$

where $\delta_i := \frac{\alpha\beta^{\frac{\beta}{1-\beta}}}{\omega_i^{\frac{1}{(1-\beta)c}} w_i^{\frac{\beta}{1-\beta}}}$ and $\kappa_i := \frac{\bar{k}_i}{\bar{k}_\varnothing}$. From (A9) it is straightforward that $\Delta x_i = 0$, if $\omega_i = \kappa_i$ or $\omega_i \bar{k}_\varnothing = \bar{k}_i$. To specify how $\alpha\gamma\Delta x_i$ responds to changes in $\bar{k}_\varnothing$ consider the derivatives of (A9)

$$\frac{d(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing} = \frac{\alpha\delta_i}{1-\beta} \cdot \bar{k}_\varnothing^{-\frac{1}{(1-\beta)c}-1} \left(\omega_i \bar{k}_\varnothing - \frac{1-b}{1-\alpha} \bar{k}_i \right) \tag{A10}$$

$$\frac{d^2(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing^2} = \frac{\alpha\delta_i}{(1-\beta)^2 c} \bar{k}_\varnothing^{-\frac{1}{(1-\beta)c}-2} \left[\left(\frac{1-\beta}{1-\alpha} + \frac{1-b}{1-\alpha} \right) \bar{k}_i - \omega_i \bar{k}_\varnothing \right]. \tag{A11}$$

These derivatives imply

$$\frac{d(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing} \gtreqless 0 \Leftrightarrow \omega_i \bar{k}_\varnothing \gtreqless \frac{1-b}{1-\alpha} \bar{k}_i \quad \text{and} \quad \frac{d^2(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing^2} \gtreqless 0 \Leftrightarrow \left(\frac{1-\beta}{1-\alpha} + \frac{1-b}{1-\alpha} \right) \bar{k}_i \gtreqless \omega_i \bar{k}_\varnothing .$$

$\frac{d(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing} = 0$ is attained for $\bar{k}_\varnothing = \frac{1-b}{1-\alpha} \cdot \frac{\bar{k}_i}{\omega_i}$ and inserting $\bar{k}_\varnothing = \frac{1-b}{1-\alpha} \cdot \frac{\bar{k}_i}{\omega_i}$ in (A11) yields $\frac{d^2(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing^2} = \frac{\alpha\delta_i}{(1-\beta)^2 c} \bar{k}_\varnothing^{-\frac{1}{(1-\beta)c}-2} \bar{k}_i \frac{1-\beta}{1-\alpha} > 0$. Hence $\alpha\gamma\Delta x_i$ attains its unique minimum at $\bar{k}_\varnothing = \frac{1-b}{1-\alpha} \cdot \frac{\bar{k}_i}{\omega_i} < \frac{\bar{k}_i}{\omega_i}$. We have shown above that $\Delta x_i = 0$, if $\bar{k}_\varnothing = \frac{\bar{k}_i}{\omega_i}$. For this value of $\bar{k}_\varnothing$ (A10) turns into $\frac{d(\alpha\gamma\Delta x_i)}{d\bar{k}_\varnothing} = \frac{\alpha\beta\gamma\delta_i}{1-\beta} \cdot \bar{k}_\varnothing^{-\frac{1}{(1-\beta)c}-1} \bar{k}_i > 0$. Therefore, at its minimum $\alpha\gamma\Delta x_i$ is negative. Moreover, since $\Delta x_i < 0$ for $\bar{k}_\varnothing = 0$ according to (A9), we conclude that $\Delta x_i > 0$ for $\bar{k}_i \in \left]0, \omega_i \bar{k}_\varnothing\right[$, $\Delta x_i = 0$ for $\bar{k}_i = \omega_i \bar{k}_\varnothing$ and $\Delta x_i < 0$ for $\bar{k}_i \in \left]\omega_i \bar{k}_\varnothing, n\omega_i \bar{k}_\varnothing\right[$.

