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Tracking the German Business Cycle

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Abstract

The German economy is an important economic driver in the Euro-area in terms of gross domestic product, labour force and international integration. We provide a state of the art estimate of the German output gap between 1995 and 2021 and present a nowcasting scheme that accurately predicts the Germany output gap up to three months prior to a gross domestic product data release. To this end, we elicit a mixed-frequency vector-autoregressive model in the spirit of Berger, Morley, and Wong (forthcoming) who propose to use monthly information to form an expectation about the current-quarter output gap. The mean absolute error of our nowcast is very small (0.25 percentage points) after only one month of observed data. Moreover, we show that international trade and labour market aggregates consistently explain large shares of variation in the German output gap.

Keywords: output gap, Germany, nowcast, mixed frequency, vector-autoregression

JEL Classification: E32, E37, C53

1 Introduction

The German economy is the fifth largest economy in the world and with a GDP weight of about 28% in 2021 the single largest economy in the Euro area. Therefore, the German economic situation is of key importance for economic flourishing and stability in the European Union. For the conduct of monetary policy in the Euro area, the European Central Bank relies on measurements of a Euro area business cycle. The latter is driven to a large extend by the German business cycle, the focus of the paper at hand.

Central banks and other policy institutions frequently use the output gap, i.e. the deviation of actual from potential real gross domestic product (GDP) as a measure of the business cycle. Being intrinsically unobserved, the output gap has to be estimated. Numerous models and filtering approaches have been proposed to this end (most prominently, Hodrick and Prescott (1997) and more recently Hamilton (2018)). However, the vast majority of available procedures only yield retrospective insights into the output gap due to a significant delay in GDP data availability. As a recent exception, Berger, Morley, and Wong (forthcoming) (BMW (forthcoming), henceforth) propose to nowcasting the output gap using a multivariate Beveridge-Nelson decomposition based on a mixed-frequency Bayesian vector-autoregressive model (VAR). They show that a model comprising economic aggregates available at monthly frequency implies a reasonable output gap for the U.S. economy well in advance current quarter GDP data is released.

Our focus is to estimate the German business cycle by means of the output gap using the approach of BMW (forthcoming). Additionally, we employ a model selection procedure to select the most relevant variables for estimating the German output gap. Using an informational decomposition allows us to quantify the relative importance of each variable. Moreover, we analyze the contribution of each variable in each month within a quarter. The accuracy of the models nowcasts are evaluated by comparing the nowcast after each month of a given quarter to the final estimate obtained using the full information set.

The contribution is twofold. First, we provide an account of the German business cycle. German business cycle fluctuations are monitored by the German council of economic experts ('Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung') at the behest of the federal government. As part of its mandate, the council presents a comprehensive summary of the main economic developments. The biannual expert reports make an important contribution to understanding the German economy. However, academic accounts of the German business cycle are sparse. Second, by tracking the German business cycle up to three months prior to GDP data release, we provide timely information about the stance of the largest economy in the Euro area that are crucial for the conduct of monetary and fiscal policy. The impact of large economic shocks, such as the COVID-19 pandemic or recent oil-market disturbances can be straightforwardly tracked within a month within a given quarter, rather than six weeks after the previous quarter.

We demonstrate that our model estimate of the output gap lines up reasonably well with established filter and production-function based measures. Furthermore, we find that fluctuations in the German business cycle are mainly transmitted by real economic aggregates and labour market fluctuations. Moreover, in line with the related literature we show that external relations play a key role for the German business cycle (see e.g. Eickmeier (2007)), whereas labour market aggregates are informative in times of large deviations from potential output. Regarding the nowcast accuracy, we find that after the first month of a quarter, the nowcast has a mean absolute error of 0.25 percentage points from the final estimate of the output gap. Even in times of substantial volatility, our model yields robust results without observing current quarter GDP data. For instance, the output gap in the COVID-19 induced recession in 2020Q2 was nowcasted to be -9.4% after the release of the June 2020 monthly data. It turned out to be -9.4% after the release of the entire quarter data (including GDP).

Sections 2 and 3 present our empirical approach and the data, respectively. In Sections 4 and 5, we discuss our results. Section 7 concludes.

2 Methodology

We estimate a model in the spirit of BMW (forthcoming), who propose a mixed-frequency bayesian vector-autoregression (MF-BVAR) to obtain the output gap c_t as the cyclical component of output from a multivariate Beveridge-Nelson decomposition. More precisely, if y_t is a $K \times 1$ vector of macroeconomic observables with $K \times 1$ drift component μ, its Beveridge-Nelson trend τ_t is given as $\lim_{h\to\infty} \mathbb{E}_t\left(y_{t+h} - h\mu\right)$ and the cycle can be obtained as $c_t = y_t - \tau_t$. Assuming stationarity and zero mean, Morley and Wong (2020) show that the cycle c_t is given by

$$c_t = -\boldsymbol{F}(\boldsymbol{I}_K - \boldsymbol{F})^{-1}\boldsymbol{X}_t, \tag{1}$$

where $\boldsymbol{I}_K$ is the identity matrix of rank K and the remaining quantities derive from the vector-autoregressive model $\boldsymbol{X}_t = \boldsymbol{F}\boldsymbol{X}_{t-1} + \boldsymbol{H}\epsilon$ with $\epsilon \sim \mathcal{N}(0, \Sigma)$ (BMW forthcoming). In particular, $\boldsymbol{X}_t$ and ϵ are $K \times 1$ vectors of macroeconomic observables and innovations in t, respectively. $\boldsymbol{F}$ is a companion-form coefficient matrix and $\boldsymbol{H}$ maps the innovations into companion form. We stack all high-frequency (HF) variables above the lower frequency (LW) variables. For instance, assume one low-frequency period can be subdivided into $d = 1, ..., D$ equidistant high-frequency-periods, then we obtain $\boldsymbol{X}_t$ as

$$\boldsymbol{X}_t = \begin{bmatrix} \boldsymbol{x}^{HF}_{t-d-1/D} \\ \vdots \\ \boldsymbol{x}^{HF}_{t-1/D} \\ \boldsymbol{x}^{HF}_{t} \\ \boldsymbol{x}^{LF}_{t} \end{bmatrix},$$

where $\boldsymbol{x}_t$ indicate partitions of $\boldsymbol{X}_t$ for the high-frequency series in $d = 1, ..., D$ in t. To contain parameter proliferation, we follow Morley and Wong (2020) in adopting bayesian shrinkage, where the shrinkage parameter λ is chosen such that the one-step-ahead root mean squared error of output is minimized.[1] Finally, we use a standard Minnesota prior for location and scale parameters.

