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# Heterogenous Skills and Homogeneous Land: Segmentation and Agglomeration

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## **Abstract**

This paper analyzes the impact of skill heterogeneity on regional patterns of production and housing in the presence of pecuniary externalities within a general-equilibrium framework assuming monopolistic competition at intermediate good markets. It shows that the interplay of heterogenous skills and relatively homogeneous land demand triggers skill segmentation and agglomeration. The core region, being more attractive to high skilled workers, has a disproportionately large share of production at all levels of the supply chain. The paper studies the effects on segmentation and agglomeration of interregional trade in intermediate goods, attachment to home, the presence of immobile unskilled workers, various conditions at local land markets, and federal taxation.

*JEL Classification:* R12, R13, R14.

*Keywords:* Skill heterogeneity, land use, segmentation, agglomeration.

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# 1 Introduction

At least since Marshall (1890) it is broadly accepted that skills and agglomeration are connected. Knowledge spillovers and local availability of specific skills act as centripetal forces. Many empirical studies have confirmed that average productivity is increasing in local market size and population density.<sup>1</sup> The city size elasticity of productivity ranges from 3-8% (Rosenthal and Strange, 2004). Productivity in large cities is extraordinarily higher than that found outside cities (Glaeser and Mare, 2001) and there is a substantial wage premium associated with the largest cities and metropolitan areas (see, for Asia and France, respectively, Fujita and Thisse, 2002; Combes et al., 2008). However, recent studies have shown that it is not only externalities and location advantages that contribute to the urban wage premium, but also quality selection processes (Lee, 2005; Fu and Ross, 2007; Combes et al., 2008). Since individual skills account for a substantial fraction of spatial wage differences, sorting by skills matters. A disproportionately large fraction of well-educated workers live and work in large metropolitan areas (Mori and Turrini, 2005). According to Bacolod et al. (2009), workers at the top and bottom of the skill distribution are attracted by large cities. However, these authors also find that average skill levels are higher in large cities, although their estimated correlation of skills and size is only of modest magnitude.

Since Melitz (2003) introduced firm heterogeneity into the new trade theory, thus allowing for segmentation on export markets<sup>2</sup> and in foreign direct investment (Helpman et al., 2004), agent heterogeneity has been on the agenda in international and regional economics. Baldwin and Okubo (2006) embedded firm heterogeneity in a standard new economic geography framework in order to show that the biggest region attracts the most productive firms,<sup>3</sup> since these firms benefit more from backward and forward linkages in

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<sup>1</sup>There is a huge body of empirical studies on the relationship between size/density and productivity/wages including Sveikauskas (1975), Ciccone and Hall (1996), Glaeser and Mare (2001), Wheaton and Lewis (2002), Syverson (2004), Lee (2005), Wheeler (2006), Fu and Ross (2007), Combes et al. (2008), and Bacolod et al. (2009). For surveys, see Rosenthal and Strange (2004) and, more recently, Strange (2009).

<sup>2</sup>See for endogenous mark-ups also Melitz and Ottaviano (2008).

<sup>3</sup>Nocke (2003) derived similar results.

the bigger market, but are less prone to the market-crowding effect.<sup>4</sup>

However, while the impact of firm heterogeneity on trade has been extensively studied, heterogenous workers have attracted less interest in the theoretical regional economics literature on migration and interregional trade despite the overwhelming evidence on spatial sorting of heterogenous workers. An exception is Mori and Turrini (2005) who showed that skill heterogeneity within a standard new economic geography framework causes spatial sorting and agglomeration whereas trade integration as a supplementary force increases the degree of agglomeration. More theoretical work on skill heterogeneity needs to be done to take into account the full spectrum of market forces analyzed within the new economic geography framework. In particular, vertical input-output linkages between firms are considered a major reason for the endogeneity of market size (see Venables, 1996; Krugman and Venables, 1995; Puga, 1999; Ottaviano and Thisse, 2004). Vertical linkages might support a bell-shaped curve of spatial development that could not occur in the original core-periphery model.

In their analysis of skill heterogeneity, Mori and Turrini (2005) neglected land use in production and housing, as well as commuting costs. Although von Thünen (1826) early on highlighted the role of land as a main force behind dispersion, the new economic geography literature initially omitted land use from analysis. However, Helpman (1998) showed that land use for housing purposes may reverse the agglomeration trend in the course of trade integration. Later, Pflüger and Südekum (2008) showed a bell-shaped curve for relationship between spatial development and land use. Recently, by setting up a monopolistic competition model with land as a productive factor, Pflüger and Tabuchi (2008) demonstrated that market-size-based agglomeration forces might be too weak to overcome the dispersion force associated with competition for land unless consumer' desire for variety is extremely strong. Agglomeration forces other than backward and forward linkages might be necessary to limit the impact of land use congestion on the spatial allocation of economic activities.

The aim of this paper is to provide a general-equilibrium-based theory of segmentation and agglomeration on a regional scale that explains two facts, namely, the existence of ag-

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<sup>4</sup>Further recent studies focussing on heterogenous firms are, among others, Bernard et al. (2007), Behrens and Robert-Nicoud (2008), and Mion and Naticchioni (2009).

glomerations despite congestion and the positive correlation of skill levels and wages with size. To this end, we compile a two-region general-equilibrium model in which land use for production and housing is essential. To allow for benefits of size, following Fujita and Hamaguchi (2001), we introduce input-output linkages between firms. The model also includes economies of scale in the intermediate good sector, benefits of variety in intermediate products, and interregional trade costs for transporting intermediate goods. Intermediate trade costs consist of transport costs, compliance costs associated with legal regulations, costs related to just-in-time production at the final good level, and so forth. Competition at the intermediate good level is assumed to be monopolistic. Rather than modeling pecuniary externalities, we could have considered other centripetal forces related to human capital such as knowledge spillovers.<sup>5</sup> However, there is some evidence that pecuniary externalities are a more prevalent source of agglomeration than knowledge spillovers (see Ellison et al., 2007). In addition, knowledge spillovers are spatially strictly bounded (see, e.g., Baldwin et al., 2008) presumably excluding larger regions as units of analysis (for a survey on human capital externalities, see Moretti, 2004). As do Mori and Turrini (2005) we include observable skill heterogeneity in the model by considering a continuum of workers with ex-ante different skills. Assuming perfect mobility, we analyze whether segmentation and agglomeration is the outcome of individual desires and market forces.

Our results can be summarized as follows:

First, the interplay of observable skill heterogeneity and (almost) homogeneous land demand triggers skill segmentation and agglomeration. The highly-skilled live in richer regions where aggregate human capital and aggregate output is higher. A symmetric long-run equilibrium without segmentation would always be unstable. Household mobility implies that wages and prices of non-tradable goods, especially land, are positively correlated across regions. Facing the choice between a high-wage-high-land-price region and a low-wage-low-land-price region, highly-skilled workers prefer the former and low-skilled workers the latter. This finding is clear without ambiguity when the individual demand for land is independent of income (and underlying preference orderings are the same for all workers). However, except when the income elasticity of demand for land is too high, pre-

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<sup>5</sup>For a comprehensive survey on urban agglomeration economies, see Duranton and Puga (2004).

ferred wages accompany skills. Mori and Turrini (2005) also established segmentation and agglomeration in a regional model with skill heterogeneity,<sup>6</sup> but in their model, where differentiated worker-sellers produce consumer goods, land is absent. Communication costs associated with interregional trade that fall more heavily on low-skill suppliers are the driving force behind segmentation. These costs lead high-skilled and low-skilled workers to make different location choices. Although Mori and Turrini's (2005) mechanism and the mechanism introduced in this paper both induce segmentation and agglomeration, it is the latter that is related to the very basic forces in spatial economics already highlighted by von Thünen (1826). Finally, we demonstrate that skill heterogeneity combined with market-size based agglomeration forces cause agglomeration of economic activities despite competition for land. Thus, lack of agent heterogeneity might explain why more standard new economic geography models are unable to establish agglomeration when land use in production and housing is taken into account (see Pflüger and Tabuchi, 2008).

