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## Wholesale Pricing with Incomplete Information about Private Label Products


#### Abstract

This article provides a theoretical model analyzing wholesale pricing tariffs set by a monopolistic manufacturer for its branded product that is sold to final customers by a monopolistic retailer. The bargaining power of the downstream retailer is strengthened by offering also a vertically differentiated private label product whose production costs are known only incompletely to the upstream manufacturer. The model shows that the manufacturer can avoid double marginalization and implement the full information outcome by combining a quantity discount with a market-share discount where only a retailer with a strong private label retroactively receives an allowance. Under these circumstances it is unprofitable for the manufacturer to impose exclusive dealing on the retailer. Keywords: Branded Products, Incomplete Information, Market-Share Discounts, Private Label Products, Wholesale Pricing JEL Classification: D42, D82, L15, L42


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## 1 Introduction

This article studies optimal wholesale pricing contracts between a manufacturer of a high-quality, branded product that is sold to final customers via a retailer who also carries a low-quality private label, which enhances the retailer's bargaining power. The model finds that in a situation where the manufacturer of the branded product is incompletely informed about the costs of the private label the manufacturer can nonetheless implement the first-best outcome when using a wholesale pricing contract that combines an incremental quantity discount and a retroactive market-share discount. This tariff prevents inefficiencies caused by double marginalization, and it prevents the branded product from being inefficiently

[^0]de-listed. The model explains certain observations from the food retail industry and has implications for competition policy as is shown in the following. Its contribution to the research literature is discussed in Section 2.

As one of the observations from practice, Villas-Boas (2007, p. 646) provides anecdotal evidence suggesting that retail supermarkets receive substantial payments from the manufacturers of branded products. Such allowances "mainly come in two forms. The first type, called slotting fees, are in return for giving the supplier's products a prominent place on the retailers shelves, or indeed any space at all. The second type, called marketing or distribution fees, are suppliers' reward to retailers when they boost sales of their products by running promotional offers on them. Such fees have been around since the 1970s. But big grocery chains began to demand much larger rebates in the recession that followed the financial crisis." ${ }^{2}$ Industry observers sometimes argue that these allowances at least partly lack a service in return because, for example, marketing fees may exceed the retailers' expenses for advertising. ${ }^{3}$ As two further observations, allowances may be paid retroactively, and it appears to be more difficult for weaker retailers to receive allowances from the manufacturers, i.e., they are more likely paid to big retailers. ${ }^{4}$ The increase in the relevance of allowances, their potentially loose connection to specific services, and their repayment especially to large retailers raises the question whether they result from retailers' buyer power and whether they impede the allocative efficiency of the market.

Villas-Boas (2007, p. 646) establishes as a further observation that the "existence of quantity discounts is common practice in this industry." This may cause wholesale list prices of food products to be even higher than their retail prices, which raises the question whether the list prices are 'astronomical' in the sense of being meaningless for everyday business as

[^1]is sometimes believed by industry experts. ${ }^{5}$ For example, in 2014 Coca Cola charged German retailers a gross list price of EUR 12.10 per crate containing twelve 1l-bottles while the retail price was only between EUR 7.99 to $8.49 .{ }^{6}$ After deducting rebates, the average wholesale price of some grocery products may be as low as $50 \%$ of the initial gross list price. ${ }^{7}$

Besides quantity discounts, one sometimes also observes market-share discounts which allow the manufacturer "to condition [the retailer's] payment on the specific quantities of [the manufacturer's] good and a second product" (Majumdar and Shaffer, 2009, p. 397), for example, a private label product. This is often equivalent to a scheme conditioning the retailer's payment on the market share of the manufacturer's product in the total sales of the retailer, that may sometimes be approximated by the number of stores operated by a retailer. For example, in 2004 the German retail chain Kaufland demanded advertising allowances using the formula number of promotions times number of stores times some multiplier. ${ }^{8}$

Analyzing the efficiency effects of quantity discounts and market-share rebates is important, for example, in the context of the increasing importance of retailers' private label products. ${ }^{9}$ Some industry observers have argued that discounts "help suppliers of branded goods by discouraging grocers from promoting their own-label products instead. Ultimately, shoppers lose out, since cheaper products are harder to find. Some countries have tried to protect consumers by making rebates illegal" ${ }^{10}$ or by putting them under antitrust scrutiny. For example, the European Commission notes in its guidance paper on the abuse of a dominant position that conditional "rebates [... can] have actual or potential foreclosure effects similar to exclusive

[^2]purchasing obligations." ${ }^{11}$
Following Jeuland and Shugan (1983), the article reinforces an efficiency justification for quantity discounts, i.e., they help to eliminate double marginalization and ultimately sell the branded product to final customers at the lowest possible price. There is no inefficient exclusion of the private label in the sense of O'Brien and Shaffer (1997, p. 758) where the private label would be "excluded from the market even though a fully integrated [...] firm would sell both goods." In the model, retroactive allowances, which are contingent on the quantity of the branded product and the private label (market-share contracts), emerge as equilibrium outcomes, and they are a means for the manufacturer of a branded product, who is incompletely informed about the cost-characteristics of a retailer, to discriminate between a strong or a weak retailer, and to extract the entire information rent from the retailer. On the contrary, tariffs that are conditional merely on the quantity of the branded product can induce the retailer to reveal its cost-type truthfully only if the manufacturer leaves the retailer an information rent. As market-share discounts help to implement the full information outcome, they also avoid the branded product from being inefficiently de-listed by a strong retailer. One may conclude that allowances are no consequence of the retailer's buyer power because they do not harm but actually benefit the manufacturer. This is the case although the manufacturer does not receive a specific service in return in terms of, e.g., better advertising.

The article is structured as follows. Section 2 reviews the related literature. The model with complete information is presented in Section 3 while the case with incomplete information is studied in Section 4. Section 5 analyzes the scope for exclusive dealing in the model, and it further generalizes the model's setup. Section 6 concludes the article. Proofs are provided in the Appendix.

## 2 Literature Review

This article contributes to several strands in the literature, these are, studies on private label products, articles on (non-linear) wholesale pricing and

[^3]market-share discounts, analyses of incomplete information, inquiries on exclusive dealing, and research on (slotting) allowances. As the related literature is vast, this review focuses on the articles presumably most related to the present study.

In terms of private label products, the present article builds on the bilateral monopoly model with vertically differentiated goods proposed by Mills (1995) where an upstream manufacturer sells a branded good to a downstream retailer who, besides reselling the branded product, may also offer a private label substitute that the retailer can obtain at a wholesale price equaling its marginal costs of production. ${ }^{12}$ His article shows that competition from private label products reduces double marginalization by strengthening the position of the retailer vis à vis the brand manufacturer, which lowers the wholesale and retail price of the branded good, raises the retailer's profit, lowers that of the manufacturer, and raises consumer surplus. ${ }^{13}$ These results are in line with the empirical results of, e.g., Draganska et al. (2010) for the German ground coffee market or of Meza and Sudhir (2010) for the US-American cereals market, who both assume a discrete choice demand structure. Narasimhan and Wilcox (1998) obtain similar results in a model with a different demand structure, allowing for customers who are loyal to the branded product irrespective of the prices charged for the products, and under the assumption of linear wholesale prices.

Villas-Boas (2007), however, finds empirically that models with nonlinear

[^4]pricing describe a regional market of yogurt in the U.S. better than a model with linear pricing. Bonnet and Dubois (2010) present further empirical evidence that market conduct in the French market for bottled water appears to be more consistent with non-linear pricing. Already Mills (1995, p. 527) suggested that instead of linear pricing, as is assumed in his model, the firms may also use non-linear pricing schemes but "uncertainty and asymmetric information [...] hamper efforts to fine tune price schedules". Non-linear pricing in case of an incompletely informed manufacturer is, thus, analyzed in the present article. Earlier attempts of Mills (1999) to extend the model of Mills (1995) in this direction were done under the more restrictive assumptions of complete information and identical marginal costs of producing the branded product and the private label. Both assumptions are relaxed in the present article.

In this context, Iyer and Villas-Boas (2003, p. 81) note that "in actual practice, both the magnitude and the incidence of two-part tariffs may be quite insignificant. In mainstream retail sectors such as grocery retailing or departmental stores, retailers do not seem to pay lump-sum fees to manufacturers." Similarly, Villas-Boas (2007, p. 646) establishes "that retail supermarkets do not often pay fixed fees to their manufacturers", which reinforces the notion that two-part tariffs appear to be uncommon. Therefore, Draganska et al. (2010, p. 70) suggested that a "fruitful avenue for future research would be to explore how to incorporate quantity discounts into the negotiation process". Kolay et al. (2004) theoretically study quantity discounts in a bilateral monopoly model. They show in line with Jeuland and Shugan (1983) that quantity discounts may be used to eliminate double marginalization and, thus, produce the same market outcomes as in the case of a two-part tariff. The present article draws on this finding, showing as a new result that the combination of a quantity discount and a market-share discount eliminates double marginalization and implements the full information outcome even if the manufacturer is incompletely informed about the retailer's costs of producing the private label product.

Market-share discounts were, for example, studied by Majumdar and Shaffer (2009) who modify the model of Kolay et al. (2004) by assuming a bilateral monopoly where a retailer sells not only one but two horizontally differentiated products. ${ }^{14}$ One is produced by a dominant upstream firm,

[^5]the other is produced by a competitive fringe and sold to the retailer at prices equaling marginal costs. Demand can either be high or low, and it is observed only by the retailer but not by the dominant manufacturer. Majumdar and Shaffer (2009, p. 394) "show that it is profitable for the [manufacturer] to condition payment on how much the retailer buys from the fringe (market-share contracts). The [manufacturer ... may be] able to obtain the full-information outcome (unlike in standard screening models, where the [retailer] earns an information rent [...])." A qualitatively similar result is obtained in the present article. Yet, the mechanisms and predictions of the two models differ. In the model of Majumdar and Shaffer (2009) with private information about demand, the manufacturer would offer a rebate to the retailer if it sells little of the fringe product because this is evidence of demand being low. In the present model with private information about the production costs of the private label, the manufacturer would offer a rebate to the retailer if it sells much of the private label product because this is evidence of this product's production costs being low. In the context of market-share rebates, there is at least one important difference between the literature on incomplete information about demand and the present article on incomplete information about the costs of a rival product. Differences in firms' relative marginal costs typically have an effect on their relative market shares, which is important when using variations of these market shares to identify rivals' cost-types, whereas different demand levels do not necessarily affect firms' market shares.

The present article is also related to Corbett et al. (2004) who study optimal wholesale contracts and, in particular, the value of information (i.e., the difference of the manufacturer's profits in the situations with complete or incomplete information) in a bilateral monopoly model where the manufacturer is incompletely informed about the retailer's selling costs. They find that "information and two-part contracts are strategic complements: The value of information is greater under two-part contracts than under one-part contracts, and the value of being able to offer two-part contracts rather than one-part contracts is greater under full information than under asymmetric information." Intuitively, the manufacturer earns higher profits
product that is produced and sold by a competitive fringe. Inderst and Shaffer (2010) explore the effects market-share discounts in the wholesale market on intrabrand and interbrand competition in the retail market. They assume a model with complete information whereas the present article is concerned with the capability of market-share discounts to solve inefficiencies arising from incomplete information.
when using a two-part tariff than by using a linear price. Therefore, incomplete information lowers the manufacturer's profit by a larger amount in the situation of a two-part tariff, and the manufacturer attributes a greater value to better information in this case. This is different in the present model where the manufacturer can use market-share contracts to implement the full-information outcome even if it is incompletely informed about the retailer's characteristics. Therefore, information has no value to the manufacturer under such circumstances.

