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Examination Rules and Student Effort

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Examination Rules and Student Effort

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Abstract

This paper contributes to the economics of examination rules. We show how rational students reallocate their learning effort as a response to a charge for the second exam attempt, a cap on the maximum resit mark, an adjustment of the passing standard, a variation of the time span between two attempts, a minimum requirement to qualify for the second attempt, and a malus points account. The effort maximizing rule is the malus account, a charge for the second attempt delivers the highest overall passing probability.

JEL-Classification: I21; I23; D81

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1 Introduction

The option of a second attempt to pass an exam creates a windfall gain, rational students respond with a decline in effort at the first attempt and thus a decline in the pass probability (Kooreman, 2013). The flip side of the coin: due to ill-prepared and/or less motivated students, the school bears (part of) the burden of the second attempt, the student windfall gain is no one-to-one gain in social welfare. The school has an incentive to substitute a more restrictive examination rule for an unconditional second attempt in order to manipulate student effort. Such rules are, for instance, a charge for a resit, a cap on the maximum resit mark, an adjustment of the passing standard, a variation of the time span between two attempts, a minimum requirement to qualify for the second attempt, and a malus points account. In a theoretical model of higher education we show how these rules affect student effort and how they differ with respect to the overall pass probability.

The determination of student learning effort is the topic of a wide body of theoretical and empirical literature (see, e.g., Bishop and Wößmann, 2004; De Fraja et al., 2010; Zubrickas, 2015; Bonesrønning and Opstad, 2015). Nonetheless, this literature says very little about how student effort is affected by examination arrangements and rules. This paper aims at filling this gap.

2 The Model

2.1 Setup

We distinguish between two groups of agents, schools and students. The school announces a passing standard $\hat{q}$, and any student with a test score $q \geq \hat{q}$ passes the exam. The outcome of the exam is assumed to be binary, pass or fail. Following Kuehn and Landeras (2014), the test score q is the sum of the student's educational attainment $\theta(e)$ and a random variable ε. The educational attainment function is supposed to be linear in the student learning effort e: $\theta(e) = e$. The test score is given by

$$q = e + \varepsilon. \tag{1}$$

The case of an attainment function with decreasing returns is considered in the numerical example of Section 4. The random variable ε is distributed according to a continuous, symmetrical and single-peaked function $F(\varepsilon)$ with $F'(\varepsilon) = f(\varepsilon)$ and $E(\varepsilon) = 0$. The probability of passing the exam is given by $prob(q \geq \hat{q}) = 1 - prob\,(\varepsilon < \hat{q} - e) = 1 - F(\hat{q} - e)$.

2.2 Setting the Passing Standard

The school's objective function is specified as

$$W = \theta(e) - \rho(\hat{q}), \tag{2}$$

where we assume that the school is interested in the student educational attainment $\theta(e)$. Since we abstract from different levels of student ability, effort is the only variable the school can affect by setting the passing standard $\hat{q}$. But improving attainment by setting a higher standard is not costfree. Such costs include improving the quality of buildings, room equipment and computer facilities, upgrading lecturers' qualifications, the assessment of a higher number of (resit) exams, the time lecturers spend on the preparation of courses, the lecturers' enthusiasm, more tutorials, and so on (see De Fraja et al., 2010). We capture the costs of setting a passing standard by the function $\rho(\hat{q})$ with $\rho'(\hat{q}) > 0$ and $\rho''(\hat{q}) > 0$.

The first-order condition for the optimal passing standard,

$$\rho'(\hat{q}) = \frac{de}{d\hat{q}}, \tag{3}$$

states that in the optimum the marginal costs of setting the standard are equal to the marginal utility gain in terms of a higher attainment. The necessary condition for an optimal passing standard is a positive impact of $\hat{q}$ on effort e. Put another way, the assumption of a costly standard setting allows us to restict the analysis to the case of $\frac{de}{d\hat{q}} > 0$.