Proposition 3iii: In Case 6 $\frac{dx_i}{dr} = w_i L_r^i - (k_i - \bar{k}_i) > 0$ for all $r \in [r_o, r_{ia}]$ is sufficient for $x_{io} < x_{ia}$. Since CES implies $L_r^i = \frac{(\sigma_i - c_i) k_i}{w_i + r q_i}$ we find that $\frac{dx_i}{dr} > 0$ holds, if for all $r \in [r_o, r_{ia}]$

$$\sigma_i > c_i + \frac{k_i - \bar{k}_i}{\phi_i k_i} \quad \text{where} \quad \phi_i := \frac{w_i}{w_i + rq_i} \quad \text{and} \quad q_i := \left(\frac{a_{ki}}{a_{\ell i}}\right)^{\sigma_i} \left(\frac{w_i}{r}\right)^{\sigma_i}.$$

Since $\frac{k_i - \bar{k}_i}{k_i}$ is decreasing and ϕ_i is increasing in *r*, it follows that $\frac{k_{io} - \bar{k}_i}{\phi_{ia} k_{io}} \geq \frac{k_i - \bar{k}_i}{\phi_i k_i}$ for all $r \in [r_o, r_{ia}]$. That proves Proposition 3iii. □

Proof of Proposition 4. The strategy of proof is similar to that of Proposition 3ii. Country *i's* income is $x_{ia} = y_{ia} = \bar{k}_i^{\alpha} m_i^{\beta}$ in autarky and

$$x_{io} = y_{io} - r_o\left(k_{io} - \bar{k}_i\right) = k_{io}^{\alpha} m_i^{\beta} - r_o\left(k_{io} - \bar{k}_i\right) \tag{A12}$$

under free trade. In the latter case profit maximization implies

$$r_o = \alpha k_{io}^{\alpha-1} m_i^{\beta}. \tag{A13}$$

Combining (A12) and (A13) yields $x_{io} = k_{io}^{\alpha} m_i^{\beta} - \alpha k_{io}^{\alpha-1} m_i^{\beta} k_{io} + \alpha k_{io}^{\alpha-1} m_i^{\beta} \bar{k}_i = (1-\alpha) k_{io}^{\alpha} m_i^{\beta} + \alpha \bar{k}_i k_{io}^{\alpha-1} m_i^{\beta}$. Hence

$$\Delta x_i = x_{io} - x_{ia} = (1-\alpha) k_{io}^{\alpha} m_i^{\beta} + \alpha \bar{k}_i k_{io}^{\alpha-1} m_i^{\beta} - \bar{k}_i^{\alpha} m_i^{\beta}. \tag{A14}$$

Next we show that $k_{io} = \bar{m}_i \bar{k}_{ø}$. From (A13) we have $k_{io} = \left(\frac{r_o}{\alpha m_i^{\beta}}\right)^{-\frac{1}{1-\alpha}}$. Invoking $\sum_j k_{jo} = \sum_j \bar{k}_j$ from (5) yields $\left(\frac{r_o}{\alpha}\right)^{-\frac{1}{1-\alpha}} \sum_j m_j^{\frac{\beta}{1-\alpha}} = \sum_j \bar{k}_j$, $\bar{k}_{ø} = \frac{1}{n}\left(\frac{r_o}{\alpha}\right)^{-\frac{1}{1-\alpha}} \sum_j m_j^{\frac{\beta}{1-\alpha}}$ and $r_o = \alpha\left(\sum_j m_j^{\frac{\beta}{1-\alpha}}\right)^{1-\alpha} \cdot \left(n\bar{k}_{ø}\right)^{-(1-\alpha)}$. Insert r_o from the last equation in $k_{io} = \left(\frac{r_o}{\alpha m_i^{\beta}}\right)^{-\frac{1}{1-\alpha}}$ to obtain $k_{io} = \bar{m}_i \bar{k}_{ø}$. Use this information to turn (A14) into

$$\Delta x_i = (1-\alpha) \bar{m}_i^{\alpha} \bar{k}_{ø}^{\alpha} m_i^{\beta} + \alpha \bar{k}_i \bar{m}_i^{\alpha-1} \bar{k}_{ø}^{\alpha-1} m_i^{\beta} - \bar{k}_i^{\alpha} m_i^{\beta}. \tag{A15}$$