In terms of its contribution to the literature on exclusive dealing, the present article builds on Yehezkel (2008) who also studies a variant of the Mills (1995)-model with a manufacturer of a branded product and a retailer who may sell a private label. ${ }^{15}$ Yehezkel (2008) shows that no equilibria with exclusive dealing exist if the firms are fully informed and can use non-linear tariffs to implement the vertically-integrated outcome. Exclusive dealing may however occur under the assumption of asymmetric information. The present article adds to his results by showing that exclusive dealing does not occur on the equilibrium path even under asymmetric information if the firms can use market-share contracts to implement the full information outcome. The different, but complementary results of the two models are caused by different assumptions that prevent the firms from implementing the full information

[^6]outcome in the model of Yehezkel (2008). In particular, he assumes a market-share contract that merely specifies how the total payment of the retailer for the branded product depends on the quantities of the branded product and the private label where the manufacturer may set a maximum quantity to be sold of the private label product. The present model allows for market-share contracts where the manufacturer sets a list price and grants a discount based on the quantity of the branded product sold by the retailer while the manufacturer may pay an allowance to the retailer depending on the minimum quantity sold of the private label. This finer grained structure of payments matches certain features of wholesale supply contracts in the food industry and is crucial for the ability to implement the full information outcome even under asymmetric information, which makes exclusive dealing unprofitable and attributes a value of nil to better information.

The present article shows that an incompletely informed manufacturer ideally pays an allowance to a retailer who sells a low-cost private label. The allowance is optimally paid although the manufacturer does not receive a specific service in return. This adds, for example, to the results obtained by Klein and Wright (2007) who show that providing preferred retail shelf space benefits the manufacturer more than the retailer, who thus receives a compensation in form of a slotting allowance, or Kim and Staelin (1999) who show that paying slotting allowances in return for merchandising support of their brands is a best response for competing manufacturers even if in equilibrium this does not translate into higher profits. In both earlier models, the payment of the allowance is conditional on a specific service delivered by the manufacturer whereas in the present model they are optimal for the manufacturer even without receiving such a service.

## 3 The Model with Complete Information

This section extends the model of Mills (1995) by allowing for nonlinear pricing, discount schemes, bargaining, and uncertainty about the production costs of the private label product. For the moment, it retains the assumption that the firms are completely informed about each other's characteristics.

### 3.1 Setup

Consider the following static model of an industry with one upstream and one downstream firm. A monopolistic downstream retailer sells two vertically differentiated products to final customers. The products have exogenously determined qualities $s_{u}$ and $s_{d}$ with $0<s_{d}<s_{u}$. The low-quality product is produced by the downstream retailer at constant marginal costs $c_{d}$. The high-quality product is produced by an upstream manufacturer at constant marginal costs $c_{u}$ with $c_{d} \leq c_{u}$. Assuming the marginal costs of the high-quality product to be weakly above those of the low-quality product is plausible because generating a higher quality may require a costlier production process or more expensive inputs. Production does not require any fixed costs. The quality differential is defined as $\Delta s=s_{u}-s_{d}>0$ and the cost differential as $\Delta c=c_{u}-c_{d} \geq 0$.

The upstream manufacturer charges a payment $T\left(q_{u}, q_{d}\right)$ from the downstream retailer if it buys a quantity $q_{u}$ of the high-quality product, and if the retailer sells a quantity $q_{d}$ of the private label.

$$
T\left(q_{u}, q_{d}\right)=\left\{\begin{array}{ccc}
0 & \text { if } & q_{u}=0  \tag{1}\\
w\left(q_{u}\right) q_{u}+f\left(q_{u}\right)-a\left(q_{d}\right) & \text { if } & q_{u}>0
\end{array}\right.
$$

The manufacturer sells the high-quality product to the downstream firm by selecting a wholesale price $w$ per unit, which may be a function of the output $q_{u}$ to allow, e.g., for quantity discounts. If the firms agree on a two-part tariff the downstream retailer also pays a fixed fee $f \geq 0$ to the upstream manufacturer. The manufacturer may also pay an allowance $a \geq 0$ to the retailer that may be contingent on the output $q_{d}$ of the private label product. Conditioning the payment on the quantities of both goods constitutes a market-share discount as was analyzed, for example, by Majumdar and Shaffer (2009) or Inderst and Shaffer (2010). The values of $w, f$, and $a$ depend on the firms' relative bargaining power as is explored in Sections 3.2 and 4, while Mills (1995) assumed $f=0$ and $a=0$.

The downstream retailer decides whether to list (i.e., acquire and re-sell) the high-quality product, and it selects the retail prices $p_{u}$ and $p_{d}$. Condition (2) represents the retailer's individual rationality constraint, i.e., the high-quality product is listed $(\ell=1)$ if the profit $\pi_{d, \ell}$ of the downstream manufacturer at profit-maximizing prices $p_{u}$ and $p_{d}$ is weakly greater than
its reservation profit $\pi_{d, n \ell}$ when selling the private label only $(\ell=0)$.

$$
\ell=\left\{\begin{array}{lll}
1 & \text { if } & \pi_{d, \ell} \geq \pi_{d, n \ell}  \tag{2}\\
0 & \text { if } & \pi_{d, \ell}<\pi_{d, n \ell}
\end{array}\right.
$$

Final customers' preference for quality is measured by the variable $\theta$ that is uniformly distributed in the interval $\theta \in[0,1]$ with mass 1 . Consumers' indirect utility function for the high-quality product is given by equation (3), and by equation (4) for the low-quality product.

$$
\begin{align*}
& v_{u}=r+\theta s_{u}-p_{u}  \tag{3}\\
& v_{d}=r+\theta s_{d}-p_{d} \tag{4}
\end{align*}
$$

Demand for the two products is defined in Lemma 1. ${ }^{16}$ The inequalities in (5) are purposefully chosen to preclude the uninteresting case where - already in the benchmark model of Mills (1995) - the retailer would optimally sell only one of the products. The inequality $r-\left(s_{d}+c_{d}\right) \leq 0$ follows from $\theta_{0, d} \geq 0$ and ensures that the market may be incompletely covered, i.e., there may be some customers who do not buy any of the two goods given the profit-maximizing price $p_{d}^{*}$ (see equation (11) below).

Lemma 1. Given assumption (5) the demand for the high-quality product can be written as in (6), and the demand for the low-quality product as in (7).

$$
\begin{gather*}
r-\left(s_{d}+c_{d}\right) \leq 0 \leq \Delta c<\Delta s  \tag{5}\\
q_{u}=1-\hat{\theta} \quad \text { with } \quad \hat{\theta}=\frac{p_{u}-p_{d}}{\Delta s} \tag{6}
\end{gather*}
$$

[^7]\[

$$
\begin{equation*}
q_{d}=\hat{\theta}-\theta_{0, d} \quad \text { with } \quad \theta_{0, d}=\frac{p_{d}-r}{s_{d}} \tag{7}
\end{equation*}
$$

\]

Proof. See Mills (1995) and the Appendix.
The profit function of the downstream retailer is given by (8) and that of the upstream manufacturer by (9).

$$
\begin{gather*}
\pi_{d}=\ell\left[q_{u}\left(p_{u}-w\right)-f+a\right]+q_{d}\left(p_{d}-c_{d}\right)  \tag{8}\\
\pi_{u}=\ell\left[q_{u}\left(w-c_{u}\right)+f-a\right] \tag{9}
\end{gather*}
$$

If the downstream retailer lists the high-quality product ( $\ell=1$ ), maximizing $\pi_{d}$ with respect to $p_{d}$ and $p_{u}$ yields the equilibrium prices shown in (10) and (11). The profit of the downstream firm can be written as in (12) where $\pi_{d, n \ell}$ denotes the profit of the retailer when not listing the high-quality product. In this case, the retailer would optimally set the price $p_{d}^{*}$ and make the reservation profit $\pi_{d, n \ell}$.

$$
\begin{gather*}
p_{u}^{*}=\frac{s_{u}+w+r}{2}  \tag{10}\\
p_{d}^{*}=\frac{s_{d}+c_{d}+r}{2}  \tag{11}\\
\pi_{d, \ell}=\pi_{d, n \ell}+\frac{\left[\Delta s-\left(w-c_{d}\right)\right]^{2}}{4 \Delta s}-f+a  \tag{12}\\
\pi_{d, n \ell}=\frac{1}{4 s_{d}}\left(s_{d}+r-c_{d}\right)^{2} \tag{13}
\end{gather*}
$$

The high-quality product is listed if $\pi_{d, \ell}-\pi_{d, n \ell} \geq 0$ with $\pi_{d, \ell}-\pi_{d, n \ell}=$ $\left[\left(\Delta s-w+c_{d}\right)^{2} /(4 \Delta s)\right]-f+a$. Raising the wholesale price $w$ or the fixed fee $f$ deteriorate the upstream manufacturer's chance of being listed. The chances improve for higher costs $c_{d}$ of producing the low-quality substitute, for a greater quality advantage $\Delta s$ of the high-quality product, or if the manufacturer pays the retailer an allowance $a$.

### 3.2 Wholesale Pricing

This subsection analyzes the equilibria of the model with (A) linear wholesale prices, (B) two-part tariffs, and (C) quantity discounts. The variables in each of these cases are indexed by $A, B$, or $C$ with two subcases $A A$ and $B B$.
A) Linear price: Mills (1995) studies a linear pricing schedule with $w>0$, $f=0$, and $a=0$. Lemma 2 characterizes the equilibrium in this situation.

Lemma 2. When using a linear tariff, a profit-maximizing manufacturer optimally chooses the wholesale price stated in (14). For $w_{A}$, the profit of the upstream firm is given by (15) and that of the downstream firm by (16).

$$
\begin{align*}
& \begin{array}{ccc}
w_{A}=c_{u}+\frac{\Delta s-\Delta c}{} & \text { if } & \hat{\theta}\left(w_{A}\right)-\theta_{0, d} \geq 0 \\
w_{A A}=\frac{s_{u} c_{d}-r^{2} \Delta s}{2} & \text { otherwise }
\end{array}  \tag{14}\\
& \pi_{u, A}=\frac{4[\Delta s-\Delta c]^{2}}{32 \Delta s}  \tag{15}\\
& \pi_{d, A}=\pi_{d, n \ell}+\frac{2[\Delta s-\Delta c]^{2}}{32 \Delta s} \tag{16}
\end{align*}
$$

Proof. See Mills (1995).
The manufacturer sets the wholesale price $w_{A}$ if, in this case, the retailer sells a non-negative quantity $q_{d}=\hat{\theta}\left(w_{A}\right)-\theta_{0, d} \geq 0$ of the private label product. If however the retailer finds it unprofitable to sell the private label product (a situation indexed by $A A$ ), the manufacturer chooses $w_{A A}>w_{A}$ such as to equalize $\hat{\theta}\left(w_{A A}\right)=\theta_{0, d}$, i.e., $w_{A A}$ keeps the retailer indifferent between offering the private label or not. The inequality $\hat{\theta}\left(w_{A}\right)-\theta_{0, d}<0$ is the same as $c_{u}<c_{u, A A} \equiv c_{d}+\Delta s \cdot\left[2\left(c_{d}-r\right)-s_{d}\right] / s_{d}$. The retailer will offer just the branded product if its cost-disadvantage is sufficiently small. Because of $w_{A A}>w_{A}$ the ensuing profits satisfy the conditions $\pi_{u, A A}>\pi_{u, A}$ and $\pi_{d, A A}<\pi_{d, A}$.