2.3 One versus Two Exam Scenario

Suppose the student has two attempts to pass the exam, the second attempt is offered unconditionally. The risk-neutral student maximizes expected utility

$$EU = p_1 R_1 - V(e_1) + D\ (1 - p_1)\ CP, \tag{4}$$

with the continuation payoff $CP \equiv p_2 R_2 - V(e_2) > 0$ (see Weinschenk, 2012). The parameter p_i is the passing probability, R_i is the reward for passing the exam (the disutility of a fail is normalized to zero), i ($i = 1, 2$) indicates the first and second attempt, and D is a dummy variable. The effort costs $V(e_i)$ are increasing and convex: $V'(e_i) > 0$ and $V''(e_i) > 0$. An inner solution is guaranteed by $V(0) = 0$ and $V'(0) = 0$. The passing probability p_2 is given by $p_2 = 1 - F(\hat{q} - e_2 - \delta e_1)$, where $\delta \in [0, 1]$ reflects the idea that resit students may have an advantage at the second attempt. They already have some basic knowledge of the course content and course material, they are more familiar with the questioning technique ("memory effect").

First consider the one exam opportunity ($D = 0$). The first-order condition (FOC) of the maximization of (4) with respect to e_1 reads $V'(e_1) = R_1\ f\ (\hat{q}_1 - e_1)$. The optimal effort is given at the point where the marginal costs are equal to the increase in the passing probability times the reward for passing. Totally differentiating this FOC with respect to $\hat{q}_1$ yields $\frac{de_1}{d\hat{q}_1} = \frac{R_1 f'(\cdot)}{-EU''}$, where $EU'' = -R_1 f'(\cdot) - V''(e_1)$. The optimal standard setting $\frac{de_1}{d\hat{q}_1} > 0$ (see (3)) requires $f'(\cdot) > 0$, which also leads to the (fullfilled) second-order conditition $EU'' < 0$. Hence, we can restrict the analysis to the left-hand side of the modal. The pass probability will thus be greater than 0.5.

Note that this result extends the result obtained by Kuehn and Landeras (2014). These authors assume zero marginal costs, $\rho'(\hat{q}_1) = 0$, so that it is optimal to set a standard such as to maximize student effort, $\frac{de_1}{d\hat{q}_1} = 0$. But this is equivalent to choosing a $\hat{q}_1$ where $f'(\cdot) = 0$, and the exam very much resembles a lottery with a fifty-fifty chance of passing and failing. The claim that it is optimal from the school's point of view to have a failure rate of 0.5, is not very convincing. From our point of view, it is an unpleasant feature of their framework resulting from the implausible assumption of costfree standard setting.

In the two exam scenario ($D = 1$), the model has to be solved by backward induction. The FOC for the optimal e_2 reads

$$V'(e_2) = R_2\ f(\hat{q}_2 - e_2 - \delta e_1). \tag{5}$$

For $\delta > 0$, the student in his second (last) attempt is not just back in the situation of a one exam opportunity. The first-attempt effort e_1 raises the passing probability p_2 for any level of e_2. Consequently, the marginal utility gain of e_2 in terms of an increase in the passing probability p_2 is lower and hence the optimal e_2 will be lower compared to the one exam scenario. Only if we switch off the memory effect by setting $\delta = 0$, does the second attempt replicate the one exam opportunity.

For the optimal e_1, the FOC is given by

$$V'(e_1) = (R_1 - CP)\ f\ (\hat{q}_1 - e_1) + \delta(1 - p_1)\ R_2\ f(\hat{q}_2 - e_2 - \delta e_1). \tag{6}$$

For $\delta = 0$, we replicate Kooreman (2013). Knowing that one has a second attempt constitutes a continuation payoff (windfall gain), implying a lower reward of effort at the first attempt and thus a lower e_1 compared to the case of the one exam scenario. Having more than two attempts reinforces this effect (Weinschenk, 2012).

For $\delta > 0$, we observe two additional effects. First, a higher e_1 may now count twice, since it now increases both p_1 and p_2, generating an incentive to invest more e_1. This effect is captured by the second summand on the right-hand side of (6). Second, CP becomes larger when $\delta > 0$, lowering the first summand on the right-hand side of (6).