The higher (in our case, monthly) frequency information is exploited in the spirit

[1] In our final specification, we obtain $\lambda = 0.29$.

of Waggoner and Zha (1999) to update the vector-autoregression for the subsequent period. To this end, we note that (by positive-definiteness) the innovation covariance $\boldsymbol{\Sigma}$ obeys a representation $\boldsymbol{\Sigma} = \boldsymbol{B}\boldsymbol{B}'$, where $\boldsymbol{B}$ is the lower-triangular Choleski factor (alternative decompositions might be employed). $\boldsymbol{B}$, by virtue of its triangular structure, is used as the contemporaneous impact multiplier for the new high-frequency information. Put differently, by pre-multiplying the relevant parameters in $\boldsymbol{B}$ to the observed high-frequency shocks, we can track their propagation through the system in time t during D high-frequency periods. More precisely, we observe the upper partitions of $\boldsymbol{\varepsilon}_{T+1}$ and use $\boldsymbol{B}$ to form an expectation about future innovations during the *entire* low-frequency period ahead. Hence, the expected innovations are non-zero *conditionally* on information observed in a given high-frequency interval. By means of the subsequent evaluation of the vector-autoregression, we obtain a forecast of $T+1$ for the entire system. Taking advantage of this technique, BMW (forthcoming) elicit a nowcast of the output gap by iterating on Eq. 1:

$$c_{t+1} \quad = \quad -\boldsymbol{s}_k\boldsymbol{F}(\boldsymbol{I}-\boldsymbol{F})^{-1}\left[\boldsymbol{F}\boldsymbol{X}_t + \boldsymbol{H}\boldsymbol{\varepsilon}_{t+1}\right],$$

where $\boldsymbol{s}$ is a selection column vector. Finally, we can trace out variation in the cycle $c_{ij,t}$ of the higher-frequency series i to surprises in variable j in time t by means of

$$c_{ij,t} \quad = \quad -\sum_{d=1}^{D}\sum_{l=0}^{t-1}\boldsymbol{s}_k\boldsymbol{F}^{l+1}(\boldsymbol{I}-\boldsymbol{F})^{-1}\boldsymbol{H}\boldsymbol{s}_j'\boldsymbol{s}_j\boldsymbol{\varepsilon}_{t-1}. \tag{2}$$

We emphasize that this 'informational decomposition' (Morley and Wong 2020) is not structural as the innovations $\boldsymbol{\varepsilon}$ are not necessarily orthogonal and economically interpretable. However, even though Eq. 2 does not permit a causal interpretation, it is a convenient instrument to shed light on the *transmitters* of variation in the German output gap.

3 Data and model selection

Currently, the output gap literature is evolving from univariate and filter-based approaches to estimation in data-rich environments. For instance, Morley and Wong

(2020) give an account of the US output gap using a similar approach as the paper at hand. Moreover, Barigozzi and Luciani (2021) estimate the output gap in a big data environment. While multivariate models have the advantage that more information can be analysed, they may be subject to over-fitting. Thus, we face a trade-off between eliciting a model that exploits all relevant information and a parsimonious specification.

Our variable selection procedure seeks to reconcile traditional economic approaches to estimating potential output with statistical information on variable relevance. To this end, we estimate a medium scale model including economic aggregates in the spirit of the production function approach to potential output estimation. That is, we assume output growth y_t (and thus, implicitly, the output gap) obeys a linearized Cobb-Douglas regime of the form

$$y_t = \alpha k_t + (1 - \alpha) l_t + a_t,$$

where a_t is the Solow residual, k_t is capital formation, l_t denotes labour inputs and α is the substitution elasticity of capital. As the Solow residual cannot be subjected to direct analysis, we treat it as stochastic innovation. However, we can approximate capital k_t and labour l_t inputs by means of observable economic aggregates. Our choice of candidate variables is inspired by the literature on production function estimation. In particular, the included labour market aggregates largely derive from the EU commission's procedure on potential output estimation (Havik, McMorrow, Orlandi, Planas, Raciborski, Roeger, Rossi, Thum-Thysena, and Vandermeulen 2014). For approximating innovations to the capital stock, we propose to use a larger set of economic indicators broadly related to capital formation (e.g. investment and industrial production) and its costs (e.g. exchange and interest rates). Thus, we include various sectors of the real economy, the German labour market, external relations and financial markets. Subsequently, we estimate a candidate model and reduce the number of variables in accordance with a statistical criterion.

Variable	**Transformation**	**Frequency**
	CAPITAL	
External Relations		
CPB World Trade Monitor: World Trade Volume	rolling demean	M
Current Account: Exports	growth rates	M
Terms of Trade	growth rates	M
Real Effective Exchange Rate of the Euro against EERK-42	rolling demean	M
Finance		
Interbank Rate for Germany (obtained from FRED database)	growth rates	M
Total Share Prices for All Shares for Germany (obtained from FRED)	rolling demean	M
Non-financial private sector credit (obtained from BIS)	growth rates	Q
Fiscal Activity		
Government Consumption	growth rates	Q
Government Investment	growth rates	Q
Real Economy		
OECD Consumer Opinion Surveys (obtained from FRED)	growth rates	M
Consumer Price Index	growth rates	M
Residential Construction	growth rates	M
Total retail sales	growth rates	M
Industrial Production	growth rates	M
Real Gross Domestic Product	growth rates	Q
	LABOUR	
Hours in Construction	growth rates	M
Labour Costs	growth rates	Q
Labour Market Stabilization Policy ('Kurzarbeit' policy)	growth rates	M
Labour Productivity	growth rates	Q
Unemployment Rate	growth rates	M
Working Population	growth rates	M

Table 1: Variable blocks and data transformations. 'Growth rates' denotes the transformation 100 $\times$ first differences of natural logarithms. 'rolling demean' denotes a rolling demean (backward moving average) filter with a 40-quarters window. 'M' and 'Q' denote monthly and quarterly frequency, respectively.

Table 1 depicts candidate variables, transformations and sampling frequencies. We sample data for the period of 1995 until 2021 in monthly frequency. If an indicator is not available on a monthly basis, we obtain it at quarterly frequency. The time period is constrained by data availability and by the German reunification (1990) which possibly caused a business cycle regime change that we omit from the model for our purposes. If not stated otherwise in Table 1, we obtain data from the Deutsche Bundesbank database. We partition our data-set into five variable blocks, which are ordered as shown above. All monthly series are ordered before all quarterly series. We emphasize that model invertibility is indispensable for our purposes (see Eq. 1). Thus, we apply convenient transformations to secure stationarity of each time series.