It is straightforward from (A15) that $\Delta x_i = 0$ if $\bar{k}_i = \bar{m}_i \bar{k}_{ø}$. To specify how Δx_i responds to changes in $\bar{k}_i$ when $\bar{k}_{ø}$ is kept constant, consider the derivatives of (A15),

$$\frac{d\Delta x_i}{d\bar{k}_i} = \alpha m_i^{\beta}\left(\bar{m}_i^{\alpha-1} \bar{k}_{ø}^{\alpha-1} - \bar{k}_i^{\alpha-1}\right) \quad \text{and} \quad \frac{d^2 \Delta x_i}{d\bar{k}_i^2} = \alpha(1-\alpha) m_i^{\beta} \bar{k}^{\alpha-2}.$$

Since the first derivative implies $\frac{d\Delta x_i}{d\bar{k}_i} \gtrless 0 \Leftrightarrow \bar{m}_i \bar{k}_\varnothing \gtrless \bar{k}_i$ and $\frac{d^2 \Delta x_i}{d\bar{k}_i} > 0$, Δx_i from (A15) attains its unique minimum at $\bar{k}_i = \bar{m}_i \bar{k}_\varnothing$. □

Proof of Proposition 5. Equation (10) is straightforward from the first-order condition of maximizing (4) with respect to t_i: $X_t^i = Y_k^i K_\rho^i + Y_l^i L_\rho^i - rK_\rho^i = (\rho_i - r) K_\rho^i + w_i L_\rho^i = t_i K_\rho^i + w_i L_\rho^i = 0$. (11) follows from combining the equation (10) with (6) and (7) and the pertaining factor demand functions. □

Proof of Proposition 6. Suppose that $L_\rho^i < 0$ and hence $t_{i\tau}$ < 0. Starting from $t_i = 0$, successive reductions of t_i decrease $\rho = r_o + t_i$ and therefore increase $L^i(r_o + t_i, w_i)$. Suppose next that $L_\rho^i > 0$ and hence $t_{i\tau}$ > 0. Starting from $t_i = 0$ successive increases in t_i increase $\rho = r_o + t_i$ and therefore also increase $L^i(r_o + t_i, w_i)$. □

Proof of Proposition 7. The proof proceeds in several steps. First we prove the

Claim: *Associated with the equilibrium E_τ is an 'auxiliary' constant-tax trade equilibrium*[19] $\tilde{E}_\tau := \left\{ \tilde{t}_{1\tau}, ..., \tilde{t}_{n\tau}, \tilde{r}_\tau, \left(\tilde{x}_{i\tau}, \tilde{y}_{i\tau}, \tilde{k}_{i\tau}, \tilde{\ell}_{i\tau} \right)_{i=1,...,n} \right\}$ *with the following properties:*

(a) $\left(\tilde{x}_{i\tau}, \tilde{y}_{i\tau}, \tilde{k}_{i\tau}, \tilde{\ell}_{i\tau} \right)_{i=1,...,n} = \left(x_{i\tau}, y_{i\tau}, k_{i\tau}, \ell_{i\tau} \right)_{i=1,...,n}$, $\left(\tilde{t}_{1\tau}, ..., \tilde{t}_{n\tau} \right) = \left(t_{1\tau} + \triangle r, ..., t_{n\tau} + \triangle r \right)$, $\triangle r = r_\tau - r_o$ *and hence* $\tilde{r}_\tau = r_\tau - \triangle r = r_o$.

(b) $\left(\tilde{t}_{1\tau}, ..., \tilde{t}_{n\tau} \right)$ *in* $\tilde{E}_\tau$ *contains positive and negative tax rates, if the tax rates* $t_{1\tau}, ..., t_{n\tau}$ *in* E_τ *differ across countries; otherwise* $\tilde{t}_{1\tau} = ... = \tilde{t}_{n\tau} = 0$.