In order to reach a given level of p_2, a memory effect allows for a reduction of e_1. The net effect of a stronger δ on e_1 is ambiguous. Our simulations suggest that $de_1/d\delta < 0$ is the most relevant scenario. The impact of a stronger memory effect on the overall pass probability $P = p_1 + (1 - p_1)p_2$ is ambiguous too. Let us summarize::

Proposition 1 *A costly and optimal standard setting combined with an optimal effort level requires $f'(\cdot) > 0$, which implies that pass probabilities will be greater than 0.5.*

Proposition 2 *(Kooreman, 2013, and Weinschenk, 2012) The utility-maximizing response to the option of an unconditional second attempt is a decline in effort e_1 and thus a decline in the pass probability p_1.*

Proposition 3 *If e_1 raises the passing probability p_2 (memory effect), then (i) e_2 declines, and (ii) e_1 declines for a wide range of parameter values. (iii) The overall pass probability P may increase or decrease.*

3 Student Effort and Examination Rules

The school is assumed to substitute one of the examination rules discussed below for the unconditional second attempt.

3.1 Charge for the Second Attempt

The student windfall gain constitutes a positive willingness to pay for a second attempt. At least in the UK, it is quite common to impose a charge for a resit exam in order to internalize the negative externality.

With Z denoting the charge for the second attempt, student expected utility is given by $EU = p_1 R_1 - V(e_1) + (1 - p_1)(CP - Z)$.The optimal e_2 is not affected by Z. Since any student who participates in the second attempt, has to pay the fee, the fee works like a lump sum tax. However, an increase in Z leads to an increase in e_1. Totally differentiating the FOC $EU' = f(\hat{q}_1 - e_1)[R_1 - CP + Z] + \delta(1 - p_1)\ R_2\ f(\hat{q}_2 - e_2 - \delta e_1) - V'(e_1) = 0$ with respect to Z yields

$$\frac{de_1}{dZ} = \frac{f(\hat{q}_1 - e_1)}{-EU''} > 0. \tag{7}$$

Avoiding the fee at the second attempt is equivalent to an increase in R_1, students work harder to meet the passing standard. The passing probabilities p_1 and P increase in Z.

By setting $Z = CP$, the school eliminates the windfall gain and thus the incentive to reduce e_1. Both attempts now replicate the one exam opportunity. Moreover, for

$Z = CP$, a positive memory effect has no incentive-destroying effect, the rise in the continuation payoff will be "taxed away" by a higher charge. The positive effect of a double use of e_1 remains, and, in contrast to Weinschenk (2012), the optimal effort declines over time ($e_1 > e_2$). We summarize:

Proposition 4 *A charge for the second attempt (i) is neutral for* e_2*, and (ii) increases* e_1*,* p_1 *and* P*. (iii) For* $Z = CP$*, the windfall gain vanishes, students exert an effort level as if each attempt were the final attempt. Due to the memory effect, (iv) effort is decreasing over time .*

3.2 Cap on the Maximum Resit Mark

Resit marks may be different from first-attempt marks. In the UK, resit marks are usually capped at the pass threshold (40-50%), so that even a resit exam with a maximum score of 100% will be graded with the mark "sufficient". A less radical proposal, implemented in Uruguay, improves the informational content of the transcript of records and/or exam report by mentioning the number of attempts needed for the pass. Communicating this number is an informative signal, and it is easy to administer.