Parsimony is the second most important priority after economic plausibility. To reduce the size of the model in the interest of parsimony, we proceed as follows. Subsequent to estimation of the model implied by Table 1, we compute the standard deviations of the informational decomposition contributions (see Eq. 2) to approximate the explanatory relevance of the variables for the output gap (following Morley and Wong (2020)). Higher standard deviations imply higher relevance of a given economic aggregate. The standard deviations are depicted in Figure 1. Thereafter, we estimate a model that includes all variables with a standard deviation of explained contribution to variance equal to or larger that of GDP growth. Thus, we drop all variables from the model with relatively low explanatory power. The reasoning behind this approach is straightforward. First of all, we emphasize the importance of a robust approach to model selection in the context of the 'non-sense output gap' debate.[2] Output gap estimates should be as least manipulable as possible. By adopting a mechanical rule to model selection, this is easily achieved. Moreover, the intuition behind this particular rule is that only variables that add explanatory content compared to GDP should be included in the model. The underlying thought is that

[2]This debate emerged on social media among policymakers. Its focus is the alleged implausibility of European periphery output gap estimates by the IMF, the OECD and the European Commission. As output gaps are policy-relevant, e.g. for computing the fiscal stance of a country, implausible and manipulable output gaps are highly problematic.

variables associated with a smaller standard deviation of explained variation add little information compared to a univariate decomposition of GDP, while concurrently inflating the amount of parameters. Nevertheless, in Section 6.1 we conduct robustness analysis to test if adding variables would alter our estimated output gap.

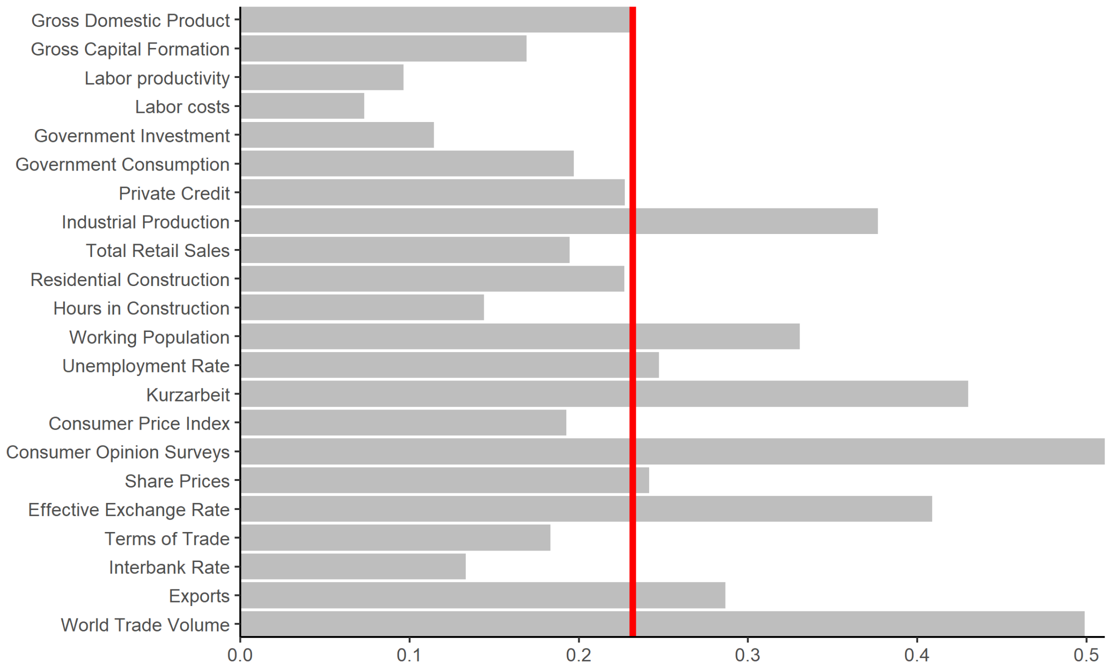

Figure 1: Standard deviations of informational decomposition. The red line indicates the standard deviation of the GDP growth contribution to output gap variation.

The 10-variable model implied by our model selection approach incorporates (in the following order): world trade volume, exports, the real effective exchange rate, share prices, consumer opinion surveys, a labour market stabilization policy ('Kurzarbeit'), the unemployment rate, the working population, industrial production and gross domestic product. We note that our model selection procedure does not imply that all variables we have dropped from the model are not important measures of the German business cycle. Instead, their informational content for the output gap is captured by the other variables included in the model.

4 The German output gap

We now turn discussing the economic properties of the proposed output gap. In a first step, we discuss our estimate in more detail and assess its plausibility. Subsequently, we examine the contributions of the individual variables to informing our estimate in a reduced-form framework.

4.1 Estimate of the output gap

Figure 2 depicts the German output gap for 1995Q1 until 2021Q3 and the nowcast for 2021Q4.

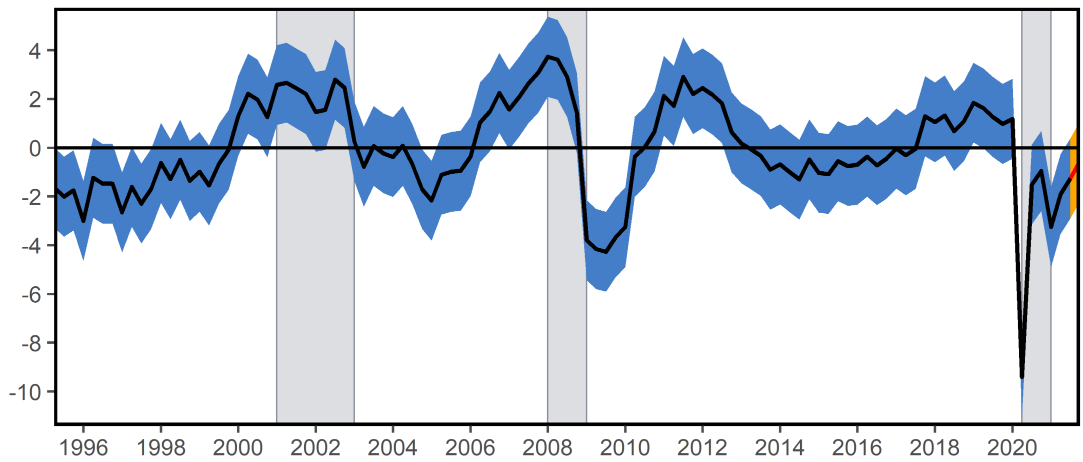

Figure 2: German output gap in percentage deviation from potential output, 1995Q1 - 2021Q3 (black) and nowcast for 2021Q4 (red). Blue and orange shaded areas indicate 90% credible sets following Kamber et al. (2018) for the mean estimate and the nowcast, respectively. Grey areas indicate recessions according to the German council of economic experts and the COVID-19 pandemic.