The wholesale pricing equation (14) with $w_{A}>c_{u}$ is at the root of the double marginalization problem. The presence of the private label product, however, restricts the manufacturer from setting its wholesale price $w$ above $c_{u}$ mostly if the two goods are positioned close to each other $(\Delta s \rightarrow 0)$. Yet, setting $w>c_{u}$ is easier if the cost advantage of the low-quality good is small $(\Delta c \rightarrow 0)$.
B) Two-part tariff with bargaining: The equilibrium with a two-part tariff is stated in Lemma 3. ${ }^{17}$

[^8]Lemma 3. When using a two-part tariff with $w_{B}=c_{u}$ and $a=0$ while splitting the profits by implementing a Nash (1950, 1953) bargaining solution to determine the fixed fee $f_{B}$, which is obtained as the maximizer of condition (17), the firms make the profits given by (18) and (19). The two-part tariff creates a Pareto improvement for the two firms and final customers when being compared to the linear price.

$$
\begin{gather*}
f_{B}=\arg \max \left[f_{B}-\pi_{u, A}\right] \cdot\left[\pi_{d, B}-\pi_{d, A}\right]=\frac{5[\Delta s-\Delta c]^{2}}{32 \Delta s}  \tag{17}\\
\pi_{u, B}=\left\{\begin{array}{c}
f_{B} \quad \text { if } c_{u} \geq c_{u, B B} \text { with } c_{u, B B}=c_{d}+\Delta s \cdot \frac{c_{d}-r}{s_{d}} \\
f_{B B} \\
\text { otherwise, with } \pi_{d, B}\left(c_{u}, f_{B B}\right)=\pi_{d, B}\left(c_{u, B B}, f_{B}\right)
\end{array}\right.  \tag{18}\\
\pi_{d, B}=\left\{\begin{array}{cc}
\pi_{d, n \ell}+\frac{8[\Delta s-\Delta c]^{2}}{32 \Delta s}-f_{B} & \text { if } \\
\pi_{d, B}\left(c_{u, B B}, f_{B}\right) & \text { otherwise }
\end{array}\right. \tag{19}
\end{gather*}
$$

Proof. See the Appendix.
To solve the double marginalization problem implied by the linear price $w_{A}$, the upstream manufacturer optimally sells the high-quality good to the downstream retailer at a wholesale price equaling its marginal $\operatorname{costs} c_{u}$. All profits are initially generated in the downstream market and split via the fixed fee $f_{B}$, which rises if the cost disadvantage $\Delta c$ of the upstream manufacturer falls and/or if its quality advantage $\Delta s$ rises.

It is optimal to charge $f_{B}$ if the retailer would want to sell both goods, i.e., $\hat{\theta}\left(c_{u}\right)-\theta_{0, d} \geq 0$. This is the case if the cost-disadvantage of the upstream manufacturer is sufficiently large, i.e., $c_{u}>c_{u, B B}$. For $c_{u}=c_{u, B B}$ the retailer would sell the branded product only, making a profit $\pi_{d, B}\left(c_{u, B B}, f_{B}\right)$ and paying a fixed fee $f_{B}\left(c_{u, B B}\right)$ to the manufacturer. For $c_{u}<c_{u, B B}$ the manufacturer can set a fixed fee $f_{B B}>f_{B}$ such that $\pi_{d, B}\left(c_{u}, f_{B B}\right)=\pi_{d, B}\left(c_{u, B B}, f_{B}\right)$ applies. This condition keeps the retailer indifferent between selling the private label or not.

Specifying the fixed fee $f_{B}$ in (17) as the solution to a Nash bargaining problem is parsimonious. Different solutions would be found if one, e.g., allowed for an asymmetric distribution of bargaining power as in Iyer and Villas-Boas (2003) and Draganska et al. (2010), if the relevant outside option to agreeing on a two-part tariff was 'not entering in business relations' rather than 'setting a linear price', or if there was perfect rent extraction by the manufacturer. The following analysis is qualitatively the same for all of these alternatives and does not depend on the exact value of $f_{B}$, which makes it convenient to use the parsimonious structure suggested in Lemma 3.

As a remark on firms' pricing strategies in practice, it appears that in the food industry two-part tariffs of the form analyzed here are less frequently used than quantity discounts. However, it is still important to study two-part tariffs because the resulting profits $\pi_{u, B}$ and $\pi_{d, B}$ are relevant benchmarks for calculating the values of the list price and a quantity discount as is shown in the following section.
C) Quantity Discounts The European Commission defines a conditional, incremental rebate as one where "the customer is given a rebate if its purchases over a defined reference period exceed a certain threshold, the rebate being granted [... on the purchases] made in excess of those required to achieve the threshold". ${ }^{18}$ In this article, a rebate of this sort where the buyer of the branded product pays a higher price on the initial units up to some threshold $\hat{q}_{u}$ and a lower price on the units purchased at or above this threshold is called a quantity discount. In a regulatory context, it would also be referred to as a decreasing block tariff. Lemma 4 shows that the objective of granting a quantity discount needs not be the exclusion of the private label from the market. The quantity discount is rather a means for raising the allocative efficiency of the market by avoiding double marginalization as was also argued by Jeuland and Shugan (1983). Lemma 4 establishes the equilibrium with quantity discounts where the rents are distributed as in the case of the two-part tariff with bargaining analyzed above.

Lemma 4. When using a quantity discount of the form displayed in (20) with a list price $w_{C}$ as shown in (21) and the threshold $\hat{q}_{u}$ defined in (22) the firms make the same profits $\pi_{u, C}=\pi_{u, B}$ and $\pi_{d, C}=\pi_{d, B}$ as in the case with a two-part tariff.

$$
\begin{gather*}
T_{C}=\left\{\begin{array}{ccc}
w_{C} q_{u} & \text { if } q_{u}<\hat{q} \\
w_{C} \hat{q}_{u}+c_{u}\left(q_{u}-\hat{q}_{u}\right) & \text { if } q_{u} \geq \hat{q}_{u}
\end{array}\right.  \tag{20}\\
w_{C}=\left\{\begin{array}{cc}
c_{u}+\frac{\Delta s-\Delta c}{16 \Gamma} & \text { if } c_{u} \geq c_{u, B B} \\
f_{B B} / \hat{q}_{u} & \text { if } c_{u}<c_{u, B B}
\end{array}\right.  \tag{21}\\
\hat{q}_{u}=\Gamma q_{u, B} \text { with } \Gamma \in(0,1) \tag{22}
\end{gather*}
$$

[^9]Proof. Lemma 4 was proven by Kolay et al. (2004) in a bilateral monopoly model with just one good. The proof with an additional private label product is as follows: The upstream manufacturer charges a wholesale price $w_{C}>c_{u}$ (the list price) for all units of the high-quality good that are purchased up to a threshold $\hat{q}_{u}$. Above this threshold (for all units $q_{u}-\hat{q}_{u}$ ) the manufacturer grants a per-unit discount $w_{C}-c_{u}$ such that the price for these incremental units of the high-quality product equals its marginal costs $c_{u}$. Charging a wholesale price of $c_{u}$ for the marginal units solves the double marginalization problem, and the retailer chooses the same price $p_{u, B}^{*}$ as in the case of a two-part tariff with $w_{B}=c_{u}$.

Charging $w_{C}>c_{u}$ for all units below $\hat{q}_{u}$ implies that the average wholesale price is above the marginal wholesale price, which allows the manufacturer to extract profits from the retailer. The value of $w_{C}$ in (21) was obtained by solving the condition $\pi_{u, C}=\left[w_{C} \hat{q}_{u}+c_{u}\left(q_{u}-\hat{q}_{u}\right)\right]-c_{u} q_{u, B}=f_{B}$ for $w_{C}$. If the retailer does not list the private label $\left(c_{u} \leq c_{u, B B}\right) w_{C}$ is chosen such that $f_{B B}=w_{C} \hat{q}_{u}$ applies. This ensures that the manufacturer makes the same profit when using the quantity discount $T_{C}$ or the two-part tariff with $w_{B}=c_{u}$ and $f_{B}\left(\right.$ or $\left.f_{B B}\right)$. The retailer's total payment $T_{C}$ thus equals the payment $T_{B}=f_{B}+c_{u} q_{u, B}$ that it would make to the manufacturer when using a two-part tariff. Moreover, the retailer sets the same prices in the downstream market and receives the same revenues in both situations such that its profits are also the same, i.e., $\pi_{d, C}=\pi_{d, B}$.

Lemma 4 establishes a link between research in economics, which often uses two-part tariffs as a solution to overcome the double marginalization problem, and practice in the food industry, where quantity discounts appear to be used more frequently than two-part tariffs. Note that the inequality $w_{C}>p_{u}^{*}\left(w=c_{u}\right)$ applies if $\Gamma<0.125-\left(p_{d}^{*}-c_{d}\right) / 8\left[p_{u}^{*}\left(w=c_{u}\right)-c_{u}\right]$, i.e., if the threshold $\hat{q}_{u}$ is sufficiently low. This matches the observation that manufacturers' list prices for food or drugstore products are sometimes above their retail prices. In terms of the present model, Lemma 4 establishes a relevant benchmark for the subsequent analysis of wholesale pricing schemes when the manufacturer is incompletely informed about the marginal costs $c_{u}$ of producing the private label product. Uncertainty about marginal costs is introduced in Section 3.3 and incomplete information in Section 4. Neither of these features has been analyzed previously in the model of Mills (1995) or its derivatives.

### 3.3 Uncertainty about Costs

This subsection adds cost-uncertainty to the model. At the beginning of the game, the marginal production costs of producing the low-quality good are drawn randomly from the set $c_{d} \in\left\{\underline{c}_{d}, \bar{c}_{d}\right\}$ with $\underline{c}_{d}<\bar{c}_{d}$. The results of the model are not sensitive to the assumption of just two cost-states. Generalized results with infinitely many states are provided as an extension in Section 5.2. Expected marginal costs are denoted by $E\left(c_{d}\right)$, and the cost differentials by $\Delta \underline{c}=c_{u}-\underline{c}_{d}$ and $\Delta \bar{c}=c_{u}-\bar{c}_{d}$. Under these assumptions, condition (5) in Lemma 1 must be modified as is shown in (23) to ensure that demand for the two goods can be written as in (6) and (7).

$$
\begin{equation*}
r-\left(s_{d}+\underline{c}_{d}\right) \leq 0 \leq \Delta \underline{c} \leq \Delta s \tag{23}
\end{equation*}
$$

If the realization of $c_{d}$ is observed by the downstream retailer and the upstream manufacturer prior to their pricing decisions, they behave as was described in Section 3.2. With two-part tariffs and quantity discounts at hand, there appears no need for further wholesale pricing schemes (e.g., tariffs including allowances) to implement the jointly profit-maximizing market outcome, i.e., to prevent double marginalization. This is different if the upstream firm is only incompletely informed about characteristics of the downstream firm. Mills (1999, p. 137) argues: "To implement an optimal two-part tariff [...] the manufacturer must have complete information about the demand the retailer confronts. If the manufacturer is uncertain about consumer demand, and the retailer has private information about that demand [...] then the manufacturer is at an informational disadvantage [... and] cannot implement an optimal two-part tariff." Section 4 illustrates that these difficulties also exist if (other than having incomplete information about demand) the manufacturer is incompletely informed about the costs of the private label product. However, Section 4 also shows as its main contribution how a market-share discount can be used to implement the full information outcome even under asymmetric information.