These measures are captured by lowering the reward for the second attempt, R_2. Totally differentiating the FOCs (5) and (6) with respect to R_2 yields

$$\frac{de_2}{dR_2} = \frac{f(\hat{q}_1 - e_1)\ F(\hat{q}_2 - e_2 - \delta e_1)}{-EU''(e_2)} > 0 \tag{8}$$

$$\frac{de_1}{dR_2} = \frac{-p_2\ f(\hat{q}_1 - e_1) + \delta(1 - p_1)\ f(\hat{q}_2 - e_2 - \delta e_1)}{-EU''(e_1)} < 0. \tag{9}$$

The decline in R_2 moves the effort levels in opposite directions. Students work less hard at the second attempt, but due to the decline in the continuation payoff, they will work harder at the first attempt. A positive memory effect minors the decline in CP by a higher p_2, but even for $\delta = 1$ the memory effect does not neutralize the effect of a lower R_2. Concerning the passing probabilities we observe $\frac{dp_1}{dR_2} < 0$ and $\frac{dp_2}{dR_2} = f(\hat{q}_2 - e_2 - \delta e_1)\left[\frac{de_2}{dR_2} + \delta\frac{de_1}{dR_2}\right] \gtreqless 0$. If there is no memory effect, the multiplier is positive, p_2 goes down. Only for a large memory effect, the dual use of e_1 may overcompensate the decline in e_2, so that p_2 goes up. The impact on P cannot be signed unambiguously. But our simulations suggest that for almost all parameter constellations the positive effect of a lower continuation payoff outweighs the negative effect of a reduction of e_2, so that P goes up.

Proposition 5 *A decline in* R_2 *induces (i) a decline in* e_2*, (ii) an increase in* e_1*, (ii)* *an increase in p_1, (iv) a decrease in p_2 (unless the memory effect is very strong). (v) The overall passing probability P increases for almost all parameter constellations.*

3.3 Adjustment of the Resit Passing Standard

By definition, the score of the resit students was in the lower tail of the distribution. If the bad score reflects low ability and/or low effort, then the resit students are a biased sample. On the other hand, we know from the literature that the resit pass rate is comparable to the first attempt pass rate (see, e.g., McManus, 1992; Scott, 2012), we do not observe the expected decline in the pass rate. Pell et al. (2009) argue that usually the resit is not undertaken in conjunction with any other assessment, so students can concentrate on this assessment alone. Resit students may receive extra classes. These factors put resit students at an unfair advantage over first attempt students. To create a level playing field, they propose a higher passing standard for the resit.

How are the optimal effort levels affected by such an adjustment of the passing standard $\hat{q}_2$? Totally differentiating the first-order conditions (5) and (6) with respect to $\hat{q}_2$ yields:

$$\frac{de_2}{d\hat{q}_2} = \frac{R_2\ f'(\hat{q}_2 - e_2 - \delta e_1)}{R_2\ f'(\hat{q}_2 - e_2 - \delta e_1) + V''(e_2)} > 0 \tag{10}$$

$$\frac{de_1}{d\hat{q}_2} = \frac{R_2\ f(\hat{q}_1 - e_1)\ f(\hat{q}_2 - e_2 - \delta e_1) + \delta R_2 F(\hat{q}_1 - e_1)\ f'(\hat{q}_2 - e_2 - \delta e_1)}{-EU''(e_1)} > 0. \tag{11}$$

The optimal response to a higher passing standard $\hat{q}_2$ is a higher level of effort at the second attempt. But the increase in e_2 and thus the increase in the expected test score q_2 is lower than the increase in $\hat{q}_2$ (note that $\frac{de_2}{d\hat{q}_2} < 1$), the second attempt pass probability p_2 declines. The increase in effort does not compensate for the increase in the passing standard.[1] The expectation of a higher e_2 combined with a lower p_2 triggers a positive side effect. Because of the lower continuation payoff, students have an incentive to exert more effort at the first attempt, e_1 and thus p_1 goes up. A memory effect mitigates the decline in p_2. The overall passing probability cannot be signed unambiguously. Our simulation results are mixed. For reasonable values of the memory effect, the increase in p_1 does not overcompensate the decline in p_2 and the overall pass rate P decreases. Only for a very strong memory effect, P may increase.

[1] Using data from the State of Florida, Clark and See (2011) analyze the impact of a toughening of high school graduation standards. They find a (small) decrease in graduation rates, which is in line with our result.

