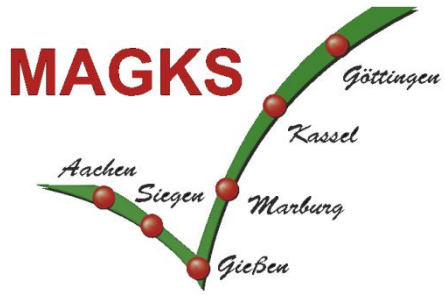

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Updating Inflation Expectations*

Michael J. Lamla† and Samad Sarferaz‡

March 2012

Abstract

This paper investigates how inflation expectations evolve. In particular, we analyze the time-varying nature of the propensity to update expectations and its potential determinants. For this purpose we set up a flexible econometric model that tracks the formation of inflation expectations of consumers at each moment in time. We show that the propensity to update inflation expectations changes substantially over time and is related to the quantity and the quality of news.

JEL classification: E31, E37, E52, D83

Keywords: inflation expectation formation, time-varying parameters, Bayesian methods, disagreement, media coverage, stochastic volatility.

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1 Introduction

Expectations of the public are an crucial ingredient in macroeconomic and microeconomic models. Despite its prominence and the ample use there is only scarce evidence about how people form their expectations and why they disagree.

Mankiw and Reis (2002) and Sims (2003) revived the interest in modelling information frictions in general macroeconomic models. The implications of allowing imperfect information for modelling responses of macroeconomic shocks were substantial and allowed to solve several puzzles (see Mankiw and Reis, 2011).[1]

In general there are two main strands of rational expectations models incorporating informational friction. In Mankiw and Reis (2002) agents do not update frequently as they face costs of absorbing and processing information. However, if they update they gain full information. Sims (2003) as well as Maćkowiak and Wiederholt (2009) belong to the class of partial information models. In these models the observed inertial reaction of economic agents arises from an inability to pay attention to all the noisy information available although people update continuously.[2] Another approach is put forward by Carroll (2003). He argues that staggered updating of expectations can be described in an epidemiological way. He applies this idea to inflation expectations. News media provide information which is received only by a share of the population. This information is then spread from period to period through the population, similar to the way diseases spread. Consequently, more

[1] For instance Ball et al. (2005) argue that the sticky-information approach is more consistent with observed inflation persistence and the effects of monetary policy. Consequently, the concept of sticky information has been extended and applied in various contributions (see, e.g., Reis, 2006b,a). Recent papers like for instance Reis (2012) and Paciello and Wiederholt (2012) convincingly demonstrate that incorporating informational frictions leads to different implications for policy making.

[2] It is an optimal choice for economic agents– internalizing their capacity constraints– to remain inattentive to some available information because incorporating all signals is impossible (Sims, 2010). In contrast to the sticky information approach agents cannot observe anything perfectly. Delays in the reaction of agents to new information in rational inattention models stem from the serial correlation and the size of the disturbances. Maćkowiak and Wiederholt (2009) model rational attention similar to Sim's approach, however, agents have to divide their attention between tracking the aggregate and the idiosyncratic component. They show that more attention is devoted to the targeted variable if the variance is large and if the variable is important.

news improve the accuracy of inflation expectations.

This paper analyzes how attentive people are, if the attentiveness changes over time and which factors might influence these movements. Specifically, our study provides evidence of staggered updating of inflation expectations of households for Europe. In contrast to Döpke et al. (2008), however, our econometric framework allows us to trace the stickiness of expectations at every moment in time. This is realized via a Bayesian state-space model featuring time-varying parameters and variances. Thus, we identify not only an average updating frequency but also extract to which extend updating is state- or time-dependent. Furthermore, we consider explanatory variables that may affect the propensity to update expectations. Motivated by a learning model we contend that not only the amount of signals received, but also their relative quality are important determinants.

Thus our paper is closely related to studies that analyze the expectations formation process empirically. Mankiw et al. (2003) provide evidence for delayed updating to new information. Carroll (2003) estimates the information updating frequency for the U.S., suggesting that consumers update inflation expectations once per year. Looking at the movements of forecast errors in relation to the variable being forecasted Coibion and Gorodnichenko (2008) and Coibion (2010) document pervasive and robust evidence consistent with information rigidities. For Europe Döpke et al. (2008) show that consumers update their inflation expectations once every 18 months. In sum these studies provide evidence for staggered updating and for information frictions. However, all of these studies either assume or do not explicitly monitor the possibility that the fraction of people that update may change over time. Hence, this is the main focus of our paper.