The existence of the constant-tax trade equilibrium $\tilde{E}_\tau := \left\{ \tilde{t}_{1\tau}, ..., \tilde{t}_{n\tau}, \tilde{r}_\tau, \left(\tilde{x}_{i\tau}, \tilde{y}_{i\tau}, \tilde{k}_{i\tau}, \tilde{\ell}_{i\tau} \right)_{i=1,...,n} \right\}$ as defined in part (a) of the Claim follows immediately from Proposition 1. Note that (5) implies $\sum_j \bar{k}_j = \sum_j K^j(r_o, w_j) = \sum_j K^j(r_\tau + t_{j\tau}, w_j) = \sum_j K^j(r_\tau - \Delta r + t_{j\tau} + \Delta r, w_i) = \sum_j K^j(r_o + \tilde{t}_{j\tau}, w_i)$. Part (b) of the Claim postulates that if tax rates $t_{1\tau}, ..., t_{n\tau}$ differ across

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[19] $\tilde{E}_\tau$ provides the information about the tax rates needed for shifting from the no-tax trade equilibrium allocation to the tax-competition equilibrium allocation while keeping unchanged the interest rate prevailing in the no-tax trade equilibrium.

countries, the tax rates $\tilde{t}_{1\tau},...,\tilde{t}_{n\tau}$ need to contain negative and positive components. Suppose not. Then sign $\tilde{t}^{\tau}_{\min}$ = sign $\tilde{t}^{\tau}_{\max}$ with $\tilde{t}^{\tau}_{\min} \neq 0$ or $\tilde{t}^{\tau}_{\max} \neq 0$, where $\tilde{t}^{\tau}_{\min}$ and $\tilde{t}^{\tau}_{\max}$ are the minimum and maximum components of $\left(\tilde{t}_{1\tau},...,\tilde{t}_{n\tau}\right)$, respectively. That obviously implies $\sum_j \overline{k}_j$ = $\sum_j K^j\left(r_o, w_j\right) \neq \sum_j K^j\left(r_o + \tilde{t}_{j\tau}, w_j\right)$ contradicting the fact that $\tilde{E}_\tau$ is an equilibrium. If the tax rates $t_{1\tau}, ..., t_{n\tau}$ are uniform across countries, the (associated) tax rates $\tilde{t}_{1\tau},...,\tilde{t}_{n\tau}$ are also uniform because by definition of $\tilde{t}_{i\tau}$ it is true that $\tilde{t}_{i\tau} := \vartriangle r + t_{i\tau}$ for all i and satisfy $\tilde{t}_{1\tau} = ... = \tilde{t}_{n\tau} = 0$ by construction of $\tilde{E}_\tau$. This completes the proof of the Claim.

With this information we proceed to establish Proposition 7. We first focus on the change in employment induced by moving from E_o to E_τ by proving the equivalence

$$\left(t_{i\tau} \cdot \tilde{t}_{i\tau}\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \iff L^i\left(r_o + \tilde{t}_{i\tau}, w_i\right) = L^i\left(r_\tau + t_{i\tau}, w_i\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} L^i\left(r_o, w_i\right).$$

Consider the case that $t_{i\tau} \leq 0$ and $\tilde{t}_{i\tau} \leq 0$. We know that $t_{i\tau} \leq 0 \iff L^i_\rho \leq 0$ and therefore $L^i\left(r_o + \tilde{t}_{i\tau}, w_i\right) \geq L^i\left(r_o, w_i\right)$. Similarly, if $t_{i\tau} \geq 0$ and $\tilde{t}_{i\tau} \geq 0$, we have $t_{i\tau} \geq 0 \iff L^i_\rho \geq 0$ and therefore $L^i\left(r_o + \tilde{t}_{i\tau}, w_i\right) \geq L^i\left(r_o, w_i\right)$ as well. Based on this information it is straightforward to show that $\left(t_{i\tau} \cdot \tilde{t}_{i\tau}\right) \leq 0 \iff L^i\left(r_o + \tilde{t}_{i\tau}, w_i\right) \leq L^i\left(r_o, w_i\right)$.

The sign of the difference $k_{i\tau} - k_{io}$ (second column of Table 2) is easily calculated as being equal to the sign of $\left(\tilde{t}_{i\tau} \cdot K^i_\rho\right)$, since $K^i_\rho < 0$.