## 4 The Model with Incomplete Information

This section analyzes wholesale pricing if the upstream firm is incompletely informed about the downstream firm's marginal costs. Subsection 4.1 illustrates that a linear wholesale price causes two inefficiencies. First, it
causes double marginalization and, second, setting a single price does not allow to discriminate between retailers with high or low costs. Subsection 4.2 shows how the manufacturer can induce the retailer to reveal its cost-type when offering a menu in the form of a market-share discount. However, only the combination of a market-share discount and a quantity discount induces the retailer to reveal its cost type and to solve the double marginalization problem, as is shown in Subsection 4.3.

### 4.1 Linear price

Assume that the realization of $c_{d}$ is observed by the downstream retailer but not by the upstream manufacturer. The retailer thus bases its decisions on the true realization of $c_{d}$ while the manufacturer relies on the expected costs $E\left(c_{d}\right)$ when setting the linear wholesale price $w_{D} .{ }^{19}$ Continuing the labeling introduced in Section 3.2, this setting is referred to as $D$ and variables are indexed accordingly. Lemma 5 characterizes the equilibrium in this situation.

Lemma 5. When using a linear tariff while being incompletely informed about the costs $c_{d}$, a profit-maximizing manufacturer optimally chooses the wholesale price stated in (24).

$$
\begin{gather*}
w_{D}=c_{u}+\frac{\Delta s-\left(c_{u}-E\left(c_{d}\right)\right)}{2} \quad \text { if } \quad \hat{\theta}\left(w_{D}\right)-\theta_{0, d}\left(w_{D}\right) \geq 0 \\
w_{D D}=\frac{s_{u} E\left(c_{d}\right)-r \Delta s}{s_{d}} \quad \text { otherwise }  \tag{24}\\
\text { One finds } \pi_{u, D}<\pi_{u, A} \forall c_{d}, \pi_{d, D}\left(\underline{c}_{d}\right)<\pi_{d, A}\left(\underline{c}_{d}\right), \text { and } \pi_{d, D}\left(\bar{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right) .
\end{gather*}
$$

Proof. See the appendix.
The optimal wholesale prices $w_{D}$ and $w_{D D}$ resemble $w_{A}$ and $w_{A A}$ from condition (14) if one replaces the true value of $c_{d}$ with $E\left(c_{d}\right)$. Besides double marginalization, which is caused by linear pricing, further inefficiencies arise from incomplete information as is shown now.

In case of $\underline{c}_{d}$ the manufacturer sets a wholesale price $w_{D}\left(\underline{c}_{d}\right)>w_{A}\left(\underline{c}_{d}\right)$ that is suboptimally high and loses on market share. This causes both the manufacturer and the retailer to earn lower profits than under complete information $\left(\pi_{u, D}\left(\underline{c}_{d}\right)<\pi_{u, A}\left(\underline{c}_{d}\right)\right.$ and $\left.\pi_{d, D}\left(\underline{c}_{d}\right)<\pi_{d, A}\left(\underline{c}_{d}\right)\right)$. Because of $\pi_{d, D}\left(\underline{c}_{d}\right)<\pi_{d, A}\left(\underline{c}_{d}\right)$ it is more difficult to satisfy the retailer's individual

[^10]rationality constraint (2), and the branded product will be de-listed if $\pi_{d, D}\left(\underline{c}_{d}\right)<\pi_{d, n \ell}\left(\underline{c}_{d}\right)$, which would be inefficient if the product had been listed under complete information, i.e., $\pi_{d, n \ell}\left(\underline{c}_{d}\right) \leq \pi_{d, A}\left(\underline{c}_{d}\right)$. In case of $\bar{c}_{d}$ the manufacturer sets a wholesale price $w_{D}\left(\bar{c}_{d}\right)<w_{A}\left(\bar{c}_{d}\right)$ that is suboptimally low and loses on its profit margin. This implies lower profits for the manufacturer $\left(\pi_{u, D}\left(\bar{c}_{d}\right)<\pi_{u, A}\left(\bar{c}_{d}\right)\right)$ and higher profits for the retailer $\left(\pi_{d, D}\left(\bar{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right)\right)$ as compared to situation $A$ with complete information. This relaxes the retailer's individual rationality constraint (2), and the branded product might be listed only under the assumption of an incompletely informed manufacturer, i.e., $\pi_{d, D}\left(\bar{c}_{d}\right) \geq \pi_{d, n \ell}\left(\bar{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right)$.

Because of $\pi_{d, D}\left(\underline{c}_{d}\right)<\pi_{d, A}\left(\underline{c}_{d}\right)$ and $\pi_{d, D}\left(\bar{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right)$ the retailer does not have an incentive to reveal the realization of its marginal costs truthfully. It optimally signals to have low marginal costs irrespective of their true value, which gives rise to the pooling equilibrium presented in Lemma 5. The linear wholesale price thus is neither capable of eliminating double marginalization nor of separating between high-cost and low-cost retailers.

### 4.2 Market-share discount

Lemma 5 showed that an incompletely informed manufacturer makes lower profits than a completely informed manufacturer for all realizations of $c_{d}$. Proposition 1 suggests a tariff $T_{E}$ making it incentive-compatible for the retailer to reveal the true realization of $c_{d}$ such that the manufacturer's profits will rise to the same level as under complete information when using a linear price. Hence, I use situation $A$ from above as a benchmark. This separating equilibrium is obtained by the manufacturer if it charges a list price that is appropriate when facing a high-cost retailer. This price is combined with a retroactive rebate where the manufacturer makes a lump-sum repayment to the retailer (equivalent interpretations are an all-units discount or a retroactively paid allowance) conditional on selling a combination of $q_{u}$ and $q_{d}$ that would only be chosen by a low-cost retailer. By conditioning the allowance not only on the quantities of the high-quality product but also on those of the low-quality product this repayment constitutes a market-share discount. As a specific characteristic of this mechanism, it allows the manufacturer to implement the full information outcome without having to leave an information rent to the retailer. All variables are labeled by the index $E$.

Proposition 1. Tariff $T_{E}$ shown in (25) - with the allowance $a_{E}$ being defined in (26) - implements the same market outcomes under incomplete information that would also be obtained under complete information when the manufacturer sets a linear wholesale price.

$$
\begin{gather*}
T_{E}=\left\{\begin{array}{c}
q_{u} \bar{w}_{A}-a_{E} \text { if } q_{u} \leq q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right) \text { and } q_{d} \geq q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right) \\
q_{u} \bar{w}_{A} \\
\text { otherwise } \\
a_{E}=q_{u} \cdot\left[\bar{w}_{A}-\underline{w}_{A}\right]
\end{array} .\right. \tag{25}
\end{gather*}
$$

One finds $\pi_{u, E}=\pi_{u, A} \forall c_{d}$ and $\pi_{d, E}=\pi_{d, A} \forall c_{d}$. This implies $\pi_{u, E}>\pi_{u, D} \forall c_{d}$. Finding $\pi_{d, A}\left(\underline{c}_{d}\right)-q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right) \cdot\left(\bar{c}_{d}-\underline{c}_{d}\right)<\pi_{d, A}\left(\bar{c}_{d}\right)$ ensures that it is incentive compatible for the retailer to reveal its cost-type truthfully.

Proof. The variable $q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ denotes the sales of the high-quality product if it is sold to a low-cost retailer with $\underline{c}_{d}$ at a wholesale price $\underline{w}_{A}=w_{A}\left(\underline{c}_{d}\right)$, which is the linear wholesale price that a completely informed manufacturer would charge from a low-cost retailer (see equation (14)). The variable $q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ denotes the sales of the private label product in this situation. Similarly, $\bar{w}_{A}=w_{A}\left(\bar{c}_{d}\right)$ denotes the linear wholesale price that a completely informed manufacturer would charge from a high-cost retailer.

The upstream manufacturer sets a high list price $\bar{w}_{A}$ and implements the allowance $a_{E}$ as a retroactive all-units discount whose value $a_{E}=q_{u} \cdot\left[\bar{w}_{A}-\underline{w}_{A}\right]$ is chosen such that the low-cost retailer's net payment $q_{u} \bar{w}_{A}-a_{E}$ to the incompletely informed manufacturer equals the payment $q_{u} \underline{w}_{A}$ it would have made to a completely informed manufacturer. The manufacturer grants $a_{E}$ at the end of the period only after observing that the downstream firm's market conduct was consistent with having had low marginal costs. This is the case for $q_{u} \leq q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ and $q_{d} \geq q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right) .{ }^{20}$ Charging an effective price of $\underline{w}_{A}$ from a low-cost retailer and $\bar{w}_{A}$ from a high-cost retailer establishes $\pi_{u, E}=\pi_{u, A} \forall c_{d}$ and $\pi_{d, E}=\pi_{d, A} \forall c_{d}$. Given $\pi_{u, D}<\pi_{u, A} \forall c_{d}$ from Lemma 5, one finds $\pi_{u, E}>\pi_{u, D} \forall c_{d}$, that is, tariff $T_{E}$ allows the incompletely informed manufacturer to raise its profits above those made when setting a single linear price based on $E\left(c_{d}\right)$.

[^11]Given $\pi_{d, E}=\pi_{d, A} \forall c_{d}$, the retailer's individual rationality constraint $\pi_{d, E} \geq \pi_{d, n \ell}$ will be satisfied under tariff $T_{E}$ with incomplete information if it is satisfied under tariff $T_{A}$ with incomplete information. Because of $\pi_{d, A}\left(\underline{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right)$ it is incentive compatible for a low-cost retailer to reveal its cost-type truthfully. It will be shown that it is also incentive compatible for a high-cost retailer to pay $\bar{w}_{A}$ and earn $\pi_{d, A}\left(\bar{c}_{d}\right)$ rather than mimicking the conduct of a low-cost retailer by setting $q_{u}=q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ and $q_{d}=q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ in order to qualify for the retroactive rebate. When misrepresenting its cost-type, the retailer would earn the profit $\pi_{d, A}\left(\underline{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right)$ minus the extra expenses of producing $q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ units of the private label product at the additional cost of $\bar{c}_{d}-\underline{c}_{d}$ per unit. The retailer truthfully reports its cost-type if its incentive compatibility constraint (27) is satisfied that can be re-stated as in (28) after plugging in the specific functional forms of $\pi_{d, A}\left(\underline{c}_{d}\right)$ and $\pi_{d, A}\left(\bar{c}_{d}\right)$ stated in (16).

$$
\begin{gather*}
\pi_{d, A}\left(\underline{c}_{d}\right)-q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right) \cdot\left(\bar{c}_{d}-\underline{c}_{d}\right)<\pi_{d, A}\left(\bar{c}_{d}\right)  \tag{27}\\
\frac{3 \Delta s}{4}-\frac{\left(\bar{c}_{d}-\underline{c}_{d}\right) \Delta s}{s_{d}}+\frac{\Delta \bar{c}-7 \Delta \underline{c}}{8}<0 \tag{28}
\end{gather*}
$$

Inequality (28) applies (i.e., it is unprofitable for the high-cost retailer to lie about its costs $\bar{c}_{d}$ ) even under the circumstances that would make such lying most profitable, which is the case if the repayment $a_{E}=q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right) \cdot\left[\bar{w}_{A}-\underline{w}_{A}\right]$ is at its maximum. Using the specific functional forms of $q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right), \bar{w}_{A}$, and $\underline{w}_{A}$, one can show that $a_{E}$ is maximized for $\max \left(\bar{c}_{d}-\underline{c}_{d}\right)$. The modeling assumption $c_{d} \leq c_{u}$ implies $\max \left(\bar{c}_{d}\right)=c_{u}$, while $\Delta \underline{c} \leq \Delta s$ from (23) implies $\min \left(\underline{c}_{d}\right)=c_{u}-\Delta s$. Plugging these terms in (28) and re-arranging yields $-1 / 8<\Delta s / s_{d}$, which is true because of $\Delta s>0$ and $s_{d}>0$, and it proves that the high-cost retailer's incentive constraint (27) is satisfied.