Second, since labor income taxes fall more heavily on cities with high productivity, high wages and high land rents, federal taxation redistributes toward peripheral regions. From the general-equilibrium trade model perspective, there is no rationale for redistribution to regions that are inefficient in the traded-good sector (see Albouy, 2008), however, in this model with pecuniary externalities, federal taxation may turn out to be welfare enhancing by reducing over-agglomeration.

Third, even though reducing intermediate trade costs increases the number of varieties available in both regions and lowers production costs of final goods, it also reduces the skill advantage of the core, thus diminishing regional differences. However, neither segmentation nor agglomeration fully disappears provided there are some, albeit, small trade costs. In contrast to the basic core-periphery model of Krugman (1991), but in agreement with Helpman's (1998) model, trade integration weakens agglomeration.

Fourth, we confirm the dispersive nature of attachment to home identified by Tabuchi and Thisse (2002). An increasing number of immobile unskilled workers who are substitutes for mobile workers in production and/or a generally decreasing willingness to leave the home region for higher pecuniary utility alleviate agglomeration either by directly reducing

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<sup>6</sup>Abdel-Rahman (1998) also establishes segmentation in a two-skill-type-two-sector model with commuting where public infrastructure is the basic agglomeration force.

differences in aggregate skills and output or by limiting the impact of a regional wage gap on migration flows. Consequently, relocating substitutable immobile workers from the periphery to the core might stem the migration flow of mobile workers. However, increasing the number of immobile workers could have the opposite effect if immobile workers are complementary production factors that reinforce the impact of human capital on output.

Fifth, if firms' demand for land has a substantial impact on regional land prices and/or if consumer land demand is income elastic, the core region, despite attracting more human capital and production than the periphery, might be less densely populated. Lack of firms and high-income workers keep land prices in the periphery relatively low, thus stemming the flow of migration.

Our model is related to Black (1999) who set up a multi-region model with knowledge spillovers embedded in the production function, two skill levels, and land use for housing. The fundamental force triggering segregation in his model is the same as in our model. Higher wages in core regions will offset higher living costs only for the highly-skilled, but not for the low-skilled. The segregation process in his model also requires that average productivity is increasing in average skills. However, his model differs in important aspects from our model. First, agglomeration externalities are not the result of market forces, they are simply imposed. Second, instead of a continuum of skill levels, there are only two types. Consequently, the number of regions is endogenous (and larger than the number of types); otherwise, the migration equilibrium conditions would be fulfilled only by chance and agglomeration would be imposed, not derived. Assuming an endogenous number of regions, is useful for a very long-run analysis of cities, but less suitable for the analysis of countries. Third, in Black's model, there is always a continuum of free-mobility equilibria, making predictions and policy analysis difficult. Accordingly, interregional redistribution policy might be allocatively neutral. Fourth, Black (1999) didn't consider imperfect mobility and variations in the local land market conditions. Finally, Black does not fully elaborate on the single good market equilibrium which is, however, critical in light of cross-region differences in productivity.

The paper is organized as follows. The next section develops the basic economic model and discusses existence, stability, and efficiency of short-run and long-run equilibria, as well as analyzing the impact on equilibria of federal taxation and trade in intermediate goods.

Section 3 extends and modifies the basic model so as to study the effects on segmentation and agglomeration of attachment to home, the presence of immobile unskilled workers, and various conditions at local land markets. Section 4 concludes.

## 2 The basic model

We consider a country comprised of two ex-ante identical regions,  $i = C, P$ . Each region is endowed with  $L$  square miles of land. The total mass of individuals living in the country is denoted by  $N$ . It is assumed that each person owns an identical share of land and that total land rents,  $R$ , are equally distributed among all citizens. Each person supplies inelastically one unit of labor in the region of residence. However, workers are heterogenous in terms of skills. The effective labor supply of a worker of type  $s$  is simply  $s$  and gross wage income is  $sw_i$ ,  $i = C, P$ , where  $w_i$  is the wage rate for a unit of normalized labor. Skills are distributed according to a continuous density function  $f(s)$  with support  $[\underline{s}, \bar{s}]$ . Hence, the total stock of skills-weighted labor, termed human capital,  $S$ , is

$$S = N \int_{\underline{s}}^{\bar{s}} sf(s)ds. \quad (1)$$

We allow for a proportional federal wage tax, where tax revenue is redistributed via a lump-sum transfer. Denoting the tax rate by  $t$ , with  $0 \leq t < 1$ , tax revenue is

$$T = t(w_C S_C + w_P S_P). \quad (2)$$

Since we assume that each worker lives on one unit of land, utility of workers of type  $s$  in region  $i$  can be written as income minus land expenses:

$$U_i(s) = (1 - t)w_i s + \frac{R + T}{N} - r_i, \quad i = C, P, \quad (3)$$

where  $r_i$  indicates land rent – including commuting costs.

Workers are perfectly mobile. They maximize utility by choice of residence. However, we begin our analysis by considering the short-run equilibrium for a given interregional allocation of workers and then, in a second step, analyze the long-run equilibrium where mobility is fully taken into consideration.



## 2.1 Short-run equilibrium

There are two types of goods, a final good,  $X_i$ , and a continuum of differentiated intermediate goods,  $q_i(j)$ . The final good is produced using human capital,  $S_{xi}$ , land,  $L_{xi}$ , and the intermediate aggregate,  $I_i$ . Following Ethier (1982), Abdel-Rahman and Fujita (1990), and Fujita and Hamaguchi (2001), the production function in region  $i$ ,  $i = C, P$ , is

$$X_i = S_{xi}^\eta I_i^\mu L_{xi}^{1-\mu-\eta}, \quad \text{where } I_i = \left[ \int_0^{n_i} q_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}. \quad (4)$$

$\sigma$ , with  $\sigma > 1$ , indicates the substitution elasticity of intermediate goods. The number of varieties,  $n_i$ , human capital,  $S_{xi}$ , and land use,  $L_{xi}$ , are determined endogenously. Since the production function exhibits constant returns to scale, the number of firms and output per firm are indeterminate. Without loss of generality, we proceed as if the total output in every region is produced by a single representative firm that behaves competitively. The final good is intra- and inter-regionally traded at no cost; its price is normalized at unity. In the basic model, intermediate goods are non-tradeable; their prices might vary by variety and region. Furthermore, we let  $r_i$  the price of land,  $p_i(j)$  the price of intermediate good  $j$ , and  $P_i$  the CES-price index of intermediate goods, with

$$P_i = \left[ \int_0^{n_i} p_i(j)^{-(\sigma-1)} dj \right]^{-\frac{1}{\sigma-1}}, \quad i = C, P. \quad (5)$$

The representative firm chooses inputs so as to maximize profits  $\Pi_i = X - w_i S_{xi} - r_i L_{xi} - P_i I_i$ . As a result, the demand functions for human capital, land, and intermediate goods in region  $i$ ,  $i = C, P$ , are

$$\begin{aligned} S_{xi} &= X \frac{r_i^{1-\mu-\eta} P_i^\mu}{w_i^{1-\eta}} \left( \frac{\eta}{1-\mu-\eta} \right)^{1-\mu-\eta} \left( \frac{\eta}{\mu} \right)^\mu, \\ L_{xi} &= X \frac{w_i^\eta P_i^\mu}{r_i^{\mu+\eta}} \left( \frac{1-\mu-\eta}{\eta} \right)^\eta \left( \frac{1-\mu-\eta}{\mu} \right)^\mu, \\ q_i(j) &= X \frac{r_i^{1-\mu-\eta} w_i^\eta}{P_i^{1-\mu}} \left( \frac{\mu}{1-\mu-\eta} \right)^{1-\mu-\eta} \left( \frac{\mu}{\eta} \right)^\eta \left( \frac{P_i^\sigma}{p_i^\sigma} \right). \end{aligned} \quad (6)$$