Proposition 6 *Consider an increase in the resit passing standard $\hat{q}_2$. This leads to (i) an increase in effort e_2, (ii) an increase in effort e_1, (iii) an increase in the passing probability p_1, (iv) a decline in the passing probability p_2, and (v) a decline in the overall pass probability P (for reasonable values of the memory effect).*

3.4 Time Span Between Attempts

The time span between the first and the second attempt differs across both schools and countries. In Germany, the second exam opportunity is, in general, about six months after the first exam. Students have to repeat all lectures/classes/seminars. In the UK, the study year ends with the main assessment in May. Students with a fail have to take a resit, which takes place in August. There is no possibility to attend lectures again. Shortening the time span between the attempts influences the learning strategy. In our model we put the shortening of the time span to the extreme, putting it to zero. Such a scenario perfectly mimics multiple choice tests in an E-Learning center.

A zero time span means that the only choice variable is e_1, which, if necessary, can be used twice. The continuation payoff now turns out to be $CP = p_2 R_2$. Since the effort costs $V(e_2)$ drop out, CP goes up, c.p. The pass probability at the second attempt is $p_2 = 1 - F(\hat{q}_2 - e_1)$, so that the FOC for the optimal e_1 reads

$$V'(e_1) = f(\cdot)[R_1 - CP + F(\cdot)R_2],$$

where we make use of the simplifying assumption of identical passing standards, $\hat{q}_1 = \hat{q}_2$. Compared to the one exam opportunity, we observe two effects. The second chance creates a continuation payoff, implying a decline in e_1. But the chance of using e_1 twice increases the expected marginal utility gain of e_1, implying an increase in e_1. Since the fail probability $F(\cdot)$ is smaller than 0.5, the former effect always dominates the latter effect, which can be seen from $CP - F(\cdot)R_2 = (p_2 - F(\cdot))R_2 = [1 - 2F(\cdot)]R_2 > 0$. In other words, the second chance immediately after the first attempt is a disincentive for exerting effort to meet the passing standard.

Compared to the case of two attempts with a substantial time span between these attempts, we obtain a strong increase in e_1, since the students have no option to increase their effort after a fail at the first attempt. Thus, p_1 increases, whereas p_2 decreases. The net effect on P is ambiguous.

Proposition 7 *Shortening the time span between the first and second attempt leads to (i) an increase in e_1 and p_1, and (ii) a decrease in p_2. (iii) The overall passing probability P may increase or decrease.*

3.5 Conditional Second Attempt

Up to now we have assumed that all students who fail at the first attempt have an unconditional second attempt for passing the exam. A fail with zero points is very much the same as a fail just below the pass rate. Especially when the maximum number of attempts is large, the continuation payoff will be large and it will be optimal to invest very little effort at the first attempts. In order to prevent students looking for more bang for their buck, the school may set a minimum requirement. For instance, students qualify for the second attempt only if their first attempt result is not less than k percent of the passing standard $\hat{q}_1$. Students with a first attempt result less than $k\hat{q}_1$ have failed definitively.

In such a scenario, the probability of attending the second attempt is given by $prob(k\hat{q}_1 \leq q < \hat{q}_1) = F(\hat{q}_1 - e_1) - F(k\hat{q}_1 - e_1)$. The expected utility reads $EU = p_1R_1 - V(e_1) + [F(\hat{q}_1 - e_1) - F(k\hat{q}_1 - e_1)]CP$. The optimal e_2 is not affected by the minimum requirement. Let us turn to the first attempt and switch off the memory effect, $\delta = 0$. Totally differentiating the FOC for the optimal effort at the first attempt, $EU' = f(\hat{q}_1 - e_1)\, R_1 - [f(\hat{q}_1 - e_1) - f(k\hat{q}_1 - e_1)]\; CP - V'(e_1) = 0$, with respect to k yields

$$\frac{de_1}{dk} = \frac{\hat{q}\; CP\; f'(k\hat{q}_1 - e_1)}{-EU''} > 0. \tag{12}$$

The higher the minimum requirement k, the lower is the probability of attending the second attempt, the lower is the expected continuation payoff, and the higher is the optimal effort e_1. The case of $k = 1$ replicates the one exam opportunity. Concerning the overall pass probability $P = p_1 + prob(k\hat{q}_1 \leq q < \hat{q}_1) \cdot p_2$, the impact of a minimum requirement cannot be signed unambiguously. But our simulations suggest that for almost all parameter constellations the negative effect of a lower probability of attending the second attempt outweighs the positive effect of the higher optimal effort e_1.