Tariff $T_{E}$ implements a first-best solution by inducing the retailer to reveal the realization of its marginal costs $c_{d}$ truthfully without having to leave an information rent to the retailer. This is because the tariff $T_{E}$ imposes a twofold punishment on lying about the retailer's cost-type. First, to receive the discount a high-cost retailer would have to buy a suboptimally low quantity of the branded product $q_{u} \leq q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)<q_{u}\left(\bar{w}_{A}, \bar{c}_{d}\right)$. Second, a high-cost retailer would have to sell a suboptimally high quantity of the private label $q_{d} \geq q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right)<q_{d}\left(\bar{w}_{A}, \bar{c}_{d}\right)$. With just one of these conditions
in place the manufacturer could sometimes induce the high-cost retailer to truthfully reveal its cost-type only by leaving it an information rent as is argued in Lemma 6.

Lemma 6. Tariff $T_{E^{\prime}}$ is the same as $T_{E}$ but for the missing condition on $q_{d}$.

$$
T_{E^{\prime}}=\left\{\begin{array}{ccc}
q_{u} \bar{w}_{A}-a_{E} & \text { if } & q_{u} \leq q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)  \tag{29}\\
q_{u} \bar{w}_{A} & \text { otherwise }
\end{array}\right.
$$

If the weak inequality $2 \Delta s /(\Delta \underline{c}+\Delta \bar{c}) \geq 3$ is satisfied (i.e., the branded product is of superior quality, possesses only a mild cost-disadvantage, and thus contributes much to the retailer's profit), it is profitable for the retailer to set a price $p_{u, E^{\prime}}>p_{u}^{*}\left(\bar{w}_{A}\right)$ to ensure $q_{u, E^{\prime}}=q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ such that the retailer receives the discount and only has to pay the lower wholesale price $\underline{w}_{A}$ instead of $\bar{w}_{A}$. In this case, the manufacturer has to leave an information rent to the retailer if the upstream firm wants a high-cost retailer to reveal $\bar{c}_{d}$ truthfully. This is achieved by charging a lower list price $w_{E^{\prime}}<\bar{w}_{A}$ instead of $\bar{w}_{A}$.

Proof. See the appendix.
To summarize, market-share contract $T_{E}$ solves inefficiencies that were caused by the incomplete information of the manufacturer about the retailer's cost-type. One finds that setting a lower price $\underline{w}_{A}$ to a strong buyer with low marginal costs $\underline{c}_{d}$ is no immediate consequence of the retailer's buyer power. It rather is a means of the upstream firm to price discriminate and extract a higher profit from the downstream firm. Yet, by implementing a pricing scheme equivalent to linear pricing under complete information tariff $T_{E}$ does not solve inefficiencies related to double marginalization. Such issues are addressed in the following section.

### 4.3 Market-share and quantity discounts

This subsection shows how the manufacturer can combine retroactive market-share rebates and incremental quantity discounts to solve both the double marginalization problem and the problem of incomplete information about the retailer's costs of producing the private label product. This situation will be indexed by $F$.

To establish additional notation, let $q_{u}\left(c_{u}, \underline{c}_{d}\right)$ and $q_{u}\left(c_{u}, \bar{c}_{d}\right)$ denote the sales of the high-quality product if it is sold to the retailer at a
wholesale price equaling marginal costs $\left(w=c_{u}\right)$, and if the retailer operates at low costs $\underline{c}_{d}$ or high costs $\bar{c}_{d}$ respectively. Similarly, $q_{d}\left(c_{u}, \underline{c}_{d}\right)$ and $q_{d}\left(c_{u}, \bar{c}_{d}\right)$ denote the sales of the private label product in these cases. A low-cost retailer would sell a lower quantity of the branded good and a higher quantity of the private label product than a high-cost retailer $\left.\left(q_{u}\left(c_{u}, \underline{c}_{d}\right)<q_{u}\left(c_{u}, \bar{c}_{d}\right), \quad q_{d}\left(c_{u}, \underline{c}_{d}\right)\right)>q_{d}\left(c_{u}, \bar{c}_{d}\right)\right)$. Proposition 2 suggests that tariff $T_{F}$ solves both the problem of double marginalization and the problem of incomplete information about the retailer's marginal costs by combining a market-share discount and a quantity discount. The tariff implements the same market outcome under incomplete information that would have been obtained under complete information when using a two-part tariff, i.e., I use situation $B$ from above as a benchmark.

Proposition 2. Tariff $T_{F}$ with $\hat{q}_{F}<q_{u}\left(c_{u}, \underline{c}_{d}\right)$ (and with $w_{F}$ and $a_{F}$ being defined in (31) and (32)) ensures that the firms make the profits $\pi_{u, F}\left(c_{d}\right)=\pi_{u, B}\left(c_{d}\right) \forall c_{d}$ and $\pi_{d, F}\left(c_{d}\right)=\pi_{d, B}\left(c_{d}\right) \forall c_{d}$. It is incentive-compatible for a low-cost retailer to reveal its cost-type, i.e., incorrect sorting of the retailer is prevented, because the inequality $\pi_{d, B}\left(\underline{c}_{d}\right)-\left(\bar{c}_{d}-\underline{c}_{d}\right) \cdot q_{d}\left(c_{u}, \underline{c}_{d}\right)<$ $\pi_{d, B}\left(\bar{c}_{d}\right)$ applies.

$$
\begin{gather*}
T_{F}=\left\{\begin{array}{ccc}
w_{F} \hat{q}_{F} & \text { if } & q_{u} \leq \hat{q}_{F} \\
w_{F} \hat{q}_{F}+c_{u}\left(q_{u}-\hat{q}_{F}\right) & \text { if } q_{u}>\hat{q}_{F} \text { and } q_{d}<q_{d}\left(c_{u}, c_{d}\right) \\
w_{F} \hat{q}_{F}+c_{u}\left(q_{u}-\hat{q}_{F}\right)-a_{F} & \text { if } q_{u}>\hat{q}_{F} \text { and } q_{d} \geq q_{d}\left(c_{u}, \underline{c}_{d}\right)
\end{array}\right.  \tag{30}\\
w_{F}=f_{B}\left(\bar{c}_{d}\right) / \hat{q}_{F} \tag{31}
\end{gather*}
$$

Proof. Tariff $T_{E}$ is depicted in Figure 1 where $L$ indicates the combination of $q_{d}$ and $q_{u}$ that a low-cost retailer sells if the wholesale price is $w=c_{u}$. The dashed line through $L$ shows all linear combinations of $q_{u}$ and $q_{d}$ that sum up to the total output $1-\theta_{0, d}\left(\underline{c}_{d}\right)$ that a low-cost retailer would sell for the optimal price $p_{d}^{*}\left(\underline{c}_{d}\right)$ and varying prices $p_{u}$. A profit-maximizing low-cost retailer would not sell a total output in the area marked by the dark gray background as this could only be achieved by reducing the price $p_{d}$ below the profit-maximizing level $p_{d}^{*}\left(\underline{c}_{d}\right)$. Similarly, point $H$ indicates the quantities $q_{d}$ and $q_{u}$ that a high-cost retailer sells if the wholesale price is equal to marginal $\operatorname{costs}\left(w=c_{u}\right)$. The dashed line through $H$ shows all linear combinations of $q_{u}$ and $q_{d}$ that sum up to the total output $1-\theta_{0, d}\left(\bar{c}_{d}\right)$ that a high-cost

Figure 1: Tariff $T_{F}$

retailer would sell for $p_{d}^{*}\left(\bar{c}_{d}\right)$ and varying prices $p_{u}$. A profit-maximizing high-cost retailer would not sell a total output in the area indicated by the light gray or the dark gray area.

To explain tariff $T_{F}$, a retailer pays the list price $w_{F}>c_{u}$ on all units of the high-quality good with $q_{u} \leq \hat{q}_{F}$ (see the widely hatched area). Above this threshold, the retailer receives an incremental quantity discount such that all additional units are priced at the marginal costs of production $\left(w=c_{u}\right.$; see the more densely hatched area). This threshold is set low enough (i.e., $\left.\hat{q}_{F}<q_{u}\left(c_{u}, \underline{c}_{d}\right)\right)$ such that both a high-cost or a low-cost retailer receive the quantity discount. They will thus both set the efficient price $p_{u}^{*}\left(w=c_{u}\right)$, which establishes that $T_{F}$ solves the double marginalization problem.

Choosing the list price according to equation (31) ensures that the retailer makes a payment to the manufacturer equaling the payment that a high-cost retailer would make to a completely informed manufacturer in case of a two-part tariff with Nash-bargaining (see equation (18) in Section 3.2). This proves $\pi_{u, F}\left(\bar{c}_{d}\right)=\pi_{u, B}\left(\bar{c}_{d}\right)$ and $\pi_{d, F}\left(\bar{c}_{d}\right)=\pi_{d, B}\left(\bar{c}_{d}\right)$, i.e., the incompletely informed manufacturer extracts the same rents $f_{B}\left(\bar{c}_{d}\right)$ from a high-cost retailer that a completely informed manufacturer would receive when using a two-part tariff with wholesale prices equaling upstream marginal costs.

However, because of $f_{B}\left(\underline{c}_{d}\right)<f_{B}\left(\bar{c}_{d}\right)$ paying the high list price $w_{F}$ puts a low-cost retailer at a disadvantage as compared to a situation of complete information. This puts the retailer's individual rationality constraint (2) under strain and might prevent the branded product from being listed. Therefore, the manufacturer promises a low-cost retailer a retroactive discount / an allowance $a_{F}=f_{B}\left(\bar{c}_{d}\right)-f_{B}\left(\underline{c}_{d}\right)$ conditional on observing market conduct that only a low-cost retailer would choose, which is selling a high quantity $q_{d} \geq q_{d}\left(c_{u}, \underline{c}_{d}\right)$ of the private label product (see the horizontally hatched triangle). This re-payment ensures that a low-cost retailer does not pay more to an incompletely informed manufacturer than to a completely informed manufacturer, which proves $\pi_{u, F}\left(\underline{c}_{d}\right)=\pi_{u, B}\left(\underline{c}_{d}\right)$ and $\pi_{d, F}\left(\underline{c}_{d}\right)=\pi_{d, B}\left(\underline{c}_{d}\right)$.

Figure 1 shows that for sufficiently low values of the threshold $\hat{q}_{F}$ a high-cost retailer might in principle also receive the allowance $a_{F}$ and possibly raise its profit by selling $q_{u}>\hat{q}_{F}$ and $q_{d}=q_{d}\left(c_{u}, c_{d}\right)$. Equivalently to the statements made in Lemma 6, this would be achieved by lowering $p_{u}$ while keeping the price of the private label product at the profit-maximizing level $p_{d}^{*}\left(\bar{c}_{d}\right)$, i.e., the retailer would sell the quantities indicated by $h$. In other words, in case of a badly designed tariff it might be profitable for a high-cost retailer to misreport its cost-type and mimic the conduct of a low-cost retailer. In the following, it will be shown that such a strategy can be made unprofitable by the manufacturer if it sets $\hat{q}_{F}=q_{u}\left(c_{u}, \underline{c}_{d}\right)-\epsilon$ with $\epsilon \rightarrow 0$. In this case, the high-cost manufacturer only receives the allowance $a_{F}$ if it sets the same prices and, thus, sells the same quantities $q_{u}\left(c_{u}, \underline{c}_{d}\right)$ and $q_{d}\left(c_{u}, \underline{c}_{d}\right)$ as a low-cost retailer (see point $L$ in Figure 1).