Hence, per-unit costs are

$$c_i = \psi w_i^\eta P_i^\mu r_i^{1-\mu-\eta}, \quad i = C, P, \quad (7)$$

with  $\psi = \mu^{-\mu} \eta^{-\eta} (1-\mu-\eta)^{-(1-\mu-\eta)}$ . At equilibrium, per-unit costs are equal to the final good price. Hence, the final good market equilibrium requires identical per-unit costs in

both regions:

$$c_C = c_P = 1. \quad (8)$$

Using Equation (8), demand functions (6) at the final good market equilibrium can be written as

$$\begin{aligned} S_{xi} &= \eta \frac{X_i}{w_i}, \quad L_{xi} = (1 - \mu - \eta) \frac{X_i}{r_i}, \\ q_i(j) &= \mu \frac{X_i P_i^{\sigma-1}}{p_i^\sigma}, \quad \text{implying } I_i = \mu \frac{X_i}{P_i}. \end{aligned} \quad (9)$$

Each intermediate good is produced using only human capital. For  $q_i(j)$  units of intermediate goods,  $S_{qi}(j) = F + aq_i(j)$  units of human capital are required.  $F$  indicates the fixed costs,  $a$  per-unit variable cost. If firms can differentiate their good without cost, at equilibrium each good will be produced by only one firm. Since there is a continuum of varieties, the intermediate sector is characterized by monopolistic competition à la Dixit and Stiglitz (1977), where all firms act as if their behavior does not affect the price level. As a consequence, each firm maximizes its profits

$$\pi_i(j) = p_i(j)q_i(j) - w_i [F + aq_i(j)], \quad i = C, P, \quad (10)$$

based on the final good sector's demand function (9) by mark-up pricing, where the mark-up is the same for all intermediate good suppliers:

$$p_i := p_i(j) = \left( \frac{\sigma}{\sigma - 1} \right) w_i a, \quad i = C, P. \quad (11)$$

Furthermore, free market entry enforces zero profits, which, along with the mark-up price policy, implies that all intermediate good suppliers produce the same quantity:

$$q := q_i(j) = \frac{(\sigma - 1)F}{a}. \quad (12)$$

The higher the substitution elasticity, the lower the price, but the greater the quantity. An increase in fixed costs and a decrease in variable costs raise the equilibrium quantity.

Taking the final good sector's demand (9) and the intermediate good sector's zero-profit condition (12) into account, the intermediate good market equilibrium determines – depending on the number of varieties – the regional final good production:

$$X_i = \frac{\sigma F w_i n_i}{\mu}, \quad i = C, P. \quad (13)$$

Higher wages, higher fixed costs, higher substitutability, and more varieties increase the regional final good production.

The number of varieties in a region is related to the stock of human capital available in the region. If the stock of human capital available in region  $i$  is denoted by  $S_i$ , a labor market equilibrium requires

$$S_{x_i} + \int_0^{n_i} [F + aq_i(j)] dj = S_i, \quad i = C, P. \quad (14)$$

Using the demand function of the final good sector, Equation (9), the intermediate good sector's zero-profit condition (12), and the formula for regional production, (13), the number of varieties can be written as

$$n_i = \frac{S_i}{\sigma F \left(1 + \frac{\eta}{\mu}\right)}, \quad i = C, P. \quad (15)$$

The number of varieties is proportional to the stock of human capital.

Land in each region is considered as a straight line with one unit of land at each location. The land market is perfectly competitive. Production takes place in the central business district (CBD) around the center.<sup>7</sup> Since the intermediate sector does not use land, the size of the CBD is calculated using the the final good sector's demand for land:  $2\hat{\rho} = (1 - \mu - \eta)X_i/r_i$ . For simplicity, we omit transport of goods and people within the CBD. Workers live to both, the right and the left of the CBD and it is assumed that each worker lives on one unit of land. Commuting costs per mile,  $k$ , are constant. The rent in the CBD is  $r_i$ , starting at  $\hat{\rho}$  it declines monotonically towards the edge of the inhabited area,  $\bar{\rho}$ . Hence, the land rent schedule can be written as a function of distance  $\rho$  from the center:

$$r_i(\rho) = \max\{r_i - k(\rho - \hat{\rho}), r_i\}, \quad i = C, P. \quad (16)$$

All workers pay for land use plus commuting effectively  $r_i$ . In the benchmark model, we assume that each region is large enough such that the land rent at the edge of the inhabited area is equal to the exogenously given opportunity cost of land,  $r_A = 0$ :  $r_i - k(\bar{\rho} - \hat{\rho}) = 0$ .

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<sup>7</sup>The urban city model simplifies the analysis a lot and makes analytical solutions possible. In the next section, we will demonstrate that most results are still valid under alternative land markets conditions.

Furthermore, since everyone must live somewhere, the inhabited area matches population:  $2(\bar{\rho} - \hat{\rho}) = N_i$ . Hence, the land rent in the CBD is

$$r_i = \frac{kN_i}{2}, \quad i = C, P. \quad (17)$$

Since land is abundant, but commuting is costly, population determines the price that the producer must pay. Taking both the final good sector's demand for land and the land rent in the CBD into account, aggregate land rent can be determined:

$$R = 2 \sum_{i=C,P} \left[ r_i \hat{\rho} + \frac{r_i(\bar{\rho} - \hat{\rho})}{2} \right] = (1 - \mu - \eta)(X_C + X_P) + \frac{k(N_C^2 + N_P^2)}{4}. \quad (18)$$

Using the definitions of the intermediate goods price index, Equation (5), the equilibrium price, Equation (11), the land rent, Equation (17), and the equilibrium number of varieties, Equation (15), the international final good market equilibrium condition, Equation (8), can be written as

$$1 = \psi w_i^\eta \left\{ \left( \frac{\sigma}{\sigma - 1} \right) w_i a \left[ \frac{S_i}{\sigma F \left( 1 + \frac{\eta}{\mu} \right)} \right]^{-\frac{1}{\sigma-1}} \right\}^\mu \left( \frac{kN_i}{2} \right)^{1-\mu-\eta}, \quad i = C, P. \quad (19)$$

Defining

$$\kappa = \left( \frac{k}{2} \right)^{1-\mu-\eta} \psi \left( \frac{a\sigma}{\sigma - 1} \right)^\mu \left[ \frac{1}{F\sigma \left( 1 + \frac{\eta}{\mu} \right)} \right]^{\frac{\mu}{1-\sigma}}, \quad (20)$$

equilibrium wages can be written as

$$w_i = \left( N_i^{1-\mu-\eta} \kappa S_i^{\frac{\mu}{1-\sigma}} \right)^{-\frac{1}{\mu+\eta}}, \quad i = C, P. \quad (21)$$

Ceteris paribus, the regional wage increases as skills increase or population declines.

We can now define the *short-run equilibrium* as, for any given interregional allocation of workers and skills, a set of prices, namely, wages, final good prices (normalized to unity), intermediate goods prices, and land-rent schedules, so that intermediate good suppliers and final good suppliers maximize profits, workers maximize utility, and supply equals demand in all markets, that is, land markets, intermediate goods markets and the final good market. Hence, the short-run equilibrium is determined by Equations (11), (17), (16), and (21). Together with regional final good production, (13), these equations also

determine aggregate land rent and, therefore, worker utility. In the case where neither population nor skills are fully agglomerated in one region, and income minus land expenses is positive in both regions, a short-run equilibrium exists for any interregional allocation of population and skills. Hence, if regions are somewhat similar, a short-run equilibrium exists.

## 2.2 Long-run equilibrium

In the long-run, workers are mobile and choose where to live so as to maximize utility. Hence, a *long-run equilibrium* is a short-run equilibrium with the additional property that no worker can increase utility by migrating to the other region, assuming that wages, prices, and rents do not change. In the long-run equilibrium, for all types  $s \in [\underline{s}, \bar{s}]$  holds that  $U_i(s) \geq U_k(s)$ ,  $k \neq i$ , if this worker lives in region  $i$ .