As can be seen, the German economy suffered from substantial slack at the beginning of the millennium. This was likely due to high adjustment costs resulting from German reunification and the global millennium recession. Subsequent to the millennium recession and after the Dotcom-bubble burst in 2002, we observe strong overheating (+4%) prior to the financial crisis (2005 – 2008). Unsurprisingly, the

European sovereign debt and banking crisis (2010 – 2015) coincides with sluggish mean-reversion tendencies of the output gap which fluctuated slightly below zero at the time.

As a result of the COVID-19 shock the output gap dropped from the pre-pandemic level of 1.18 to -9.4% at the end of 2020Q2. Thus the -10.58% decline of the output gap is similar to the decline of German GDP growth in 2020Q2. This implies that the COVID-19 shock only marginally affected potential output but is accounted for by a massive decline in the output gap. Interestingly, even the second and third 'lock-down' episodes (2020M11 – 2021M3 and 2021M4 – 2021M5) exerted – relatively – small contractionary pressures of about -2% to -4% on the German output gap. In the second half of 2021, the German output gap is still well below the zero mean. As of December 2021, our model predicts the closing of the output gap towards 2022.

4.2 Is our output gap estimate plausible?

We turn to discussing the plausibility of the our results. To this end, Figure 3 depicts the comparison of our output gap estimate (black) with the GDP-based output gaps implied by the Hodrick and Prescott (1997) filter (green) with the smoothing parameter set to 1600 (as is common for quarterly data) and the Hamilton (2018) filter (orange) with $p = 4$ lags. Moreover, Figure 3 depicts a comparison of our estimate (black) to the official output gaps estimated by the German council of economic experts (dashed red) and AMECO (dashed blue). The latter two estimates are obtained from models that use a production function approach to approximate potential output. Both are only available at yearly frequency.

First of all, we note that all estimates are reasonable similar during most of the sample period. Our estimate differs in magnitude compared to the Hamilton regression filter during and prior to the two large recessions (2008 and 2020), as do all alternative estimates. In particular, we note that the magnitudes for the Hamilton-filtered estimate appear relatively large. For instance during 2016 – 2018, the Hamilton filter indicates an overheating almost as substantial as prior to the Great Recession in

2008. Given that no economic narrative is available to support this conjecture, this seems surprising. Thus, the Hamilton filter seems to yield an 'upper bound' estimate of the German business cycle.

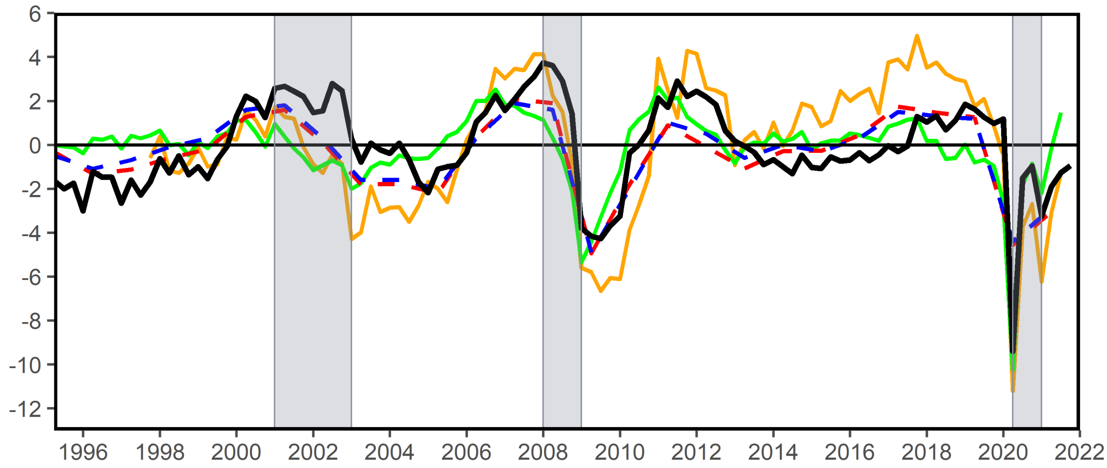

Figure 3: Comparison of our output gap measure (black) with established alternatives: One-sided Hodrick and Prescott (1997) filter (green) and Hamilton (2018) filter (orange). All estimates are reported in growth rates. Red and blue dashed lines are the (yearly-frequency) estimates by the German council of economic expert and AMECO, respectively. For further notes see Figure 2.

Moreover, we observe that the HP-filtered estimates have a significant tendency to indicate recessions and more subtle economic downturns earlier (about one year, generally) than alternative estimates. This is exemplified prior to the financial crisis in 2008 and even more in advance of the COVID-19 health crisis. In case of the latter, the HP-filter produces results that indicate mean-reversal in mid 2017, which seems debatable (at least with respect to the shown magnitudes) in absence of a plausible economic narrative. For instance, German GDP grew between 2017 and 2018 about 0.5% to 1% almost each quarter compared to the previous quarter (except 2018Q1 and 2018Q3 with about -0.4% each). Given the HP-filtered negative growth rates of the output gap, this would imply substantial and implausibly high growth rates for potential GDP in 2017 and 2018. Morley and Wong (2020) suggest that a decent output gap estimate should be correlated positively with future inflation and negatively with future output growth. Thus, we compute correlations between the output gap estimate and the future year-on-year growth rates of output and consumer price index. We find that for the Hamilton filter, results are inconclusive (Pearson correlation coefficients of -0.05 for inflation and -0.44 for output growth). The coefficients of the HP-filtered estimate line up well with the economic expectation (coefficients of 0.12 and -0.18 with inflation and output growth, respectively). Our model compares well to the HP-filter (correlations of 0.12 and -0.28 with inflation and output growth, respectively). Summing up, we conclude that our estimate is economically at least as plausible as the output gaps obtained from filtering GDP by means of the procedures proposed in Hamilton (2018) and Hodrick and Prescott (1997).