In this case, the high-cost retailer would make the profit of a low-cost retailer minus the extra costs of producing the output $q_{d}\left(c_{u}, \underline{c}_{d}\right)$ at high instead of low costs, i.e., $\pi_{d, B}\left(\underline{c}_{d}\right)-\left(\bar{c}_{d}-\underline{c}_{d}\right) \cdot q_{d}\left(c_{u}, \underline{c}_{d}\right)$. It can be shown that incentive constraint (33) applies that can be re-stated as in (34), i.e., lying about its cost-type is unprofitable for a high-cost retailer.

$$
\begin{align*}
& \pi_{d, B}\left(\underline{c}_{d}\right)-\left(\bar{c}_{d}-\underline{c}_{d}\right) \cdot q_{d}\left(c_{u}, \underline{c}_{d}\right)<\pi_{d, B}\left(\bar{c}_{d}\right)  \tag{33}\\
& \frac{10 \Delta s+3 \Delta \bar{c}-13 \Delta \underline{c}}{8}-\frac{\left(\bar{c}_{d}-\underline{c}_{d}\right) \Delta s}{s_{d}}<0 \tag{34}
\end{align*}
$$

The proof of (33) is equivalent to the proof of (27) in Section 4.2. Given $\partial a_{F} / \partial \underline{c}_{d}<0$ and $\partial a_{F} / \partial \bar{c}_{d}>0$, lying about its cost-type yields a high-cost retailer the maximum allowance $\max \left(a_{F}\right)$ at $\max \left(\bar{c}_{d}-\underline{c}_{d}\right)$, which is the case
for $\max \left(\bar{c}_{d}\right)=c_{u}$ and $\min \left(\underline{c}_{d}\right)=c_{u}-\Delta s$. Plugging these terms in (34) and re-arranging yields $-3 / 8<\Delta s / s_{d}$, which is true because of $\Delta s>0$ and $s_{d}>0$. Hence, tariff $T_{F}$ makes it unprofitable for a high-cost retailer to misreport its cost-type even if it could earn the maximum benefit from doing so.

To summarize, tariff $T_{F}$ ensures that high-cost and low-cost retailers sort correctly into the appropriate options of the tariff. By combining a quantity discount and a retroactive market-share rebate this occurs without any efficiency losses. Tariff $T_{F}$ avoids the inefficiency caused by double marginalization and the inefficiency caused by the retailer's incomplete information. The allowance $a_{F}$ is only paid to the low-cost retailer, whose total sales are higher than those of a high-cost retailer. Yet, the allowance is no consequence of the large retailer's buyer power although the manufacturer does not receive a specific service in return. This is because the retailer receives profits no higher than those made without asymmetric information where no allowance was used. In the present model, it is only the manufacturer who benefits from the allowance, because it is a means for the manufacturer to set discriminatory prices and avoid inefficiencies caused by incomplete information about the retailer's costs of producing the private label product.

## 5 Extensions

This section extends the results obtained in the main model. Based on the finding that the firms can implement the full information outcome even under asymmetric information, Subsection 5.1 illustrates that in this case it is unprofitable for the manufacturer to offer an exclusive dealing contract to the retailer. Subsection 5.2 proves that the full information outcome is also obtained by the combination of a market-share rebate and a quantity discount if the costs of the private label may take on more than two states.

### 5.1 Exclusive Dealing

This subsection studies whether it would be profitable for the manufacturer to offer the retailer an exclusive dealing contract specifying that the retailer must sell just the branded product while de-listing the private label, and the subsection analyzes whether the retailer would agree to such a contract. In
line with O'Brien and Shaffer (1997), the manufacturer is assumed to use a two-part tariff because this tariff implements the Pareto-efficient market outcome equivalent to vertical integration (see Lemma 3), and this market outcome would be no different under other forms of non-linear pricing such as quantity discounts (see Lemma 4). Moreover, the section assumes complete information because Propositions 1 and 2 proved that an incompletely informed manufacturer can use market-share discounts to implement the full information outcome obtained in case of a two-part tariff.

In case of a two-part tariff with $w=c_{u}$, the retailer finds it most profitable to sell only the branded product if $\hat{\theta}\left(c_{u}\right)<\theta_{0, d}$ applies. In this case, exclusion of the private label is allocatively efficient because at the prevailing prices $p_{u}^{*}\left(c_{u}\right)$ and $p_{d}^{*}\left(c_{d}\right)$ the customer with the lowest preference for quality $\theta_{0, d}$ who would still buy the product receives a greater net-utility from buying the branded product than the private label, i.e., $v_{d}\left(\theta_{0, d}\right)<v_{u}\left(\theta_{0, d}\right) .{ }^{21}$ It would be in the best interest of the firms and the customers not to sell the private label even in the absence of any exclusive dealing restrictions imposed by the manufacturer. Exclusive dealing would, however, be inefficient if it occurred for $\hat{\theta}\left(c_{u}\right) \geq \theta_{0, d}$, i.e., if a fully integrated firm would sell both goods (O'Brien and Shaffer, 1997, p. 758).

The basic structure of the following exclusive dealing game is as in Bernheim and Whinston (1998): In the first stage, the manufacturer $u$ decides among the actions $a_{u} \in\{X, n X\}$. Action $X$ means offering an exclusive-dealing contract where the retailer must sell solely the high-quality product and make a payment to the manufacturer according to the non-linear tariff $T_{X}=c_{u} q_{u}+f_{X}$. If the manufacturer does not offer an exclusive-dealing contract $(n X)$ the retailer is free to sell a positive quantity of the private label product, and the manufacturer uses the non-linear tariff $T_{B}=c_{u} q_{u}+f_{B}$ as was defined in Section 3.2. In the second stage, the retailer decides among the actions $a_{d} \in\{R, n R\}$, i.e., rejecting the exclusive dealing offer $(R)$ or not $(n R)$. In the third stage, the retailer chooses the price(s) in the product market, and the firms make the profits $\pi_{u, X}$ and $\pi_{d, X}$ (see equations (35) and (36)) if the private label is excluded or $\pi_{u, B}$ and $\pi_{d, B}$

[^12]if not. ${ }^{22}$
\[

$$
\begin{gather*}
\pi_{u, X}=f_{X}  \tag{35}\\
\pi_{d, X}=\frac{\left(s_{u}+r-c_{u}\right)^{2}}{4 s_{u}}-f_{X} \tag{36}
\end{gather*}
$$
\]

The fees $f_{B}$ and $f_{X}$ differ because the manufacturer must 'bribe' the retailer to refrain from selling the private label in order to choose a higher quantity of the branded product. It will be shown that Proposition 3 applies.

Proposition 3. Under both complete or incomplete information about the costs $c_{d}$, the manufacturer never benefits from exclusive dealing and would not choose to offer an exclusive dealing contract $(X)$.

Proof. The retailer would accept an exclusive dealing contract offered by the manufacturer if the inequality $\pi_{d, X}>\pi_{d, B}$ applied that can also be expressed as is shown in (37), i.e., the retailer must be compensated for the lower operating profits by being charged a fixed fee $f_{X}$ that satisfies $f_{X}<f_{B}$ as follows from (37).

$$
\begin{equation*}
f_{X}<f_{X, c r i t} \equiv f_{B}-\frac{\left[\left(r-c_{d}\right) s_{u}-\left(r-c_{u}\right) s_{d}\right]^{2}}{4 \Delta s\left(s_{u} s_{d}\right)} \tag{37}
\end{equation*}
$$

However, $f_{X}<f_{B}$ implies $\pi_{u, X}<\pi_{u, B}$, which proves that it is unprofitable for the manufacturer to offer an exclusive dealing contract in the first place.

A related result was obtained by Yehezkel (2008, p. 122) who showed "that under full information, [the manufacturer] never benefits from imposing exclusive dealing [...]." However, once he assumed the manufacturer to possess imperfect information about the state of demand, exclusive dealing could occur on the equilibrium path. My model adds to these earlier results by showing that exclusive dealing does not occur even under asymmetric information, which is the case if the manufacturer can use market-share contracts to implement the full-information outcome as was proven by Propositions 1 and $2 .{ }^{23}$

[^13]To summarize, in the present model neither allowances nor market-share contracts specifying a minimum market share of the private label are devices for discouraging private label marketing by the retailer. This addresses a "concern among policymakers [...] that dominant suppliers might use market-share contracts to foreclose their competitors" (Inderst and Shaffer, 2010, p. 709), which however mainly relates to market-share contracts specifying a maximum share of the private label, which would then function as a softer version of exclusive dealing. My model supports a conclusion in the fashion of the Chicago School: If market-share contracts can be used to implement the full information outcome, allocatively inefficient exclusion does not occur because this would lower both the retailer's profits and the fixed fee that can be extracted by the manufacturer.

### 5.2 Distribution of Costs

A combination of market-share rebates and quantity discounts also solves the problems related to double marginalization and imperfect information if there are not only two but infinitely many states of the private label's costs in the interval $c_{d} \in\left[c_{d}, \bar{c}_{d}\right]$, as is shown by Proposition 4 in this section. Under such circumstances, it is harder for the manufacturer to design an optimal tariff, because if the retailer's conduct is found to be inconsistent with having high costs $\bar{c}_{d}$ the manufacturer cannot immediately infer that the retailer is necessarily operating at costs $\underline{c}_{d}$ and set the value of the allowance $a_{G}$ accordingly.

Proposition 4. If the costs $c_{d}$ are distributed in the interval $\left[\underline{c}_{d}, \bar{c}_{d}\right]$ according to some probability density function, the full information outcome can be obtained (i.e., $\pi_{u, G}=\pi_{u, B} \forall c_{d}$ and $\pi_{d, G}=\pi_{d, B} \forall c_{d}$ ) if the manufacturer and the retailer get together at the beginning of the period, and the retailer reports some value $c_{d, G} \in\left[\underline{c}_{d}, \bar{c}_{d}\right]$ to the manufacturer, who then sets a tariff $T_{G}$ as is shown in (38) with a list price $w_{G}$ and an allowance $a_{G}$ as defined in (39) and (40).

$$
\begin{gather*}
T_{G}=\left\{\begin{array}{ccr}
w_{G} \hat{q}_{F} & \text { if } & q_{u} \leq \hat{q}_{G} \text { with } \hat{q}_{G}=q_{u}\left(c_{u}, c_{d, G}\right)-\epsilon \\
w_{G} \hat{q}_{F}+c_{u}\left(q_{u}-\hat{q}_{G}\right) & \text { if } & q_{u}>\hat{q}_{G} \text { and } q_{d}<q_{d}\left(c_{u}, c_{d, G}\right) \\
w_{G} \hat{q}_{F}+c_{u}\left(q_{u}-\hat{q}_{G}\right)-a_{G} & \text { if } & q_{u}>\hat{q}_{G} \text { and } q_{d} \geq q_{d}\left(c_{u}, c_{d, G}\right)
\end{array}\right.  \tag{38}\\
w_{G}=f_{B}\left(\bar{c}_{d}\right) / \hat{q}_{G} \tag{39}
\end{gather*} a_{G}=f_{B}\left(\bar{c}_{d}\right)-f_{B}\left(c_{d, G}\right)=\frac{5\left(\bar{c}_{d}-c_{d, G}\right)\left(2 \Delta s-\Delta \bar{c}-\left(c_{u}-c_{d, G}\right)\right)}{32 \Delta s} .
$$

It is incentive compatible for the retailer to report its costs truthfully $\left(c_{d, G}=c_{d}\right)$.