Obviously, a *perfectly symmetric long-run equilibrium* always exists: If half of each type of workers live in region  $C$  and the other half in region  $P$ , the regions are identical not only ex-ante, but also ex-post. Wages, prices, rents, and, therefore, utility for every type of worker are the same in both regions. All markets clear and there is no incentive to migrate. However, as Mori and Turrini (2005) stress, this type of equilibrium is essentially unstable if the adjustment is assumed to be myopic, that is, if workers move smoothly to the region where utility is highest. For example, if for some reason the composition of skills changed so that aggregate skills in region  $C$  are higher than aggregate skills in region  $P$ , but the regions are still identical in terms of pure numbers, normalized wages in region  $C$  will exceed those in region  $P$ , but land prices in the CBD will be identical. Thus, high-skilled workers would migrate to region  $C$  and low-skilled workers would prefer to move to region  $P$ , reinforcing the initial difference in skills, and ultimately resulting in full segmentation. The symmetry-breaking result in Mori and Turrini's (2005) model is driven by the interaction of skill heterogeneity and communication costs; here, however, symmetry is broken by the interplay of skill heterogeneity and land demand. Labor income depends on skills, but land demand does not. As will be shown in an extension, the crucial point is not that land demand is fully independent of skills, but that an increase in skills raises labor income more than the demand for land.

In addition to the symmetric equilibrium, there exists a *segmented long-run equilibrium* where all high-skilled workers live in one region and all low-skilled workers in the other. More precisely, a segmented long-run equilibrium is a long-run equilibrium where all workers of type  $s > \hat{s}$  live in region  $i$  and all workers of type  $s < \hat{s}$  live in region  $k$ , with  $k \neq i$ , where

$$U_C(\hat{s}) = U_P(\hat{s}). \quad (22)$$

Without loss of generality, we consider only those segmented equilibria where skilled workers live in region  $C$ , labeled the core, and unskilled workers live in region  $P$ , called the periphery. Hence,

$$S_C = N \int_{\hat{s}}^{\bar{s}} s f(s) ds, \quad S_P = S - S_C, \quad N_C = N \int_{\hat{s}}^{\bar{s}} f(s) ds, \quad \text{and} \quad N_P = N - N_C. \quad (23)$$

Segmentation occurs because variation in land expenses is the same for all workers, but labor income differences depend on skills.  $U_C(s) - U_P(s)$  is increasing in  $s$  if  $w_C > w_P$ . Given that wages in the core exceed those in the periphery, it is obvious that no worker has an incentive to migrate. High-skilled workers are better off in the core and low-skilled workers prefer to live in the periphery. Note that the critical skill level must be somewhere in the interior, that is,  $\underline{s} < \hat{s} < \bar{s}$ , since otherwise the price of land in one region's CBD will be zero and land demand by final good producers cannot be defined.

To compensate workers in the periphery for lower wages, land prices in the CBD need to be lower, too:  $r_P < r_C$ . Taking the relation between population and land prices, i.e., Equation (17), into account, lower land prices imply that the periphery is not only poor in skills but also less densely populated than the core:  $N_P < N_C$ . Since the average worker in the periphery has less human capital than the average worker in the core, the periphery's smaller population size leads to less aggregate skills, i.e.,  $S_P < S_C$ . Since the size of the intermediate good market is chiefly determined by labor supply, Equation (15) makes it evident that lower aggregate human capital results in a smaller number of intermediate good varieties:  $n_P < n_C$ . Along with lower wages, the smaller number of intermediate good varieties according to Equation (13) leads to a smaller final good sector in the periphery:  $X_P < X_C$ . This is true, even though, due to the mark-up pricing rule (11), intermediate good prices are comparatively low in the periphery:  $p_P < p_C$ . According to Equation (7),

per-unit costs are the same in both regions because a larger set of varieties compensate final-good producer in the core for higher wages, higher land rents, and higher intermediate good prices. Scarcity of land and heterogeneity of workers not only lead to segmentation, but also to agglomeration. The core is more productive and also larger. However, because land in the periphery is useful for final good production, agglomeration is only partial.

Furthermore, a segmented long-run equilibrium exists if for some  $\hat{s}$  that fulfills Equation (22) regions are still sufficiently similar in terms of population and skills. Finally, Equation (21) implies

$$\frac{dw_i}{d\hat{s}} = \frac{w_i}{\mu + \eta} \left[ \frac{(1 - \mu - \eta)(\sigma - 1)S_i - \mu\hat{s}N_i}{N_i S_i (1 - \sigma)} \right] \frac{dN_i}{d\hat{s}}, \quad i = C, P, \quad (24)$$

when  $dS_i/d\hat{s} = \hat{s}dN_i/d\hat{s}$  is taken into consideration. Suppose that  $1 - \eta - \mu > \mu/(\sigma - 1)$  holds which calls for a large partial output elasticity of land, a small partial output elasticity of the intermediate composite good, and a large elasticity of substitution of intermediate varieties. Land is decisive relative to any intermediate good variety. Then, acknowledging  $dN_C/d\hat{s} = -f(\hat{s})$  and  $\hat{s}N_C < S_C$ , it is evident that  $dw_C/d\hat{s}$  is positive. The wage in the core increases as the critical skill level rises and, therefore, population and aggregate skills shrink. By contrast, the wage in the periphery, where labor force, aggregate skills, and average productivity increase, may change in either direction, since  $\hat{s}N_P > S_P$ . Whether the wage differential increases or not, depends on the relative size of  $S_C/(\hat{s}N_C)$  and  $(N_P/N)/(S_P/S)$ . On the other hand, migration from the core to the periphery unambiguously implies a declining land rent differential. Putting wage effects and land rent effects together, the segmented long-run equilibrium will be stable if

$$(1 - t)(w_C - w_P) + \frac{f(\hat{s})N}{(\mu + \eta)(\sigma - 1)N_C N_P S_C S_P} \times \quad (25)$$

$$\left\{ (1 - \mu - \eta)(\sigma - 1)S_C S_P N \left[ (1 - t)w_C \hat{s} - \frac{N_C}{N} (N_C - N_P) \frac{k}{2} \right] \right.$$

$$\left. - \mu \hat{s} N_C N_P S \left[ (1 - t)w_C \hat{s} - \frac{S_C}{S} (N_C - N_P) \frac{k}{2} \right] \right\} + k f(\hat{s}) N > 0,$$

where the left-hand side is  $d[U_C(\hat{s}) - U_P(\hat{s})]/d\hat{s}$ . If this inequality is fulfilled, which is likely, provided that  $1 - \eta - \mu > \mu/(\sigma - 1)$ , an increase in  $\hat{s}$  raises the utility differential for the critical type  $\hat{s}$ . A lower skill type of worker would be indifferent between living in the core and living in the periphery. Migration flows restore the equilibrium.

Proposition 1 summarizes our results on long-run equilibria:

**Proposition 1** (i) *A perfectly symmetric long-run equilibrium always exists, but is essentially unstable.*

(ii) *If a segmented long-run equilibrium exists, wages, land rents, and intermediate good prices are higher in the core than in the periphery. The core is larger than the peripheral region in terms of aggregate skills, population, intermediate good production, and final good production. Stability of the segmented long-run equilibrium is dependent on condition (25).*

Due to pecuniary externalities, long-run equilibria are presumably spatially inefficient. Migration of certain groups of workers could increase welfare. Aggregate welfare, i.e., labor income plus land rents minus commuting costs,

$$W = w_C S_C + w_P S_P + R - r_C N_C - r_P N_P, \quad (26)$$

simplifies to

$$W = \frac{w_C S_C + w_P S_P}{\mu + \eta} - \frac{r_C N_C + r_P N_P}{2}, \quad (27)$$

when the properties of short-term equilibria are fully taken into consideration.