Proposition 8 *A minimum requirement for the second attempt (i) is neutral for e_2, and (ii) increases e_1 and p_1. (iii) The overall passing probability P decreases for almost all parameter constellations.*

3.6 Malus Points Account

In Germany, many higher education institutions make use of a malus points account. Take for instance the economics department of the University of Cologne, where each Bachelor student with a study program of 180 credit points has a scope of 60 permissible malus points. Should module exams carrying more than 60 credit points in total have been failed, the Bachelor's examination has been failed definitively. Within the scope of the permissible malus points, the student has the right to repeat a failed module.

Suppose the student takes two tests, module A and module B. The student now splits the learning effort into first attempt effort for module A and first attempt effort for module B, e_{A1} and e_{B1}, respectively. To ensure that the maximum number of fails is lower than the number of modules, we permit one fail. To qualify for a second attempt in module A (B), the student must have passed module B (A) in the first attempt. The expected utility is given by $EU = p_{A1}R_{A1} + p_{B1}R_{B1} - V(e_{A1}) - V(e_{B1}) + (1 - p_{A1})p_{B1}CP_A + (1 - p_{B1})p_{A1}CP_B$ with $CP_A \equiv p_{A2}R_{A2} - V(e_{A2})$ and $CP_B \equiv p_{B2}R_{B2} - V(e_{B2})$. To allow for a meaningful comparison with the already discussed examination rules, the effort costs have to be assumed as additive separable in e_{A1} and e_{B1}.

Suppose that the student fails the first time in module A (B) and passes in module B (A), so that s/he is allowed to take part in the resit of module A (B). In this case, the student will be back in the situation of the one exam opportunity.

Turn to the first attempt. Let us switch off the memory effect, $\delta = 0$, and let us focus on module A. The considerations for module B are analog. The FOC for the optimal e_{A1} is

$$V'(e_{A1}) = f(\hat{q} - e_{A1})\ [R_{A1} - p_{B1}CP_A + (1 - p_{B1})CP_B]. \tag{13}$$

The lower the passing probability for module B, p_{B1}, the lower is the expected continuation payoff of module A. This is an incentive to exert more effort to pass module A already at the first attempt, e_{A1} increases compared to the case of an unconditional second attempt. The increase in e_{A1} will be reinforced by the fact that a pass in module A is a precondition for a repeat of module B. Qualifying for a repeat of module B is part of the utility gain of a higher effort level e_{A1}. The higher e_{A1}, the higher is p_{A1}, and the higher is the probability of getting the expected continuation payoff of module B, $(1 - p_{B1})CP_B$. The increase in e_{A1} leads to a higher p_{A1}, but that cannot outweigh the negative effect of the requirement to pass module B at the first attempt. Thus, the overall pass probability declines.

Proposition 9 *Suppose that a pass of module B (A) is a precondition for a resit in module A (B). Then (i) e_{A1} as well as p_{A1} increase, whereas (ii) e_{A2} and p_{A2} remain unaffected. (iv) The overall pass probability declines compared to the case of an unconditional second attempt for module A.*

4 A Numerical Example

To gain some intuition for the differential effects of the examination rules, we provide a numerical example. Effort is stated as percentage share of learning time in total time, e_i is thus bounded between zero and one. The educational attainment function is specified as $\theta(e_i) = e_i^{\beta}$ with $\beta \in (0,1]$. The effort costs are given by $V(e_i) = e_i^{\gamma}$ with $\gamma > 1$. For the reward, we choose $R_1 = R_2 = 2$. The random variable ε follows a logistic distribution $F(\varepsilon)$ with zero mean. The school sets the passing standard at $\hat{q}_1 = \hat{q}_2 = 0.5$. Table 1 presents the results for $\beta = 0.5$ and $\gamma = 2$.