Proof. Tariff $T_{G}$ shares many similarities with tariff $T_{F}$ specified in Proposition 2 for $c_{d} \in\left\{\underline{c}_{d}, \bar{c}_{d}\right\}$. The manufacturer sets a list price $w_{G}$ that is appropriate to extract the surplus of a retailer with the highest costs $\bar{c}_{d}$, and it grants an incremental quantity discount for $q_{u}>\hat{q}_{G}$ to solve the double marginalization problem. Additionally, if $q_{d} \geq q_{d}\left(c_{u}, c_{d, G}\right)$ the manufacturer retroactively grants the retailer an allowance $a_{G}$ such that the retailer's individual rationality constraint (2) will not be impaired as compared to benchmark situation $B$ where the costs of the private label match the reported costs $c_{d, G}$ while the manufacturer is completely informed about $c_{d}$ and uses a two-part tariff.

A central feature of $T_{G}$ is the reliance of the manufacturer on the self-reported costs $c_{d, G}$ of the manufacturer, who might, in principle, opportunistically misstate its costs. Tariff $T_{G}$ makes it however unprofitable for the retailer to lie about its true costs $c_{d}$ as is shown in the following. First, overstating the costs of the private label $\left(c_{d, G}>c_{d}\right)$ is unprofitable because this reduces the retroactive discount $a_{G}$, i.e., the gain from lying. This follows from $\partial a_{G} / \partial c_{d, G}<0$. Moreover, to act consistently with $c_{d, G}>c_{d}$ the retailer would have to sell a suboptimally high quantity of the branded product to receive the incremental quantity discount because the threshold $\hat{q}_{G}=q_{u}\left(c_{u}, c_{d, G}\right)-\epsilon$ rises in $c_{d, G}$.

Second, understating the costs of the private label $\left(c_{d, G}<c_{d}\right)$ in order to receive a higher allowance $a_{G}$ is equally unprofitable. To see this, consider a retailer with true costs $c_{d}=\bar{c}_{d}$, who would have the greatest incentive to understate its costs because it is the only cost-type who does not receive an allowance when revealing its costs truthfully $\left(a_{G}\left(\bar{c}_{d}\right)=0\right)$. Because of $\partial a_{G} / \partial c_{d, G}<0$ the repayment $a_{G}$ would be maximized if the retailer claimed $c_{d, G}=\underline{c}_{d}$. With $c_{d}=\bar{c}_{d}$ and $c_{d, G}=\underline{c}_{d}$ the retailer's incentive compatibility constraint is the same as (33) from Section 4.3 where it had been shown that it is unprofitable for a manufacturer with $c_{d}=\bar{c}_{d}$ to lie about its costs and claim $c_{d, G}=\underline{c}_{d}$. Showing this for a retailer who would gain most from lying implies that retailers with $c_{d}<\bar{c}_{d}$ would not want to understate their costs, either. ${ }^{24}$

[^14]These findings imply that tariff $T_{G}$ makes it incentive compatible for the retailer to report its costs truthfully. It then follows from Proposition 2 that tariff $T_{G}$ suffices for implementing the full information outcome with $\pi_{u, G}=\pi_{u, B} \forall c_{d}$ and $\pi_{d, G}=\pi_{d, B} \forall c_{d}$ for infinitely many states of $c_{d}$, which proves Proposition 4.

## 6 Conclusion

This article studies the effects of a private label product on the wholesale price charged for a branded product whose manufacturer is incompletely informed about the marginal costs of producing the private label. The model confirms the intuition that the private label may be a source of countervailing buyer power. However, the observation of slotting or advertising allowances that are retroactively paid to large retailers is no particular consequence of this buyer power. In the model, these allowances rather result from the upstream firm's desire to price discriminate among low-cost and high-cost retailers, and to perfectly extract their information rent which can be achieved by combining market-share rebates and quantity discounts.

The model suggests that incomplete information about the costs of the retailer does not impair the monopolistic manufacturer's profits as long as it can use market-share contracts. The value of better information would be zero under such circumstances. Future research might analyze under which circumstances this result extends to an upstream oligopoly. In this case, the exchange of information between upstream manufacturers about the characteristics of downstream retailers might potentially have no anti-competitive effects. This is relevant for competition policy because, for example, the German Bundeskartellamt had considered the conduct of several producers of drugstore products a violation of competition laws, who had exchanged information about the retailers in the downstream market. Yet, the economic effects of this information exchange deserve greater study, for example, in an extension of the present model.

As a second extension, one might add not only further upstream
that $\Delta \pi_{d, G}\left(c_{d}, c_{d}\right)=0, \partial \Delta \pi_{d, G}\left(c_{d, G}, c_{d}\right) / \partial c_{d, G}>0$, and $\partial^{2} \Delta \pi_{d, G}\left(c_{d, G}, c_{d}\right) / \partial c_{d, G}^{2}=0$, i.e., truthfully revealing its costs gives the retailer additional profits of zero, with these additional profits falling linearly if the retailer reports $c_{d, G}<c_{d}$. This proves $\Delta \pi_{d, G}\left(c_{d}, c_{d}\right)<0 \forall c_{d, G}<c_{d}$, implying that it is incentive compatible for the retailer to report its costs truthfully.
manufacturers but also additional downstream retailers. This allows studying the foreclosure-effects of different wholesale pricing schemes in the presence of private label products and incomplete information. A model with an upstream and a downstream oligopoly would also be relevant for antitrust practice when analyzing the effects of buyer power in retail markets, for example, in the context of a merger of retail chains or in the context of collusion among upstream firms. A third extension also applies to the downstream market. Allowing for discriminatory pricing in the downstream market (e.g., season sales) helps in making more refined statements about consumer surplus if elements in the upstream market change.

## Appendix

Proof of Lemma 1: Equating $v_{u}=v_{d}$ as defined in (3) and (4) yields the location $\hat{\theta}$ of the indifferent consumer that is shown in equation (6). For $\hat{\theta}>1$ not even the consumer with the highest preference for quality $(\theta=1)$ buys the high-quality product, which would result in $q_{u}=0$. This case is ruled out by $\Delta c<\Delta s$ in assumption (5). To see this, consider that $\hat{\theta}>1$ can be re-arranged as follows after plugging in the profit-maximizing prices $p_{u}^{*}$ and $p_{d}^{*}: w_{u}-c_{d}>\Delta s$. Plugging the optimal linear wholesale price $w_{A}$ from (14) in this inequality and re-arranging yields $\Delta c>\Delta s$, which is ruled out by assumption (5).

All consumers would buy the high-quality product $\left(q_{u}=1\right)$ if $\hat{\theta}<0$. Re-arranging this inequality yields $p_{u}<p_{d}$. Using the retail prices $p_{u}^{*}$ from equation (10), the linear wholesale price $w_{A}$ from (14), and $p_{d}^{*}$ from equation (11) one finds $p_{u}<p_{d}$ if $3 \Delta s<-\Delta c$. When using a two-part tariff in the wholesale market with $w=c_{u}$, one finds $p_{u}<p_{d}$ if $\Delta s<\Delta c$. Neither of these two conditions would be satisfied under assumption (5). This rules out the existence of a situation with $q_{u}=1$.

Consumers derive a positive net-utility from buying the private label product if $v_{d} \geq 0$. Let $\theta_{0, d}$ define the critical value of $\theta$ such that $v_{d}\left(\theta_{0, d}\right)=0$ applies. Consumers with $\theta<\theta_{0, d}$ would not buy the private-label product, i.e., there is only partial market coverage. Therefore, demand for the private label product can be written as in (7).

Proof of Lemma 3: The optimal value $f_{B}$ is straightforward to determine. The critical value $c_{u, B B}$ as defined in (18) is found by solving the equality $\hat{\theta}\left(c_{u}\right)-\theta_{0, d}=0$ for $c_{u}$. Making the present model consistent with Yehezkel (2008) by setting $r=0$ shows that the condition $c_{u} \geq c_{u, B B}$ is the same as his [p. 122] condition $c_{u} \geq c_{d} /\left(s_{d} / s_{u}\right)$. Because of $\pi_{u, B}>\pi_{u, A}$ and $\pi_{d, B}>\pi_{d, A}$ the two firms benefit from the two-part tariff. Because of $p_{d, B}^{*}=p_{d, A}^{*}$ and $p_{u, B}^{*}<p_{u, A}^{*}$ the customers also benefit from the two-part tariff, which proves the second part of Lemma 3 concerning the Pareto improvement.

Proof of Lemma 5: In the situation with incomplete information, the manufacturer would choose the linear wholesale price $w_{D}$ such as to maximize (41). The second line follows from the first because $\gamma\left(1-\hat{\theta}\left(w, \underline{c}_{d}\right)\right)+(1-\gamma)\left(1-\hat{\theta}\left(w, \bar{c}_{d}\right)\right)=1-\hat{\theta}\left(w, E\left(c_{d}\right)\right.$ applies.

$$
\begin{array}{rlr}
E\left(\pi_{u}\right) & =\ell\left[\gamma\left(w-c_{u}\right)\left(1-\hat{\theta}\left(w, \underline{c}_{d}\right)\right)+(1-\gamma)\left(w-c_{u}\right)\left(1-\hat{\theta}\left(w, \bar{c}_{d}\right)\right)\right] \\
& = & \ell\left[\left(w-c_{u}\right)\left(1-\hat{\theta}\left(w, E\left(c_{d}\right)\right)\right]\right. \tag{41}
\end{array}
$$

For $w_{D}$ and in case of $\ell=1$, the profit of the upstream firm is given by (42). The inequality $\pi_{u, D}<\pi_{u, A}$ follows from $\left(c_{u}-E\left(c_{d}\right)\right)^{2}>0$.

$$
\begin{equation*}
\pi_{u, D}=\pi_{u, A}-\frac{4\left(c_{u}-E\left(c_{d}\right)\right)^{2}}{32 \Delta s} \tag{42}
\end{equation*}
$$

For $w_{D}$ and in case of $\ell=1$, the profit of the downstream firm is given by (43).

$$
\begin{equation*}
\pi_{d, D}=\pi_{d, A}-\frac{2\left(E\left(c_{d}\right)-c_{d}\right)\left[2 \Delta s-2 \Delta c-\left(E\left(c_{d}\right)-c_{d}\right)\right]}{32 \Delta s} \tag{43}
\end{equation*}
$$

Consider that the high-quality product is listed if the weak inequality $\Delta s-\Delta c-\left(E\left(c_{d}\right)-c_{d}\right) \geq 0$ applies. This implies $2 \Delta s-2 \Delta c-\left(E\left(c_{d}\right)-c_{d}\right)>0$. Because of $E\left(c_{d}\right)-\underline{c}_{d}>0$ one finds $\pi_{d, D}\left(\underline{c}_{d}\right)<\pi_{d, A}\left(\underline{c}_{d}\right)$. The inequality $\pi_{d, D}\left(\bar{c}_{d}\right)>\pi_{d, A}\left(\bar{c}_{d}\right)$ follows from $E\left(c_{d}\right)-\bar{c}_{d}>0$.