Equation (27) makes it easy to see that the symmetric long-run equilibrium is spatially inefficient. For this purpose, we compare the symmetric equilibrium with an allocation of workers, where workers are segmented according to skills, but both regions still have equal population. In this exercise, the right term in Equation (27) cancels out and the difference in labor income can be written as

$$\begin{aligned} & \left(\frac{N}{2}\right)^{-\frac{1-\mu-\eta}{\mu+\eta}} \kappa^{-\frac{1}{\mu+\eta}} \\ & \times \left\{ S_C^{1+\frac{\mu}{(\mu+\eta)(\sigma-1)}} - \left(\frac{S}{2}\right)^{1+\frac{\mu}{(\mu+\eta)(\sigma-1)}} - \left[ \left(\frac{S}{2}\right)^{1+\frac{\mu}{(\mu+\eta)(\sigma-1)}} - S_P^{1+\frac{\mu}{(\mu+\eta)(\sigma-1)}} \right] \right\}. \end{aligned} \quad (28)$$

Since  $1 + \mu/[(\mu + \eta)(\sigma - 1)] > 1$ , the difference in labor income is strictly positive given that  $S_C > S/2 > S_P$ . In contrast to many economic geography models, in the presence of agent heterogeneity full symmetry is clearly inefficient. Agglomeration of human capital without agglomeration of population raises welfare above the welfare level at the perfectly symmetric equilibrium. On the one hand, congestion and commuting costs are minimized



by symmetry in population. On the other hand, pecuniary externalities are exploited by agglomeration. However, this segmented symmetric allocation of workers would not be a long-run equilibrium, since workers in the periphery were not compensated for their lower wages by lower land rents.

The segmented long-run equilibrium asymmetrically allocates not only skills but also people. As a result, commuting costs are not minimized. Not surprisingly, the segmented long-run equilibrium is generically also spatially inefficient. Calculating the impact on welfare of a change in the critical skill type at the long-run equilibrium where Equation (22) holds, yields

$$\begin{aligned} \frac{dW}{d\hat{s}} \Big|_{U_C(\hat{s})=U_P(\hat{s})} &= \frac{1}{\mu + \eta} \left( S_C \frac{dw_C}{d\hat{s}} + S_P \frac{dw_P}{d\hat{s}} \right) \\ &\quad - \frac{1}{2} \left( N_C \frac{dr_C}{d\hat{s}} + N_P \frac{dr_P}{d\hat{s}} \right) - \left( \frac{1}{2} - \frac{1}{(1-t)(\mu + \eta)} \right) f(\hat{s})N(r_P - r_C). \end{aligned} \quad (29)$$

For some parameters, this term is clearly positive, but a negative sign cannot be ruled out analytically. As confirmed by simulations, over-agglomeration is the most likely, but under-agglomeration appears to be possible. Whether the core will shrink or grow depends on the relative size of commuting costs and pecuniary externalities.

Proposition 2 summarizes our inefficiency results:

**Proposition 2** (i) *The perfectly symmetric long-run equilibrium is spatially inefficient.*  
(ii) *The segmented long-run equilibrium is also generically spatially inefficient.*

### 2.3 Federal taxation

Although interpersonal redistribution is the intended effect of progressive federal taxation, it also has a strong spatial dimension. The federal tax is highly unequal in a geographical sense (see Albouy, 2008), federal taxation redistributes from high-wage-high-rent regions to low-wage-low-rent regions. Federal tax not only redistributes across regions, it also affects the interregional allocation of workers and production. In our model, federal taxation changes the critical skill level in the segmented long-run equilibrium. From the migration equilibrium equation (22), it immediately follows that federal taxation increases the critical

skill level, provided the long-run equilibrium is stable, since

$$\frac{\partial [U_C(\hat{s}) - U_P(\hat{s})]}{\partial t} = (w_P - w_C)\hat{s} < 0. \quad (30)$$

Hence, federal taxation reduces regional differences in population, aggregate skills, and production. The periphery grows at the expense of the core. Whether federal taxation raises aggregate welfare depends on the direction of spatial inefficiency at the no-tax equilibrium, and also on the tax rate. Simulations suggest that low tax rates often raise welfare by averting over-agglomeration. By extension, too high a rate can result in under-agglomeration.

## 2.4 Trade in intermediate goods

To this point, trade in final goods has been taken as frictionless, but prohibitively high barriers have foreclosed the possibility of trade in intermediate goods. In this subsection, we look at a more realistic situation and allow trade in intermediate goods. For this purpose, we assume iceberg costs  $\tau - 1$  for trade in intermediate goods, with  $\tau > 1$ . Intermediate good supplier export part of their output albeit at higher costs. The intermediate good quantity and price indices in region  $i$  are

$$I_i = \left[ \int_0^{n_i} q_{ii}(j)^{\frac{\sigma-1}{\sigma}} dj + \int_0^{n_k} q_{ik}(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad (31)$$

$$P_i = \left[ \int_0^{n_i} p_i(j)^{-(\sigma-1)} dj + \int_0^{n_k} \phi p_k(j)^{-(\sigma-1)} dj \right]^{-\frac{1}{\sigma-1}}, \quad (32)$$

for  $k \neq i$ , where  $q_{ik}(j)$  denotes region  $i$ 's demand for quantity  $j$  produced in region  $k$ .  $\phi = \tau^{1-\sigma}$  indicates "trade freeness", with  $0 \leq \phi < 1$ . Profit maximization of final good producing firms determine the demand for local and foreign intermediate goods

$$q_{ii}(j) = \mu \frac{X_i P_i^{\sigma-1}}{p_i^\sigma} \quad \text{and} \quad q_{ik}(j) = \mu \frac{X_i P_i^{\sigma-1}}{(\tau p_k)^\sigma}, k \neq i. \quad (33)$$

Intermediate good suppliers in region  $i$  have demand

$$q_i(j) = q_{ii}(j) + \tau q_{ki}(j), \quad k \neq i. \quad (34)$$

However, as is well known, mill pricing is optimal and the mark-up rule is the same as without trade, i.e. as determined by (11). Furthermore, trade also leaves individual in-

intermediate good quantities per firm unaltered; Equation (12) still holds. But, since intermediate good supply meets demand from both regions, the intermediate good market equilibrium conditions become

$$\frac{(\sigma - 1)F}{a} = \mu \frac{X_i P_i^{\sigma-1}}{p_i^\sigma} + \phi \mu \frac{X_k P_k^{\sigma-1}}{p_i^\sigma}, \quad k \neq i. \quad (35)$$

Substituting for prices and price indices, yields final good demand

$$X_i = \frac{F\sigma (w_i^\sigma - \phi w_k^\sigma) (n_i w_i^{1-\sigma} + \phi n_k w_k^{1-\sigma})}{\mu(1 - \phi^2)}, \quad k \neq i. \quad (36)$$

This type of equilibrium, where both regions produce intermediate and final goods, requires a moderate wage differential, i.e.  $\phi < (w_C/w_P)^\sigma < 1/\phi$ . Existence calls for sufficiently high trade costs.<sup>8</sup> Since final good demand is related to wages and quantities in both regions, the number of varieties cannot be explicitly calculated:

$$n_i = \frac{S_i - \frac{\eta X_i}{w_i}}{\sigma F}. \quad (37)$$

What can be said, though, is that, in contrast to separated markets for intermediate goods, the number of varieties is not simply proportional to regional aggregate skills. Zero-profit conditions of final good producers can be written as

$$\kappa \left[ \left( 1 + \frac{\eta}{\mu} \right) F\sigma \right]^{\frac{\mu}{1-\sigma}} N_i^{1-\mu-\eta} w_i^\eta (n_i w_i^{1-\sigma} + \phi n_k w_k^{1-\sigma})^{\frac{\mu}{1-\sigma}} = 1, \quad k \neq i. \quad (38)$$

Finally, it should be stressed that trade in intermediate goods has no direct impact on land markets.

Just like in the case without trade in intermediate goods, a symmetric long-run equilibrium always exists and a segmented long-run equilibrium exists for a wide range of parameters. Unfortunately, Equations (37) and (38) do not allow for explicit solutions for number of varieties and regional wages and we are thus not able to fully characterize short-run and long-run equilibria analytically. However, the segmented long-run equilibrium for both the case with and without trade in intermediate goods are very similar. Wages and land rents in the CBD of the core region exceed those in the periphery, the

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<sup>8</sup>We neglect corner equilibria where final good production or intermediate good production are concentrated in one region.

core is more densely populated, and the core has a larger stock of human capital:  $w_C > w_P$ ,  $r_C > r_P$ ,  $N_C > N_P$ , and  $S_C > S_P$ . Furthermore, from Equation (38) it follows that  $n_C w_C^{1-\sigma} + \phi n_P w_P^{1-\sigma} > n_P w_P^{1-\sigma} + \phi n_C w_C^{1-\sigma}$ , which results in  $n_C > n_P$ . Together with Equation (36), it also implies  $X_C > X_P$ . Hence, regardless of whether trade in intermediate goods is feasible, the core produces more final and intermediate goods despite the fact that intermediate good mill prices are higher in the core than in the periphery, i.e.  $p_C > p_P$ . The following proposition summarizes these findings.