We show that the share of consumers that have outdated expectations varies substantially over time. We identify periods where consumers pay much attention and times where consumers are reluctant to update their expectations. Our estimates suggest that the updating frequency can vary from 2-3 months to 33 months. Moreover, the quality

of the news received matters a great deal. If, for instance, professional forecaster disagree strongly on their views on inflation expectations consumers retain the views held in the last period and inflation expectations of the consumers become more persistent and sticky. Furthermore, there is some evidence that a greater share of people update if more news are received.

This paper proceeds as follows. In Section 2 we introduce the Bayesian learning model and discuss the influence of the amount of news and its quality on the formation of inflation expectations. Section 3 comprises the description of the data used. In Section 4 we describe the econometric methodology applied and in Section 5 we present the results from the econometric analysis. Finally, Section 6 concludes.

2 Modeling updating

To motivate our empirical setup we borrow from the signal extraction literature. Put more specifically we use the concept of Bayesian updating.[3]Assume that at the beginning of month t agent i has an initial prior belief about the future inflation rate (prior forecast). The prior belief $\Pi_{i,t}$ is normally distributed with mean $\pi_{i,t}$ and variance σ_a^2:

$$\Pi_{i,t} \sim N(\pi_{i,t}, \sigma_a^2).$$

During each month the agent absorbs a number V of news. We assume that each news contains a noisy information about future inflation. Thus they receive a noisy signal $\psi_{v,t} \sim N(\theta_t, \sigma_{\psi,t}^2)$ and face a signal extraction problem. This news is likely to be an inflation forecast of a professional forecaster. θ_t can than be consequently interpreted as the average inflation forecast of all experts ($\theta_t = 1/N \sum_{j=0}^{N} \theta_{j,t}$). Lamla and Lein (2008) even show

[3]This model is an adapted version of the political ability of DellaVigna and Kaplan (2007). See also Lamla and Maag (2012). For a DSGE model with endogenous and time-varying sticky information see also Dräger (2011).

that one could add a normally distributed media bias on top of that which would not affect the following mechanism and its implication.

Given the prior belief about future inflation and V units of noisy media reports ψ the agent has to infer θ. The agent updates his prior belief according to Bayes' rule:

$$k_i(\pi_{i,t+1}|\psi_{v,t}) \propto \Pi_{v=1}^{V} f_i(\psi_{v,t}|\pi_{i,t})h(\pi_{i,t}). \tag{1}$$

where $h(.)$ is the prior density, $f_i(.)$ the conditional density of the observed public information given the prior belief $\pi_{i,t}$ and $k_i(.)$ the resulting posterior density given media reports V. Under the normality assumptions the posterior distribution is again normal with mean:

$$E(\pi_{i,t+1}|\psi_{v,t}) = \rho_t \pi_{i,t} + (1-\rho_t)\bar{\psi}, \tag{2}$$

where $\bar{\psi} = V^{-1}\sum_{v=1}^{V}\psi_{v,t}$. The mean of the posterior distribution (henceforth posterior forecast $\pi_{i,t+1}$) is a weighted average of the prior mean and the average noisy signal obtained from the media. The weight on the prior mean is given by:

$$\rho_t = \frac{\frac{1}{V}\sigma_\psi^2}{\sigma_a^2 + \frac{1}{V}\sigma_\psi^2}. \tag{3}$$

Rearranging this equation yields:

$$\rho_t = \frac{\frac{1}{\sigma_a^2}}{\frac{1}{\sigma_a^2} + V\frac{1}{\sigma_\psi^2}}. \tag{4}$$

From this equation is can be easily inferred that more signals increase the weight people put on the public signal:

$$\partial\rho_t/\partial V < 0.$$

In the limit:

$$\lim_{V \to \infty} \rho_t = 0.$$

Thus people will always update on the public signal.

However, the relative precision between the public and the a priori expectations plays an important role. The higher the variance of the public signal (σ_ψ^2) relative to the variance of the prior expectations of the consumers (σ_a^2) the higher is the weight people put on the prior forecast:

$$\partial \rho_t / \partial \sigma_\psi^2 > 0.$$

This can be directly inferred from Equation (4). On the other hand the higher the variance of the prior of the consumers (σ_a^2) relative to the variance of the professional forecaster (σ_ψ^2) the lower the weight the people put on their own forecast:

$$\partial \rho_t / \partial \sigma_a^2 < 0.$$

Note that although Equation (1) is derived by solving a signal extraction problem, it looks very similar as the updating equation in Carroll (2003). The difference is here that we allow for time-varying updating parameters and explicitly explain the updating process using both the amount of signal and its precision. Note also that this representation is also related to the partial information literature. Like in Sims (2003) rational inattention framework only infinity amount of signals lead to being able to extract the full signal.