Moreover, we compare our proposed output gap to production-function based estimates. Our estimate implies about the same overheating tendencies prior to the financial crisis (2007) and prior to the COVID-19 shock (2019) as do both output gaps by AMECO (blue dashed) and the German council of economic experts (red dashed). Furthermore, all three models indicate sluggish regression to the mean in 2020 and 2021 at about the same pace and to about the same levels. We take this as evidence that our model yields reasonably similar approximations to production-function based approaches. However, note that our model indicates slightly more overheating during and less slack before the financial crisis of 2008. We conjecture that both differences are partly be explained with reference to the underlying conceptions of the output gap. Whereas our estimate includes information on financial markets, this is not incorporated in the production function approaches. The aforementioned alternative output gaps focus on real economic activity (without considering financial transactions and imbalances), whereas our estimate is best understood as a real indicator that takes into account *all* economic activity in Germany, including finance. Nevertheless, incorporating financial information in the course of estimating the output gap is important (as pointed out by Borio, Disyatat, and Juselius (2013), Berger, Richter, and Wong (2022)) when it comes to judging the sustainability of output growth, e.g. due to financial imbalances. Overall, we are confident that our model yields a plausible estimate for the German output gap.

4.3 Informational decomposition

The results described above raise a number of questions regarding the key determinants of German business cycle fluctuations. The German output gap is much less researched than, say, the United States output gap. Therefore, even non-structural information is valuable to understand German business cycle fluctuations. Subsequently, we aim to contribute towards closing this research gap. Figure 4 shows the informational decomposition of the German output gap.

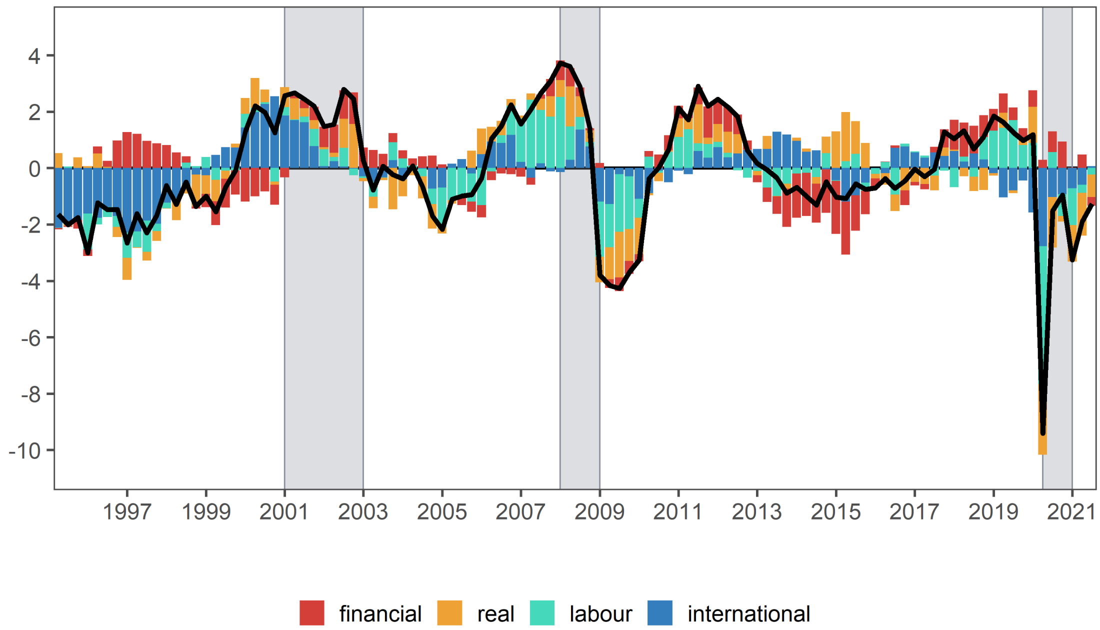

Figure 4: Informational decomposition of the German output gap. The financial block comprises share prices, the real economy block summarizes contributions from consumer opinion, industrial production and gross domestic product, the labour block is made up of Kurzarbeit as well as unemployment and the working population and the international block contains the exchange rate, world trade and exports. For further notes see Figure 2.

Although we refrain from drawing causal conclusions, Figure 4 yields interesting insights into the reduced-form contributions of our five variable blocks to business cycle variation. The capital inputs block accounts for roughly 75% of the variation (external relations 30%; the real economy 22%; share prices 23%), whereas the labour block explains about 25% of the variation in the German output gap. The high relevance of the international aggregates does not come as a surprise, as the (very open) German economy is shaped by its external relations (Eickmeier 2007). In fact, Figure 4 unambiguously shows that international economic aggregates shape the output gap dominantly throughout the entire sample period. That is, their importance does not seem to be regime-dependent.

Financial market and real economy information is mostly relevant in times of large overheating (2006-07) or substantial economic slack (e.g. during the late European banking and sovereign debt crisis in 2012-15), less so when the output gap reverts to its mean. Similar results for the US have been reported by Berger, Richter, and Wong (2022). However, this does not imply that the underlying *structural* real or financial shocks are irrelevant for the German business cycle, as the informational decomposition remains silent on this issue. Furthermore, we observe that labour market innovations contribute large amounts of variation in times of large and spontaneous contraction (e.g. 2020Q2). To a non-trivial extent, this is due to the 'Kurzarbeit' policy. The purpose of 'Kurzarbeit' is to absorb shocks to the extensive margin of the labour market: Once fewer labour is demanded by firms, the German government steps in with salary replacement payments in order to keep labour participation high, instead reducing the intensive margin of labour supply. This substitution is exactly what the data shows for the case of Germany during the COVID-19 period. However, we emphasize that this interpretation does point to labour market shocks in a structural sense. Instead, we leave the task to examine the link between 'Kurzarbeit' policy and the output gap to future (structural) research.[3]

To assess the cyclicality of the variable blocks' contributions to the German output gap more broadly, we compute correlations of the contributions to explained variation and the estimated output gap. The results are summarized in Table 2. Any sensible

[3] We present a structural historical decomposition on Figure 9 in Appendix A.

output gap estimate should be correlated positively on average with its explained variation by both capital and labour inputs, as displayed in Figure 4. As we see in Table 2, this is the case. In fact, the correlation coefficients are relatively large. In particular, the shares of variation in the output gap explained by the labour market and international economic aggregates behave (strongly) pro-cyclically. Moreover, if expansionary shocks are transmitted via these variable blocks, the next-quarter output gap will increase on average. Vice versa, as we cannot infer to causal chains at this point: As the output gap widens (closes), larger (smaller) shocks are transmitted by international, labour and real economic aggregates. However, as the coefficient between financial transmission and the current output gap is borderline-significant, a systematic pattern in the timing of shock transmission through financial markets to the output gap can only be conjectured and needs further investigation.