**Proposition 3** *At the segmented long-run equilibrium, even with trade in intermediate goods, the core will be larger than the periphery in terms of aggregate skills, population, and production. Wages, land rents, and intermediate good prices will be higher, too.*

Trade effects can be analyzed numerically. To this end, we use two density functions,  $f_1 = 1/(\bar{s} - \underline{s})$  and  $f_2(s) = 1/(s \ln \bar{s} - s \ln \underline{s})$ . Both distributions, uniform and hyperbolic, lead to similar simulation results.<sup>9</sup> An increase in  $\phi$  raises wages in both regions, but scales down the wage differential, and also diminishes the interregional differences in aggregate skills, population, land rents, and production. Moreover, the efficiency gain from reduction in interregional trade barriers causes an increase in final good production in both regions.

Opening interregional markets for intermediate goods increases the number of varieties available in both regions, which, in turn, lowers the unit costs of final goods. Production becomes more efficient, income and utility go up across the board. However, trade integration at the intermediate good level reduces the core's skill advantage, since the periphery is no longer cut off from the variety of intermediate goods produced in the core. Regional differences diminish, but they do not disappear. Although trade integration slightly dampens agglomeration, even if trade were almost free, agglomeration forces would continue to be substantial. Dispersion forces are always too weak to overcome the strong forces that cause segmentation and, as a result, agglomeration. Skill heterogeneity combined with far-reaching land-demand homogeneity breeds segmentation, regardless of whether there is trade in intermediate goods.

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<sup>9</sup>Simulations are available from the author on request.

### 3 Modifications and extensions

In this section, the basic model is modified and extended in a number of ways as a way of checking its robustness. We analyze attachment to home, the presence of immobile workers, and alternative land use. For the sake of simplicity, we ignore trade in intermediate goods throughout this section.

#### 3.1 Attachment to home

In the basic model, all workers are perfectly mobile. We begin the analysis of imperfect mobility by allowing for one important source of immobility attachment to the home region. Following, Wellisch (2000) and others, we assume that workers differ in their psychic attachment to their home region and that the psychic utility component is additively separable. Types are two-dimensional, a skill dimension,  $s$ , and a regional preference dimension,  $\nu$ , uniformly distributed on  $[0, 1]$ . For each  $(s, \nu)$ -type preferences are given by

$$V(s, \nu) = \begin{cases} U_C(s) + \beta(s)(1 - \nu) & \text{if she lives in } C. \\ U_P(s) + \beta(s)\nu & \text{if she lives in } P, \end{cases} \quad (39)$$

where  $\beta(s) \geq 0$ . For each skill type, the pecuniary utility  $U_i(s)$  is the same. Workers with a small  $\nu$  have a preference for living in region  $C$ ; workers with a large  $\nu$  prefer region  $P$ .  $\beta$  measures the degree of household mobility. If  $\beta(s) = 0$ , the worker is perfectly mobile,  $\beta(s) > 0$  means imperfect mobility. If  $\beta(s) \rightarrow \infty$ , the worker becomes perfectly immobile. For each type, a critical spatial preference could be calculated:

$$\hat{\nu}(s) = \max \left\{ \min \left\{ \frac{1}{2} + \frac{U_C(s) - U_P(s)}{2\beta(s)}, 1 \right\}, 0 \right\}. \quad (40)$$

At the long-run (weakly) segmented equilibrium with attachment to home, all workers of type  $(s, \nu)$  live in region  $C$  if  $\nu < \hat{\nu}(s)$  and in region  $P$  if  $\nu > \hat{\nu}(s)$ . The core's population and aggregate skill at the long-run equilibrium is given by

$$N_C = N \int_{\underline{s}}^{\bar{s}} \hat{\nu}(s) f(s) ds \quad \text{and} \quad S_C = N \int_{\underline{s}}^{\bar{s}} s \hat{\nu}(s) f(s) ds. \quad (41)$$

We assume first that the degree of mobility is independent of the skill level:  $\beta(s) = \beta$ . For uniform household mobility and  $w_C > w_P$ , the critical spatial parameter  $\hat{\nu}$  is a monotonically non-decreasing function of the skill level, since the pecuniary utility differential

$U_C(s) - U_P(s)$  is increasing in  $s$ . The fraction of highly-skilled workers living in the core is large compared to the number of low-skilled workers living there. Furthermore, an increase in attachment to home raises the fraction of low-skilled workers, with  $U_C(s) < U_P(s)$ , in the core, but reduces the fraction of the highly-skilled, where  $U_C(s) > U_P(s)$ . If attachment to home is rather small, there is segmentation within skill groups only at an interval around some critical skill level (see figure 1). Within this group, some workers live in the core and others in the periphery. By contrast, all workers with rather high skill levels live in the core, and all workers with quite low skills reside in the periphery. The fact that the long-run equilibrium with attachment to home does not exhibit perfect stratification of skill levels is pretty consistent with empirical observations.

If  $\beta \rightarrow 0$ , the long-run equilibrium converges to the long-run segmented equilibrium of the basic model. If, however,  $\beta \rightarrow \infty$ , the equilibrium approaches the perfectly symmetric equilibrium of our basic model, since  $\hat{v}(s) \rightarrow 1/2$  for all skill levels. Starting at full mobility, an increase in  $\beta$  leads in the core to immigration of low-skilled and emigration of high-skilled workers. The most likely outcome is a decrease in aggregate skills in the core and an increase of same size in the periphery. Simulations show an increasing degree of equalization. As a result, wages in the core shrink, while wages in the periphery rise – provided that the implied changes in population do not overcompensate skill effects. Agglomeration forces become weaker and regional differences gradually disappear.<sup>10</sup> In other words, increasing mobility typically amplifies agglomeration.

If attachment to home and skills were negatively correlated, i.e. if  $\beta'(s) < 0$ , low-skilled workers would be uniformly distributed, but the highly-skilled would be skewed toward the core. Compared to homogenous attachment to home, negative correlation of skills and immobility should enlarge the core.

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<sup>10</sup>Since both changes in population size and changes in aggregate skills affect the pecuniary utility differential and, therefore, the critical spatial preference level for all types, it cannot be ruled out analytically that for some levels of immobility a small increase in the attachment to home strengthens agglomeration forces. However, simulations have always shown monotonic changes.

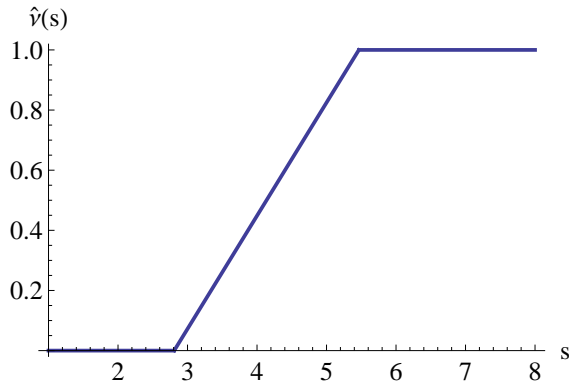


Figure 1: Skills and residence choices for  $\beta = 0.05$ :  $\hat{v}(s)$

### 3.2 Comparable immobile workers and asymmetry

We have already seen that immobility weakens agglomeration. In this subsection, we will test the robustness of this hypothesis by considering a country that contains, along with perfectly mobile workers, a group of perfectly immobile workers with a shared skill level. In addition, we analyze ex-ante differences in size.