It remains to prove the signs in the last column of Table 2. Set $\rho_i = r_o + \tilde{t}_{i\tau}$ and differentiate $x_i = y_i - r_o k_i + r_o \overline{k}_i$ with respect to $\tilde{t}_{i\tau}$:

$$\frac{dx_i}{d\tilde{t}_{i\tau}} = Y^i_k K^i_\rho + Y^i_\ell L^i_\rho - r_o K^i_\rho = \tilde{t}_{i\tau} K^i_\rho + w_i L^i_\rho = \left(\tilde{t}_{i\tau} + \frac{w_i L^i_\rho}{K^i_\rho}\right) K^i_\rho. \tag{A16}$$

From $\frac{dx_i}{d\tilde{t}_{i\tau}}\Big|_{\tilde{t}_{i\tau}=0} = w_i L^i_r$ follows sign $\frac{dx_i}{d\tilde{t}_{i\tau}}\Big|_{\tilde{t}_{i\tau}=0} = \text{sign}\, L^i_r$ and if $dx_i / d\tilde{t}_{i\tau}$ is monotone in $\tilde{t}_{i\tau}$ we are able to determine the sign of the difference $x_{i\tau} - x_{io}$. Combine $w_i L^i_\rho / K^i_\rho$ from (A16) with $r_o + \tilde{t}_{i\tau} = r_\tau + t_{i\tau}$ to obtain $\frac{w_i L^i_\rho}{K^i_\rho} = \frac{w_i L^i_\rho\left(r_o + \tilde{t}_{i\tau}, w_i\right)}{K^i_\rho\left(r_o + \tilde{t}_{i\tau}, w_i\right)} = \frac{w_i L^i_\rho\left(r_\tau + t_{i\tau}, w_i\right)}{K^i_\rho\left(r_\tau + t_{i\tau}, w_i\right)} = -t_{i\tau}$ and hence

$$\frac{dx_i}{d\tilde{t}_{i\tau}} = \left(\tilde{t}_{i\tau} - t_{i\tau}\right) K_\rho^i = \left(r_\tau - r_o\right) K_\rho^i . \tag{A17}$$

We conclude from (A17) that

$$x_{i\tau} \begin{Bmatrix} > \\ < \end{Bmatrix} x_{io} \quad \Leftrightarrow \quad \begin{cases} r_\tau > r_o \text{ and } \tilde{t}_{i\tau} < 0 \text{ (Case 1) or } r_\tau < r_o \text{ and } \tilde{t}_{i\tau} > 0 \text{ (Case 7)} \\ r_\tau > r_o \text{ and } \tilde{t}_{i\tau} > 0 \text{ (Cases 2 and 3) or } r_\tau < r_o \text{ and } \tilde{t}_{i\tau} < 0 \text{ (Cases 6 and 8).} \end{cases}$$

The Case 4 in Table 2 is obvious. □

Proof of Proposition 8. Proposition 8i follows from combining Case 1 of Table 1 with Case 4 of Table 2 and Proposition 3ii. Proposition 8ii follows from combining Case 4 of Table 1 with Case 2 of Table 2. The first part of Proposition 8iii follows from combining Case 6 of Table 1 with Case 8 of Table 2 and Proposition 3iii. The second part of Proposition 8iii follows from combining Case 6 of Table 1 with Case 4 of Table 2 and Proposition 3iii. □

Proof of the claim: $n > \left[\frac{\left(\sum_j w_j\right)^{\beta c}}{\sum_j w_j^{\beta c}} \right]^{\frac{1}{1+\beta c}} > 0 \quad \Rightarrow \quad \tilde{w} > w_{\varnothing}$.

We rearrange the inequality $\tilde{w} - w_{\varnothing} \gtreqless 0$ and obtain

$$\tilde{w} - w_{\varnothing} \gtreqless 0 \;\Leftrightarrow\; \left(\sum_j \frac{w_j^{-\beta c}}{n} \right)^{-\frac{1}{\beta c}} \gtreqless \frac{\sum_j w_j}{n} \;\Leftrightarrow\; \sum_j \frac{w_j^{-\beta c}}{n} \lesseqgtr \left(\frac{\sum_j w_j}{n} \right)^{-\beta c} \;\Leftrightarrow$$

$$\frac{\left(\sum_j w_j\right)^{\beta c}}{\sum_j w_j^{\beta c}} \lesseqgtr n^{1+\beta c} \;\Leftrightarrow\; \left[\frac{\left(\sum_j w_j\right)^{\beta c}}{\sum_j w_j^{\beta c}} \right]^{\frac{1}{1+\beta c}} \lesseqgtr n . \tag{A18}$$

With $w_i > 0$ for $i = 1, \ldots, n$ the inequality (A18) proofs the claim. □