	Capital			Labour
	External Relations	*Finance*	*Real economy*	
c_t	0.60*	0.21*	0.68*	0.79*
c_{t+1}	0.60*	0.11	0.67*	0.79*
c_{t+2}	0.45*	0.03	0.30*	0.40*
c_{t+3}	0.45*	-0.05	0.20*	0.29*
c_{t+4}	0.43*	-0.17	0.02	0.08

Table 2: Pearson correlations between average contributions to explained variation with the current and next-quarter German output gaps c_t and c_{t+1}. * marks correlation coefficients in excess of $2/\sqrt{T}$, which roughly corresponds to a significance level of 95%. The unmarked coefficients are not significant at conventional levels.

Interestingly, shock transmission from the international block exhibits quite substantial correlations with the four-quarters-ahead output gap. This finding is unique to international aggregates. For example, shock transmission from the labour market (which is associated with the largest contemporary and one-quarter-ahead correlations) are not too informative about the output gap beyond two horizons. We interpret this finding to point to the special relevance – and potential vulnerability – of the German economy to shocks transmitted by international aggregates. With regard to finance and the real economy, correlations at farther horizons confirm our previous conclusions.

5 Nowcasting performance

In times of economic disruption, a fast economic policy response is asked for. Waiting after GDP data has been released one month after the end of a quarter can be economically costly. Therefore, a timely estimate of the output gap is needed. We assess the nowcasting abilities of our approach. As Orphanides and van Norden (2002) point out, real-time estimates of the output gap are chronically unreliable. We extensively discuss whether our model is subject to this charge.

We begin with assessing our model's nowcasting abilities in a real-world setting. The COVID-19 health crisis has brought about the most devastating economic shock since World War II to many advanced economies across the world. Germany is no exception. According to our estimates, the German output gap was at historical low (-9.4%). In this situation it is crucial for policy makers to obtain real-time insight into the state of the economy to adjust or maintain stabilization measures. In times of such large (exogenous) shocks neither the unconditional forecast nor the ex post estimate are particularly useful for the conduct of stabilization policy. The former can -by construction- not forecast large shocks, and the latter is obtained far too late for a timely policy response. The mixed frequency sampling of our approach allows us to nowcast the output gap in a timely matter. Figure 5 compares the unconditional forecast, the nowcasts after each month in 2020Q2, and the final estimate of the output gap. The solid orange line is the unconditional forecast after observing all data until 2020Q1. As can be seen, it is mean-reverting – and distant from the final estimate. However, once data for April 2020 is brought in (red dashed line), we note that the nowcast is is with -7.64% not far off the final estimate. Once the nowcast incorporates data for May 2020, the difference is even smaller and with all monthly, but no quarterly information (blue dashed line), the real-time estimate is 9.39% and almost identical to the final (ex post, i.e. full sample) estimate. Thus the model provided a reasonable quantification of the COVID-19 shock after observing all April 2020 monthly data.

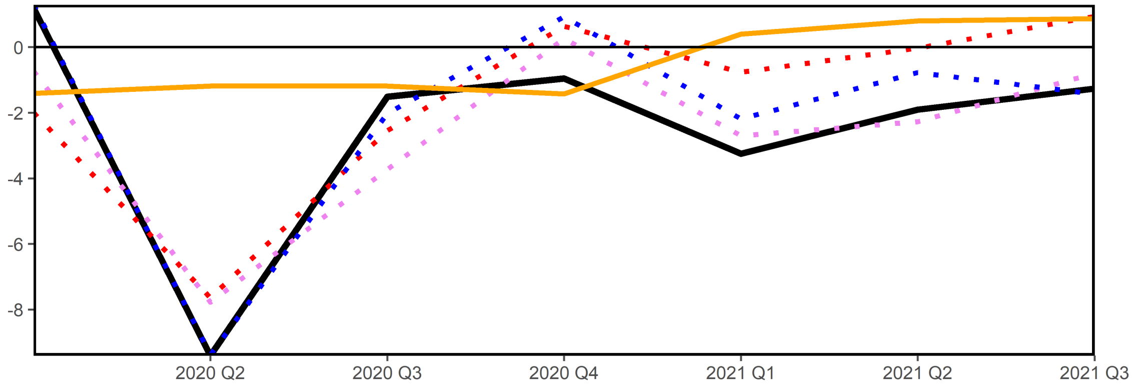

Figure 5: Nowcasts around COVID-19 (2020Q1 – 2021Q2). Ex post estimate (black), nowcasts from the perspective of month 1 (red), month 2 (pink), month 3 (blue) for real-time parameters (dashed). The solid orange line is the unconditional real-time forecast.

We proceed by analysing the nowcasting qualities of our model more rigorously. Table 3 shows the mean absolute forecast errors (MAE) for our model and given monthly indicators (first row for each indicator) compared to the end-of-quarter output gap estimate. Moreover, in order to assess the relevance of the individual variables, we compare two forecasts by means of the Diebold-Mariano procedure (Diebold and Mariano 1995): one forecast obtained from a unrestricted model with all variables and another forecast from a restricted model with the monthly indicator of interest omitted. More precisely, we test whether the forecast of the latter is superior in terms of mean absolute error than the forecast of the former model.

From Table 3 we see that the mean absolute forecast error is moderate throughout compared to an unconditional forecast with within-quarter information. The unconditional forecast implies a mean absolute forecast error of 0.45. Effectively, the additional information incorporated throughout the course of the quarter cuts the nowcast error almost in half (from 0.45 to 0.25 percentage points). At the end of the third month, the mean absolute forecast error is negligible (0.03). We emphasize that this still is several weeks prior to a GDP data release in the subsequent quarter. Observing the world trade volume, share prices, the Kurzarbeit policy,[4] the working population, unemployment and consumer opinion help to improve forecast precision in the first month. Subsequently, consumer opinion, industrial production and Kurzarbeit help to improve the forecast in the second month, whereas consumer opinion and the working population do so in the third month of a given quarter. Only exports and the exchange rate are not helpful in nowcasting.