The option of an unconditional second attempt lowers e_1 by about one third. A memory effect ($\delta = 0.5$) reinforces the decline, but raises the overall pass probability. To allow for a relative comparison of the rules, the size of the "shock" caused by the substitution of a new examination rule for the unconditional second attempt has to be the same across the rules. For given e_1 and e_2 from the unconditional second attempt, the switch to the malus account, the introduction of a charge Z, the reduction of R_2, the increase of $\hat{q}_2$, and the introduction of a minimum requirement have to be identical in terms of the decline in student expected utility. For $\delta = 0$, this requires $Z = 0.34CP$,

			e		P			EU
One exam scenario			.574		.820			1.31
Two exam scenario								
Examination rule		δ	e_1	e_2	p_1	p_2	P	CP	EU
Unconditional		0	.374	.574	.658	.820	.938	1.31	1.62
second attempt		.5	.304	.384	.575	.911	.962	1.67	1.77
Charge	$Z = 0.34CP$	0	.466	.574	.745	.820	.954	.86	1.49
	$Z = 0.43CP$	.5	.453	.348	.734	.924	.980	.99	1.53
	$Z = CP$	0	.574	.574	.820	.820	.968	0	1.31
	$Z = CP$	.5	.574	.323	.820	.933	.988	0	1.31
Cap	$R_2 = 1.453$	0	.464	.512	.744	.781	.944	.87	1.50
	$R_2 = 1.219$	.5	.453	.293	.734	.902	.974	1.01	1.53
Passing	$\hat{q}_2 = 0.692$	0	.457	.714	.738	.712	.925	.91	1.51
standard	$\hat{q}_2 = 0.858$	.5	.434	.595	.718	.807	.945	1.26	1.60
Zero time span		–	.453	–	.735	.735	.930	1.47	1.65
Conditional	$k = 0.536$	0	.474	.574	.752	.820	.892	1.31	1.50
2nd attempt	$k = 0.589$	.5	.450	.346	.740	.925	.893	1.73	1.55
Malus account		0	.491	.574	.766	.820	.913	1.31	1.52
Module A		.5	.474	.343	.752	.926	.925	1.74	1.60

Table 1: Effort and passing probabilities under different examination rules

$R_2 = 1.453$, $\hat{q}_2 = 0.692$, and $k = 0.536$. In the zero-time-span scenario the policy parameter is already fixed to zero, there is no room for manipulating this parameter to ensure a given EU. Subsequent to the introduction of the new rule, (the next generation of) students reoptimize.

From the student point of view, a zero time span between two attempts is the superior examination rule. Because of the drop of the effort costs $V(e_2)$, the lowering of EU is at its minimum. Compared to the unconditional second attempt, even an increase in EU is possible. Focussing on already implemented rules (i.e. excluding the conditional second attempt), the malus account maximizes effort and educational attainment, but at the same time it minimizes the overall pass probability P. Compared to the minimum requirement for a resit, the malus account system results in higher levels of effort, educational attainment, pass probabilities, and EU, whereas the conditional second attempt leads to the lowest P. The charge for the second attempt delivers the highest P, but it also delivers the lowest EU.

5 Conclusions

In this paper it is shown how rational students reallocate their learning effort as a response to a charge for the second exam attempt, a cap on the maximum resit mark, an adjustment of the passing standard, a variation of the time span between two attempts, a minimum requirement to qualify for the second attempt, and a malus points account. By setting such rules, the school is able to manipulate student effort towards the first attempt. The effort maximizing rule is the malus account, a charge for the second attempt delivers the highest overall passing probability. Because of a lack of a well-defined social welfare function, we cannot provide a clear-cut ranking in terms of welfare. This is an important issue for future research.

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