Suppose that region  $i$  is occupied by  $N_{Ii}$  immobile workers with common skill level  $s_I$ . Apart from the different skill levels and the different degrees of mobility, immobile and mobile workers are interchangeable. In particular, they are perfect substitutes at work. As a consequence, their presence alters only population size and aggregate skills. In the presence of segmentation among mobile workers, population and aggregate skills in the core are:

$$N_C = N_M \int_{\hat{s}}^{\bar{s}} f(s) ds + N_{Ii} \quad \text{and} \quad S_C = N_M \int_{\hat{s}}^{\bar{s}} s f(s) ds + s_I N_{Ii}, \quad (42)$$

where  $N_M$  is the number of mobile workers. The definitions for the periphery correspond accordingly. The impact of immobile workers on the segmented long-run equilibrium is inferred from the migration equilibrium condition, Equation (22). From Equation (42) and (21), we obtain

$$\frac{dw_i}{dN_{Ii}} = \frac{w_i}{\mu + \eta} \left[ \frac{(1 - \mu - \eta)(\sigma - 1)S_i - \mu s_I N_{Ii}}{N_i S_i (1 - \sigma)} \right]. \quad (43)$$

Starting at ex-ante symmetry, a small increase in the number of immobile workers in region  $i$  reduces the wage in that region if three conditions are satisfied. First, the long-run

equilibrium must be stable, i.e., the stability condition (25) is fulfilled. Second,  $1 - \eta - \mu > \mu/(\sigma - 1)$  holds. This condition also relaxes stability requirements. Third, the skills of the immobile workers are below the average skills of mobile workers in the region, meaning that additional immobile workers raise the number of workers, but not their average productivity. Land prices in the CBD, being proportional to population size, unambiguously rise. Income net of land rents and commuting expenses declines. The region becomes less attractive to mobile workers and they emigrate.<sup>11</sup>

**Proposition 4** *Starting at ex-ante symmetry, a small increase in the number of immobile workers in one region leads to emigration of mobile workers if (a) the stability condition (25) is fulfilled, (b)  $1 - \eta - \mu > \mu/(\sigma - 1)$  holds, and (c) skills of immobile workers are below average skills of mobile workers in the respective region.*

Interestingly, it is not only in the periphery that the immobile population grows at the expense of the mobile population, but also in the core.

If the number of immobile workers grows simultaneously in both regions, the land-rent differential remains unchanged. Only the wage differential would shrink. Were the number of immobile workers ultimately quite large compared to the number of mobile workers and were skill differentials between mobile and immobile workers not too large, mobile workers would have only a minor influence on wages. Differences between the two regions regarding wages, population, aggregate skills, and production would all but disappear.

### 3.3 Complementary immobile workers and asymmetry

The effects on wages and agglomeration forces of the presence of immobile workers that lead to complementary inputs might be quite different from those when workers are perfect substitutes in production processes. To show this, we assume again that region  $i$  is occupied by  $N_{Ii}$  immobile workers with skill level  $s_I$ . We define  $S_i$  as regional aggregate skills of mobile workers exclusive of immobile workers and  $N_i$  as the regional mobile population. Since mobile and immobile workers have different inputs to offer, the production function

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<sup>11</sup>If  $s_I$  were equal to  $\hat{s}$ , dislocation of immobile workers from the periphery to the core ultimately would raise both the share of immobile workers and total population in the core.



at the final good level will be  $X_i = S_{xi}^\eta (s_I N_{Ii})^\gamma I_i^\mu L_{xi}^{1-\mu-\eta-\gamma}$ . Therefore, demand for immobile workers' skills will be  $s_I N_{Ii} = \gamma X_i / w_{Ii}$ , where  $w_{Ii}$  is the immobile worker's wage. Land markets operate as in the basic model and tax revenue is  $T = t \sum_{i=C,P} (w_i S_i + w_{Ii} s_{Ii} N_{Ii})$ . Assuming that intermediate good production still relies on skilled mobile labor only, the mark-up pricing rule of intermediate good suppliers (11), the equilibrium output per variety, Equation (12), the demand rule for intermediate goods (13), and the equilibrium number of varieties, Equation (15), are still valid. Defining

$$\tilde{\kappa} = \left(\frac{k}{2}\right)^{1-\mu-\eta-\gamma} \psi \left(\frac{a\sigma}{\sigma-1}\right)^\mu \gamma^\gamma (\mu+\eta)^{-\gamma-\frac{\mu}{1-\sigma}} \left(\frac{\mu}{F\sigma}\right)^{\frac{\mu}{1-\sigma}}, \quad (44)$$

where  $\psi = \mu^{-\mu} \eta^{-\eta} \gamma^{-\gamma} (1-\mu-\eta)^{-(1-\mu-\eta)}$ , the final good market equilibrium condition (8), can be written as

$$1 = (N_i + N_{Ii})^{1-\mu-\eta-\gamma} \tilde{\kappa} S_i^{\gamma+\frac{\mu}{1-\sigma}} w_i^{\mu+\eta+\gamma} (s_I N_{Ii})^{-\gamma}, \quad i = C, P. \quad (45)$$

Hence, equilibrium wages are

$$w_i = \left[ (N_i + N_{Ii})^{1-\mu-\eta-\gamma} \tilde{\kappa} S_i^{\gamma+\frac{\mu}{1-\sigma}} (s_I N_{Ii})^{-\gamma} \right]^{-\frac{1}{\mu+\eta+\gamma}}, \quad i = C, P. \quad (46)$$

Thus,

$$\frac{dw_i}{dN_{Ii}} = \frac{w_i [\gamma (N_i + N_{Ii}) - (1-\mu-\eta-\gamma) N_{Ii}]}{(\mu+\eta+\gamma) (N_i + N_{Ii}) N_{Ii}}, \quad i = C, P. \quad (47)$$

An increase in the immobile worker population raises the wage of mobile workers if there are many mobile workers, the output elasticity of immobile workers is large, the output elasticity of land is small, and/or the number of immobile workers is small. Then, under conditions of stability, i.e., provided that  $d[U_C(\hat{s}) - U_P(\hat{s})]/d\hat{s} > 0$ , mobile workers immigrate. Otherwise, mobile workers see their pay checks shrink and emigrate. The following proposition summarizes this result.

**Proposition 5** *Suppose the segmented long-run equilibrium is stable. A small increase in the number of immobile workers in one region leads to immigration of mobile workers if  $\gamma(N_i + N_{Ii}) > (1-\mu-\eta-\gamma)N_{Ii}$  and to emigration of mobile workers otherwise.*

Similarly, uniform growth of the immobile worker population in both regions may or may not strengthen agglomeration. Simulations show that it is easy to find parameters that will support either agglomeration or dispersion.

### 3.4 Fixed area

The basic model assumed that land is abundant and thus that land prices in the CBD were determined through commuting costs by resident demand and, therefore, directly by population size. In this subsection, we discuss whether binding constraints on available land change the basic pattern of segmentation and agglomeration. When all land is in use, the final good suppliers' demand for land, along with population size, determines land prices in the CBD. Since  $L - N_i$  units of land are available for production, the demand for land, Equation (9), dictates the CBD land price:

$$r_i = (1 - \mu - \eta) \frac{X_i}{L - N_i}, \quad i = C, P. \quad (48)$$

Increases in population and production will raise the price of land. Importantly, the price of land in the CBD is no longer simply proportional to population; the relationship is now far more complicated compared to the basic model. Multiplying land area and subtracting commuting costs yields the aggregate regional land rent and thus

$$R = (1 - \mu - \eta)L \left( \frac{X_C}{L - N_C} + \frac{X_P}{L - N_P} \right) - \frac{k(N_C^2 + N_P^2)}{4}. \quad (49)$$

Neither the intermediate goods market nor the labor market are directly affected by the changes in the land market, but the zero-profit conditions of final good suppliers must be modified. Defining

$$\hat{\kappa} = \psi \left( \frac{a\sigma}{\sigma - 1} \right)^\mu \left( \frac{1 - \mu - \eta}{\mu + \eta} \right)^{1 - \mu - \eta} \left( \frac{\mu}{F\sigma(\mu + \eta)} \right)^{\frac{\mu}{1 - \sigma}}, \quad (50)$$

the final good market equilibrium condition (8), can be written as

$$1 = \left( \frac{1}{L - N_i} \right)^{1 - \mu - \eta} \hat{\kappa} S_i^{1 - \mu - \eta + \frac{\mu}{1 - \sigma}} w_i, \quad i = C, P. \quad (51)$$