	1^{st} **month**	2^{nd} **month**	3^{rd} **month**
Share prices Germany	0.38^{*}	0.24	0.11
Unemployment	0.29^{**}	0.15	0.04
Kurzarbeit	0.33^{**}	0.15^{**}	0.05
Consumer Opinion Survey	0.34^{*}	0.17^{**}	0.05^{**}
Effective Exchange Rate	0.39	0.2	0.12
Exports	0.39	0.24	0.12
Industrial Production	0.25	0.12^{**}	0.03
Working Population	0.25^{**}	0.15	0.03^{**}
World Trade Volume	0.40^{**}	0.24	0.12

Table 3: Within-quarter mean absolute forecast errors associated with monthly variables rounded to two decimal places for all four models, given the full-sample parameters. ** and * indicate Diebold-Mariano p-values (based on mean absolute error) equal to or smaller than 0.10 and 0.05, respectively. Variables are ordered by expected release.

Following BMW (forthcoming), Table 4 depicts correlations of the within-quarter output gap nowcasts (unconditionally and after a given month) and the final estimate (left block) as well as model-implied and realized output growth (right block).

[4] The importance of Kurzarbeit lines up well with Berger, Boll, Morley, and Wong (forthcoming), who underline that the US labour market is not fully approximated by traditional unemployment measures.

As can be seen, the model benefits from the high degree of persistence in the output gap (see first row of Table 4), but our estimate substantially improves upon an unconditional forecast after only a single month. Observing GDP hardly increases the correlations after seeing data for three complete months. This picture hardly changes when we consider correlations between model-implied and realized output growth: Our specification implies plausible output growth rates and observing GDP data after three month adds only little information.

	Output gap	Output growth
No information	0.96	0.96
First month	0.99	0.97
Second month	1.00	0.98
Third month	1.00	0.98

Table 4: Correlations of the within-quarter nowcasts with the final estimate, model-implied and real-time output growth

6 Robustness

In this section we examine the robustness of our results with respect to an extended data set as well as the reliability of the output gap in real time.

6.1 Larger information set

First of all, we investigate the effects of including a larger set of variables in our model. We estimate a model that comprises all variables of the baseline specification and all variables with a standard deviation of explained shares of variation larger than the median standard deviation of explained shares of variation of the aggregates that are not included in the baseline model. That is to say, in addition to the baseline variables, we include exports, terms of trade, consumer price index, private credit, residential construction, government consumption and total retail sales. Figure 6 depicts the German output gap estimated from the alternative model.

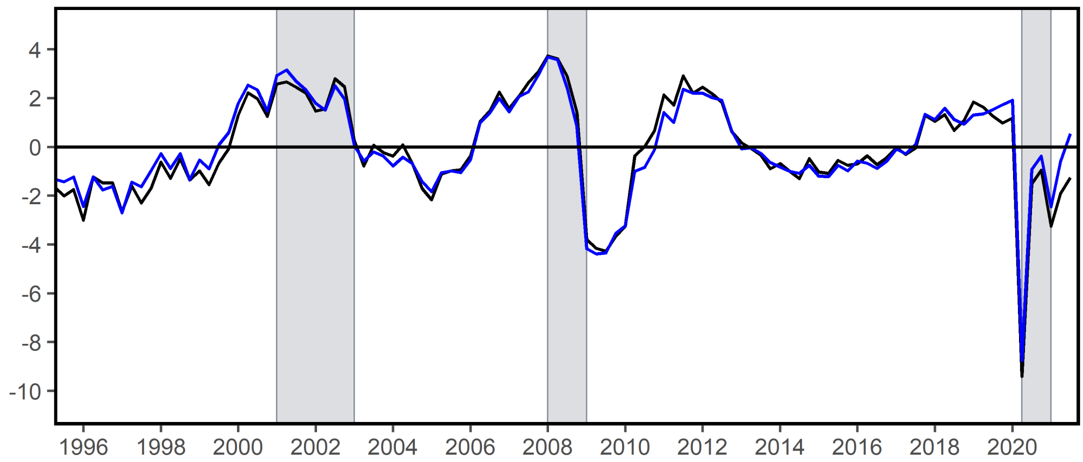

Figure 6: Alternative output gap estimate (blue) and baseline estimate (black). For further notes, see Figure 2.

We note that for most of the period under examination, the two estimates are very similar. The exception is the COVID-19 episode, where a the relative changes in both models are quite similar, but the mean of the alternative specification is larger. We hypothesize that this is due to the redundant, much larger information set. Interestingly, we do not observe this during the Great Recession in 2008/09.

6.2 Parameter revisions in pseudo real-time

After the previous quarter has been observed, our model needs updating in order to incorporate all available information. That is to say, the parameters of the autoregressive vector as well as the shrinkage factor λ need to be estimated again. In the following, we examine the effect of parameter revisions on our estimate and nowcasting performance. First of all, we examine the general effect of parameter revisions on our ex post estimate.

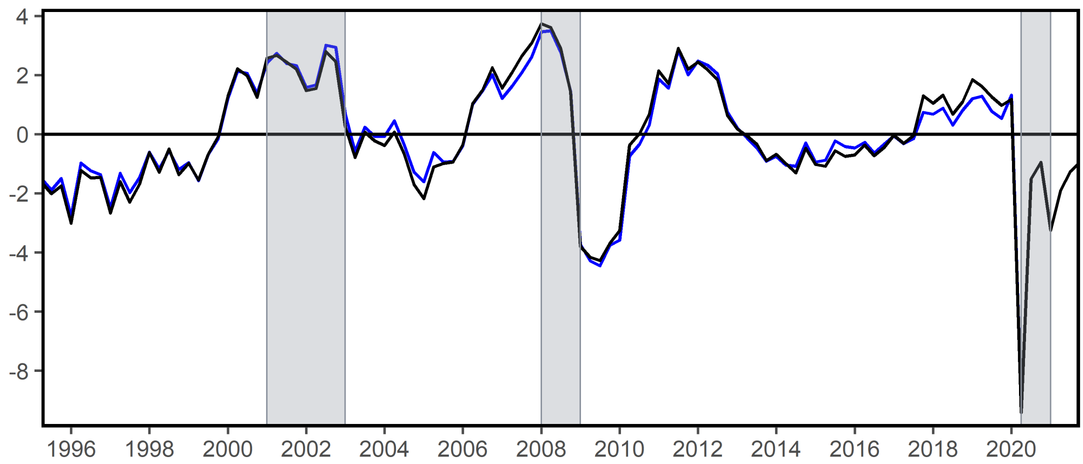

Figure 7: Real-time output gap estimate (blue) and baseline estimate (black). For further notes, see Figure 2.