Proof of Lemma 6 If a high-cost retailer truthfully reveals its costs $\bar{c}_{d}$, tariff $T_{E^{\prime}}$ implies that the retailer pays $\bar{w}_{A}$. It charges $p_{d}^{*}\left(\bar{c}_{d}\right)$ and $p_{u}^{*}\left(\bar{w}_{A}\right)$, sells $q_{d}\left(\bar{w}_{A}, \bar{c}_{d}\right)$ and $q_{u}\left(\bar{w}_{A}, \bar{c}_{d}\right)$, and makes the profit $\pi_{u, A}\left(\bar{c}_{d}\right)$. Alternatively, the retailer could also charge a price $p_{u, E^{\prime}}=(3 \Delta s+\Delta \underline{c}) / 4+p_{d}$, which ensures that the output of the branded product is $q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ for any price $p_{d}$. The retailer would thus sell a lower output $q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)<q_{u}\left(\bar{w}_{A}, \bar{c}_{d}\right)$ to receive the discount and pay only $\underline{w}_{A}<\bar{w}_{A}$ for the branded product. It can be shown that in this case the high-cost retailer would optimally keep the price of the private label at the level $p_{d, E^{\prime}}\left(\bar{c}_{d}\right)=p_{d}^{*}\left(\bar{c}_{d}\right)$, which in equilibrium causes a price $p_{u, E^{\prime}}\left(\bar{c}_{d}\right)=p_{u}^{*}\left(\bar{w}_{A}\right)+\left(\bar{c}_{d}-\underline{c}_{d}\right) / 4$, outputs $q_{u, E^{\prime}}\left(\bar{c}_{d}\right)=q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)=$ $q_{u}\left(\bar{w}_{A}, \bar{c}_{d}\right)-\left(\bar{c}_{d}-\underline{c}_{d}\right) / 4 \Delta s$ and $q_{d, E^{\prime}}\left(\bar{c}_{d}\right)=q_{d}\left(\bar{w}_{A}, \bar{c}_{d}\right)+\left(\bar{c}_{d}-\underline{c}_{d}\right) / 4 \Delta s$, and it makes a profit $\pi_{d, E^{\prime}}\left(\bar{c}_{d}, \bar{w}_{A}\right)$. Some straightforward calculations show that $\pi_{d, E^{\prime}}\left(\bar{c}_{d}, \bar{w}_{A}\right) \geq \pi_{u, A}\left(\bar{c}_{d}\right)$ applies if $2 \Delta s /(\Delta \underline{c}+\Delta \bar{c}) \geq 3$. Because of $\partial \pi_{u} / \partial w<0$ a reduction of the list price (i.e., charging $w_{E^{\prime}}<\bar{w}_{A}$ instead of $\bar{w}_{A}$ ) may be warranted to ensure $\pi_{d, E^{\prime}}\left(\bar{c}_{d}, w_{E^{\prime}}\right)>\pi_{u, A}\left(\bar{c}_{d}\right)$, i.e., the manufacturer may have to leave a high-cost retailer an information rent to ensure that it reveals its costs truthfully. This proves Lemma 6.

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    Lebensmittelzeitung. "E. Leclerc muss Rekordstrafe zahlen." 10 July 2015, https: //goo. gl/qVJmwA (accessed on 8 June 2017).
    Lebensmittelzeitung. "So schnell geben wir nicht auf." 9 October 2015, https://goo.gl/ pmL186 (accessed on 3 May 2017).

[^2]:    ${ }^{5}$ Lebenmittelzeitung, see fn. 3.
    ${ }^{6}$ Lebensmittelzeitung. "Händler nervt Preisschere bei Coke." 10 January 2014, https: //goo.gl/pxqJbM (accessed on 3 May 2017).
    ${ }^{7}$ Lebensmittelzeitung. "Die Suche nach dem Point of Speciality." 20 September 2002, https://goo.gl/Oekicv (accessed on 3 May 2017).
    ${ }^{8}$ Lebensmittelzeitung. "..und läuft und läuft und läuft..." 20 August 2004, https: //goo.gl/08Rn0Y (accessed on 3 May 2017).
    ${ }^{9}$ According to the Private Label Manufacturers' Association, private label "products encompass all merchandise sold under a retailer's brand. That brand can be the retailer's own name or a name created exclusively by that retailer. In some cases, a retailer may belong to a wholesale group that owns the brands that are available only to the members of the group" (https://goo.gl/NrMmEJ, accessed on 24 May 2017).
    ${ }^{10}$ The Economist, see fn. 2 above.

[^3]:    ${ }^{11}$ Communication from the Commission - Guidance on the Commission's enforcement priorities in applying Article 82 of the EC Treaty to abusive exclusionary conduct by dominant undertakings, Official Journal of the European Union, 2009/C 45/02 (here: para. 37)

[^4]:    ${ }^{12}$ Assuming that the private label product can be obtained at marginal costs either refers to a situation where it is produced by the retailer itself or, alternatively, by a competitive fringe. In "contrast with national brands, private label suppliers have very little market power. They are much less concentrated than national brand manufacturers and operate in a competitive market with no product differentiation. As a result, they may sell to retailers at a price close to their marginal cost" (Ailawadi, 2001, p. 308). This industry structure was assumed, e.g., by Inderst and Shaffer (2010) in a related model that, however, assumed horizontally instead of vertically differentiated products.
    ${ }^{13}$ Other studies such as Bontemps et al. (2008) have analyzed the effect that the introduction of a private label had, for example, on the price and brand-positioning of the branded product (also see Ailawadi (2001) for an early literature review). Scott Morton and Zettelmeyer (2004) empirically and theoretically analyzed retailers' incentives whether to introduce a private label and where to position it in the product market. These dynamic (or: comparative-static) effects of private labels are beyond the focus of the present article that mainly uses the costs of private label products as a source of uncertainty and incomplete information in a static model where the wholesale price of the branded good may be conditional also on the sales of the private label (market-share contracts).

[^5]:    ${ }^{14}$ Inderst and Shaffer (2010) provide another model of market-share discounts where a monopolistic manufacturer sells its product via two retailers, who also carry a competing

[^6]:    ${ }^{15}$ The focus of the present article is somewhat different from the well-known contributions on exclusive dealing of Mathewson and Winter (1987), O'Brien and Shaffer (1997), and Bernheim and Whinston (1998). They study a situation where two manufacturers of horizontally differentiated products sell at either linear (Mathewson and Winter, 1987) or nonlinear (O’Brien and Shaffer, 1997) prices to a (local) monopolist. As a fundamental difference to the present model, these upstream firms may both offer the retailer exclusive dealing contracts, and the monopolistic retailer has no further outside options but dealing with either one or both of the manufacturers. This is different in the model studied here, where the retailer may use the private label strategically to enhance its bargaining power if the single upstream manufacturer offers the retailer an exclusive dealing contract. The article of Calzolari and Denicolò (2013) on exclusive dealing when firms can use market-share contracts is also a more distant relative of the present article. The models are only somewhat related because Calzolari and Denicolò (2013) assume horizontally differentiated goods, and buyers' love of variety plays an important role in their analysis, whereas this element is absent in the present model that is concerned with vertically differentiated goods. Chen and Shaffer (2016) also study the exclusionary effects of market-share discounts. Yet, they consider a setting with three types of players, an incumbent seller, a potential entrant, and a set of homogeneous buyers. The seller and the entrant supply a homogeneous good, and the entrant incurs sunk costs of entry. Their model and the present one are sufficiently dissimilar for not being directly related.

[^7]:    ${ }^{16}$ The present model uses the notation introduced by Tirole (1988). When setting $r=0, s_{u}=1, s_{d} / s_{u}=\alpha, a=1$, and $b=1$, this notation is consistent with Mills (1995), who assumes $v_{u}=\theta-p_{u}, v_{d}=\theta \alpha-p_{d}, \theta \in[0, a]$ with density $1 / b$. While Mills (1995) assumes identical marginal costs of production $c_{u}=c_{d}=c$, the present model allows for $c_{u} \neq c_{d}$ as in Yehezkel (2008). When setting $r=0$ and $s_{d} / s_{u}=\gamma$ the notation used in the present model is consistent with Yehezkel (2008). The demand model was introduced by Mussa and Rosen (1978) and is in line with the discrete choice specifications used in the empirical studies of, for example, Draganska et al. (2010) or Meza and Sudhir (2010). As in Yehezkel (2008, p. 121), "I consider [the] specific consumer preferences [shown in (3) and (4)] instead of a more general demand function because the analysis reveals that the question of whether [the retailer] sells [the low-quality product] and whether [the manufacturer] imposes exclusive dealing or market share restriction depends on market parameters such as the degree of vertical differentiation, the asymmetries in production costs, and the degree of asymmetric information."

[^8]:    ${ }^{17}$ A two-part tariff had also been employed by Mills (1999) in a, however, simpler model with $c_{u}=c_{d}$. In his model, it would be profitable to sell only the high-quality product because of $s_{u}>s_{d}$. This is different in the present model with $c_{d} \leq c_{u}$ where both products may be profitably sold even under the assumption of a two-part tariff.

[^9]:    ${ }^{18}$ Communication from the Commission - Guidance on the Commission's enforcement priorities in applying Article 82 of the EC Treaty to abusive exclusionary conduct by dominant undertakings, Official Journal of the European Union, 2009/C 45/02 (here: para. 37)

[^10]:    ${ }^{19}$ Qualitatively similar results will be obtained if the manufacturer does not use $E\left(c_{d}\right)$ as a prior about the costs of the private label but any other number in the interval $\left[\underline{c}_{d}, \bar{c}_{d}\right]$.

[^11]:    ${ }^{20}$ The conditions $q_{u} \leq q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ and $q_{d} \geq q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right)$ could also be expressed in terms of the market share $m_{u}=q_{u} /\left(q_{u}+q_{d}\right)$ of the high-quality good, such that the inequality $m_{u} \leq q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right) /\left[q_{u}\left(\underline{w}_{A}, \underline{c}_{d}\right)+q_{d}\left(\underline{w}_{A}, \underline{c}_{d}\right)\right]$ would have to be satisfied. That market-share contracts can be equivalently specified in terms of the exact quantities or in terms of the market shares themselves was pointed out, for example, by Inderst and Shaffer (2010, p. 716).

[^12]:    ${ }^{21}$ To see this, consider that $v_{d}(\hat{\theta})=v_{u}(\hat{\theta})$ applies by definition and that $\partial v_{d} / \partial \theta=s_{d}<$ $s_{u}=\partial v_{u} / \partial \theta$ follows from the definition of $v_{u}$ and $v_{d}$ in equations (3) and (4). Therefore, $0=v_{d}\left(\theta_{0, d}\right)<v_{u}\left(\theta_{0, d}\right)$ must apply if $\hat{\theta}<\theta_{0, d}$. In case of complete information and a two-part tariff this is the case for $c_{u}<c_{u, B B}$, i.e., the private label is efficiently excluded if its quality-adjusted costs are above those of the branded product as was also shown, for example, by Yehezkel (2008).

[^13]:    ${ }^{22}$ Choosing the tariff $T_{B}$ is optimal for the manufacturer both if it does not offer exclusive dealing or if the retailer rejects the exclusive dealing offer. Alternatively, not selling to the retailer or choosing any other tariff would never be optimal for the manufacturer.
    ${ }^{23}$ Proposition 3 would remain equally valid if exclusive dealing provisions were (somewhat unnecessarily) implemented in alternative forms. Here, one might think of a contract where the manufacturer charges the retailer a fixed fee $f_{B}$ and grants it an allowance $a_{X}$ such that instead of (37) the inequality $f_{B}-a_{X}<f_{X, \text { crit }}$ applies (Mills, 1999).

[^14]:    ${ }^{24}$ This proof can also be stated as follows: Consider a retailer with $c_{d} \in\left[\underline{c}_{d}, \bar{c}_{d}\right]$ who states some $c_{d, G} \in\left[\underline{c}_{d}, c_{d}\right]$ and earns an additional profit $\Delta \pi_{d, G}\left(c_{d, G}, c_{d}\right)=\pi_{d, B}\left(c_{d, G}\right)-$ $\left(c_{d}-c_{d, G}\right) \cdot q_{d}\left(c_{u}, c_{d, G}\right)-\pi_{d, B}\left(c_{d}\right)$ over revealing its costs truthfully. It can be shown