Hence, equilibrium wages are

$$w_i = (L - N_i)^{1 - \mu - \eta} \hat{\kappa}^{-1} S_i^{-\left(1 - \mu - \eta + \frac{\mu}{1 - \sigma}\right)}, \quad i = C, P. \quad (52)$$

The basic structure of the short-run equilibrium remains unchanged. However, changes in the land market have substantial implications for the long-run equilibrium. When land is abundant, the land-price differential  $r_C - r_P$  is directly determined by the difference

in population size; however when land is constrained, the land-price differential  $r_C - r_P$  is determined by the demand for land and thus via wages and number of varieties by population size and aggregate skills. This has several implications: First, the stability condition becomes more complicated. Second and most important, the core region, albeit larger in terms of human capital and production, might be less densely populated than the peripheral region. Land prices in the core are higher not only due to resident demand, but also because of final good producers' demand. Thus, compensating low land prices in the periphery do not require a lower population density. Third, any homogenous variation in population normally alters the land-rent differential. However, the basic results continue to hold true. Typically, both the perfectly symmetric long-run equilibrium and a segmented long-run equilibrium will exist. Both are generically spatially inefficient. Simulations show that federal taxation reduces regional differences as to population size, aggregate skills, and production. Trade in intermediate goods has effects on agglomeration similar to those in the basic model.

### 3.5 Endogenous land demand

Since the interplay between resident land demand and human capital supply is the basic force behind segmentation, we test the robustness of our result for endogenous individual land demand. For this purpose, we assume fixed area of land and disregard commuting costs. Assuming Cobb-Douglas utility defined on final good consumption and as a proxy for housing lot size, indirect utility is

$$U_i(s) = \frac{(1-t)w_i s + \frac{R+T}{N}}{r_i^{1-\alpha}}, \quad i = C, P, \quad (53)$$

where  $1 - \alpha$  is the weight of land, with  $0 < \alpha < 1$ . Cobb-Douglas utility implies the land-demand function  $L_i = (1 - \alpha) [(1-t)w_i s + (R+T)/N] / r_i$  which exhibits unit income elasticity. From regional land-market equilibrium conditions

$$L = (1 - \mu - \eta) \frac{X_i}{r_i} + (1 - \alpha) \frac{(1-t)w_i S_i + N_i \frac{R+T}{N}}{r_i}, \quad i = C, P, \quad (54)$$

and the definition of aggregate land rent  $R = (r_C + r_P)L$  and tax revenue, Equation (2), land prices and aggregate land rent can be determined:

$$\begin{aligned}
r_i &= \frac{1}{\alpha L(N_i + N_k)} \{M_i(1 - \mu - \eta)(N_i + \alpha N_k) + (1 - \alpha)[M_k(1 - \mu - \eta)N_i \\
&\quad + S_i(N_i + \alpha(1 - t)N_k)w_i + N_i S_k(1 - \alpha(1 - t))w_k]\}, \quad i = C, P, k \neq i, \\
R &= \frac{1}{\alpha} \sum_{i=C,P} [(1 - \mu - \eta)X_i + (1 - \alpha)w_i S_i].
\end{aligned} \tag{55}$$

Twofold interregional redistribution from the periphery to the core is responsible for the complexity of the land price formula. First, equal land sharing redistributes from the high-rent region to the low-rent region. Second, federal taxation diverts income from the high-wage region to the low wage-region. As a result, the aggregate land rent is larger than the respective expenditure share of land calculated for gross labor income.

Intermediate goods markets and labor markets are not directly affected by consumers' endogenous land demand, but the zero-profit conditions at the final good level are different from those conditions in the basic model. Defining,

$$\bar{\kappa} = \psi \left( \frac{a\sigma}{\sigma - 1} \right)^\mu \left( \frac{1}{\alpha(\mu + \eta)} \right)^{1 - \mu - \eta} \left( \frac{\mu}{F\sigma(\mu + \eta)} \right)^{\frac{\mu}{1 - \sigma}}, \tag{56}$$

the final good market equilibrium condition (8), can be written as

$$\begin{aligned}
1 &= \bar{\kappa} S_i^{\frac{\mu}{1 - \sigma}} w_i^{\mu + \eta} \left( \frac{1}{LN} \right)^{1 - \mu - \eta} (S_i \{[1 - \alpha(\mu + \eta)](N_i + \alpha N_k) - (1 - \alpha)\alpha \\
&\quad (\mu + \eta)N_k t\} w_i + (1 - \alpha)N_i S_k [1 - \alpha(\mu + \eta)(1 - t)]w_k)^{1 - \mu - \eta} \quad i = C, P, k \neq i.
\end{aligned} \tag{57}$$

Essential non-linearity makes explicit solutions for regional wages impossible. However, numerical simulations show results qualitatively similar to those employing exogenous land use by residents. Segmentation is the likely outcome of worker migration, accompanied by agglomeration of skills and production, though the core region is sparsely populated relative to the peripheral region.

Segmentation occurs even though high-skilled workers buy larger lots than do low-skilled workers. To clarify the relationship between the demand for land and the preference for the high-wage-high-rent region, suppose that (indirect) utility could be written as  $V[sw, r(w)]$ , where  $r(w)$  captures the empirical cross-region-relationship between wages

and rents, with  $dr(w)/dw > 0$ . For the critical skill type, using Roy's identity,

$$\frac{dV[sw, r(w)]}{dw} = \frac{\partial V[sw, r(w)]}{\partial y} s + \frac{\partial V[sw, r(w)]}{\partial r} \frac{dr(w)}{dw} = \frac{\partial V[sw, r(w)]}{\partial y} \left[ s - l \frac{dr(w)}{dw} \right] \quad (58)$$

is zero, where  $y$  is individual income and  $l$  is individual land demand. Assuming a negative second derivative with respect to  $w$ , the more highly-skilled prefer a higher wage if

$$\left. \frac{d^2V[sw, r(w)]}{dw ds} \right|_{\frac{dV}{dw}=0} = \frac{\partial V[sw, r(w)]}{\partial y} \left[ 1 - \frac{dl(s)}{ds} \frac{dr(w)}{dw} \right] > 0. \quad (59)$$

Hence, if

$$\frac{dl(s)}{ds} \frac{s}{l} < 1, \quad (60)$$

that is, if the skill elasticity of the demand for land is lower than 1, skills and preferred wages are positively correlated. Taking rent income into account, Cobb-Douglas utility meets this requirement.

## 4 Concluding remarks

This paper analyzed regional patterns of production and housing in the presence of pecuniary externalities within a general-equilibrium framework with monopolistic competition at intermediate good markets. First, it showed that the interplay of heterogeneous skills and comparatively homogeneous land demand triggers skill segmentation and agglomeration. The core region, being more attractive to high-skilled workers, gains a disproportionately large share of production at all levels of the supply chain. Second, the paper demonstrated that federal taxation that automatically redistributes toward the periphery might be welfare enhancing by averting over-agglomeration. Third, the paper showed that a reduction in intermediate trade costs weakens agglomeration by narrowing the interregional wage gap. Fourth, it was demonstrated that rising immobility alleviates interregional differences, provided that immobile workers are not complementary factors in production. This implies that relocating substitutable immobile workers from the periphery to the core will initiate a countervailing migration flow of mobile workers. Fifth, the paper showed that whenever agglomeration of labor income and production raises land prices per se, the core may become less densely populated than the periphery.

For clarity of exposition, the model was set-up as a two-region model, but it could be easily extended to a multi-region model. Segregation and agglomeration would still be triggered by wage gaps and land rent differences. In the basic model without trade in intermediate goods, all major results would hold true. The perfectly symmetric equilibrium would be unstable. At the segmented long run-equilibrium, regions could be ranked according to aggregate skills, population size, output, wages, and land rents. Migration would lead to a strictly monotonic relationship between average skills and size in terms of industry and population.

Certain limitations of the model open the door to future research. Further agglomeration forces, such as knowledge spillovers, could be integrated into the model. Including imperfect competition at the final good sector, thus inducing a market-access effect and a cost-of living effect, would increase our understanding of the forces of segmentation and agglomeration.

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