To this end, we estimate the model until 2004Q4 as an initialization. Then we elicit a pseudo real-time update (ignoring data-revisions for the moment) by adding full-quarter new information of the subsequent quarters in sequentially. Thus, we re-estimate the model every quarter conditionally on ex post data and the entire-quarter information. Figure 7 depicts the pseudo real-time output gap obtained from this procedure. Clearly, the difference between the real-time estimate and the ex post output gap is small. For the second half of the sample – including the COVID-19 episode with the associated large economic disturbances – it is negligible.

6.3 GDP data revisions in pseudo-real-time

In the final robustness analysis, we investigate the relevance of data revisions. Unfortunately, we cannot obtain real-time data for the majority of the monthly indicators from Deutsche Bundesbank. Thus, we limit ourselves to investigating the effects of GDP data revisions, which are available since 2005. We re-estimate the model for each quarter, given the full-sample monthly information. Figure 8 depicts the results.

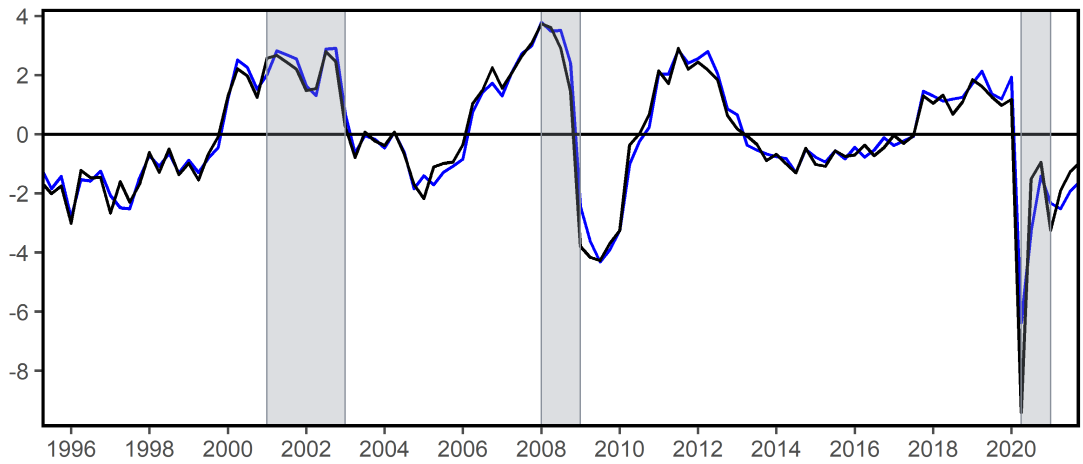

Figure 8: Black: Ex post estimate. Blue: Estimate with GDP revisions. For further notes, see Figure 2.

Note that there are two sources of revisions in GDP. First of all, as we use GDP data chained in previous year prices, the level of GDP in the past years is adjust in each first quarter of a given year. We expect this effect to be small, as we employ GDP in growth rates. Moreover, since 2005, GDP data is revised in the next quarter after the initial release. From Figure 8, we clearly see that these revisions do not play a big role for our model with the remarkable exception of 2020, where a substantial revision in GDP data occurred.

7 Conclusion

We have provided an in-depth analysis of the international and domestic determinants of the German output gap between 1995 and 2021 by means of a medium-size mixed-frequency vector-autoregressive model that exploits monthly information to evaluate the expectation associated with a multivariate Beveridge-Nelson decomposition. We showed that substantial shares of variation in the German business cycle are explained by the real economy, the labour market and international economic aggregates. Moreover, we demonstrated that our model fairly accurately predicts the German output gap up to three months prior to a German gross domestic product data release. In particular, observing consumer sentiment and the labour market allows to produce a decent nowcast of the German output gap.

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Appendix

A Structural historical decomposition

In this Appendix, we briefly present a structural historical decomposition. Structural identification of our model is a challenging task, as the mere size due to the mixed-frequency setup, complicates the analysis tremendously. Traditional identification schemes based on sign restrictions and more recent data-driven alternatives can hardly cope with a system of this size. For instance, the rotation space for identification based on sign restrictions is vast even for a relatively small number of restrictions. Thus, we base our brief analysis on a Choleski factor of the reduced form error covariance and leave more sophisticated structural identification to future research. Figure 9 depicts a structural decomposition.

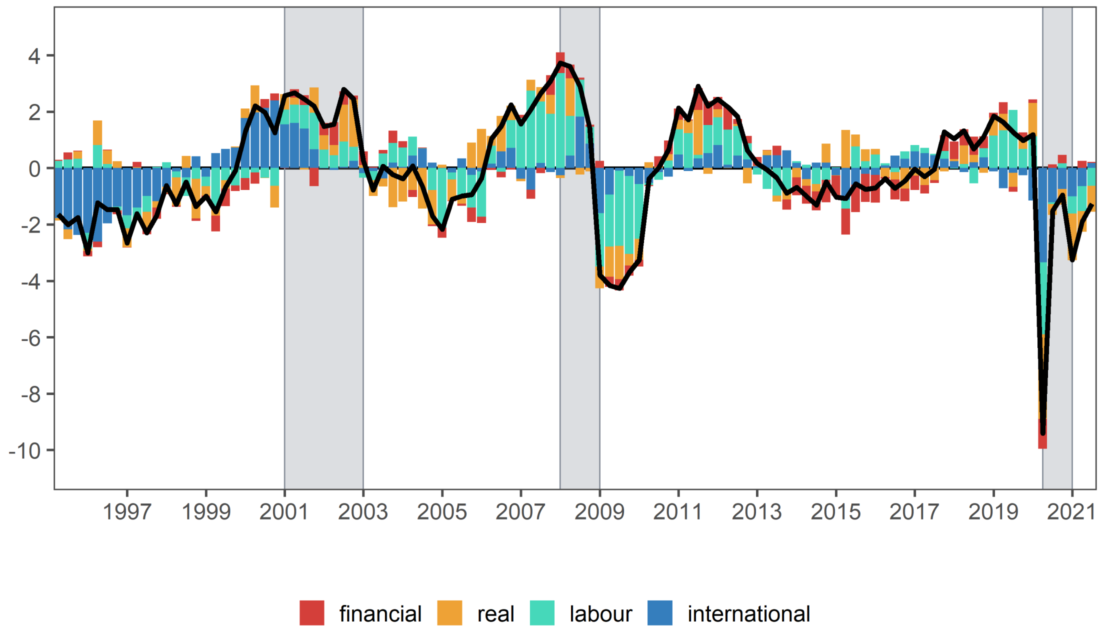

Figure 9: Structural historical decomposition based on Choleski decomposition of the German output gap. For further notes see Figure 4.