It might be also linked to the model implications of Maćkowiak and Wiederholt (2009). Similar to their model the variance of the variable of interest increases the likelihood that people pay attention. In our representation this is true for the a priori variance of the consumers. If their own forecast precision becomes less reliable they are more inclined to get the input from the public signal. This can be captured by the dispersion of the

consumers forecasts but also by the idiosyncratic component, the stochastic volatility, which is accounted for in the empirical specification.

Given the imposed time-varying nature we need a flexible econometric framework to test whether indeed inflation expectations are updated differently over the year and whether the amount of news and the precision of news may influence the speed of updating. For this purpose we estimate a Bayesian state-space model allowing for for time-varying parameters and stochastic volatility. The data used for this purpose as well as the empirical model will be described in the upcoming sections.

3 Data

In this section we introduce the data we will be using to test our hypothesis as well as the methodology applied.

3.1 Media Data

We rely on data kindly provided by the media research institute `Mediatenor`.[4] The data comprise articles and media releases on a monthly basis for the time span 01/1998 to 09/2007 in Germany, covering statements dealing with inflation that are at least five lines long in the case of printed media and last at least five seconds for television broadcasts.[5] The coding is based on the standards of media content analysis, and the data contain different specifications.[6] We are provided with the overall number of reports in a given

[4]See www.mediatenor.de for details on the coding of media articles and TV broadcasts on the basis of media content analysis. See also Holsti (1969).

[5]In detail, the following news sources are analyzed: Daily press: Frankfurter Allgemeine Zeitung, Welt, Süddeutsche Zeitung, Frankfurter Rundschau, Tageszeitung, Bild, Neue Zürcher Zeitung, Berliner, Volksstimmer, Sächsische, Westdeutsche Allgemeine Zeitung, Kölner Stadt-Anzeiger, Rheinischer Merkur; daily TV-News: ARD Tagesschau, Tagesthemen, ZDF Heute, Heute Journal, RTL Aktuell, SAT.1 18:30, ProSieben Nachrichten; Weekly Press: Spiegel, Focus, Die Woche, Wochenpost, Welt am Sonntag, Bild am Sonntag, Die Zeit.

[6]Media Content Analysis is a scientific method to capture the content of text passages. Several especially trained persons, called coders, read the news items and code them according to several characteristics,

period, the amount of reports dealing with rising or falling inflation, the tone of the report, the tense of the report (i.e., whether the content was mainly related to the present, the past or the future), the source of the media report (i.e., whether it was distributed via TV or via newspaper) and the visibility of the report.

Given our model we need a number that captures the overall news on inflation called V. V is the number of inflation reports within a given month. Following Carroll, this variable should improve the accuracy of inflation forecasts.

However we will also extract the tone of each report and summarize the tone into three variables: good, bad, neutral. Lamla and Lein (2008) have shown that the tone has also impact on the quality of inflation expectations and may point towards some kind of media bias. The variable *good* denotes all news which are judged to be good news, bad represents all bad news and neutral is a category where no clear explicit judgement had been made by the writer of the news article.

We obviously assume that news printed contain new information. The main reason for that is that Media outlets can only sell if they contain stories that are exiting and more importantly new (Hamilton, 2004).

3.2 Data on Inflation Expectations of Consumers and Professionals

In this section we will describe the data used to identify the expectations of consumers and professionals.

Data on consumers' inflation expectations are taken from the EU business and consumer

such as the topic, the tone or the visibility of a news item. The characteristics coded are discussed in the main text below. Inter-coder reliability tests guarantee the high quality of the data. One main advantage of these tests is that the coding is at least done by two different coders, after which the tests check whether the coders came up with the same results. If not, unequally coded passages are recoded. This allows the capturing of the objective content of each statement and ensures reproducibility. Media Tenor's homepage provides more details on media content analysis.

survey and are available on a monthly basis. German consumers are asked whether they expect prices to rise, fall or remain unchanged in the upcoming 12 months (expected inflation). To obtain quantitative measures of inflation expectations from the qualitative survey data, we rely on data kindly provided by the Bundesbank.[7] The underlying method used is the probability approach put forward by Carlson and Parkin (1975), which has been extended by Berk (1999). It assumes that expectations are normally distributed and do not impose unbiasedness *ex ante*. One advantage of this quantification method is that it directly links the expected inflation rates to the currently perceived inflation rates.

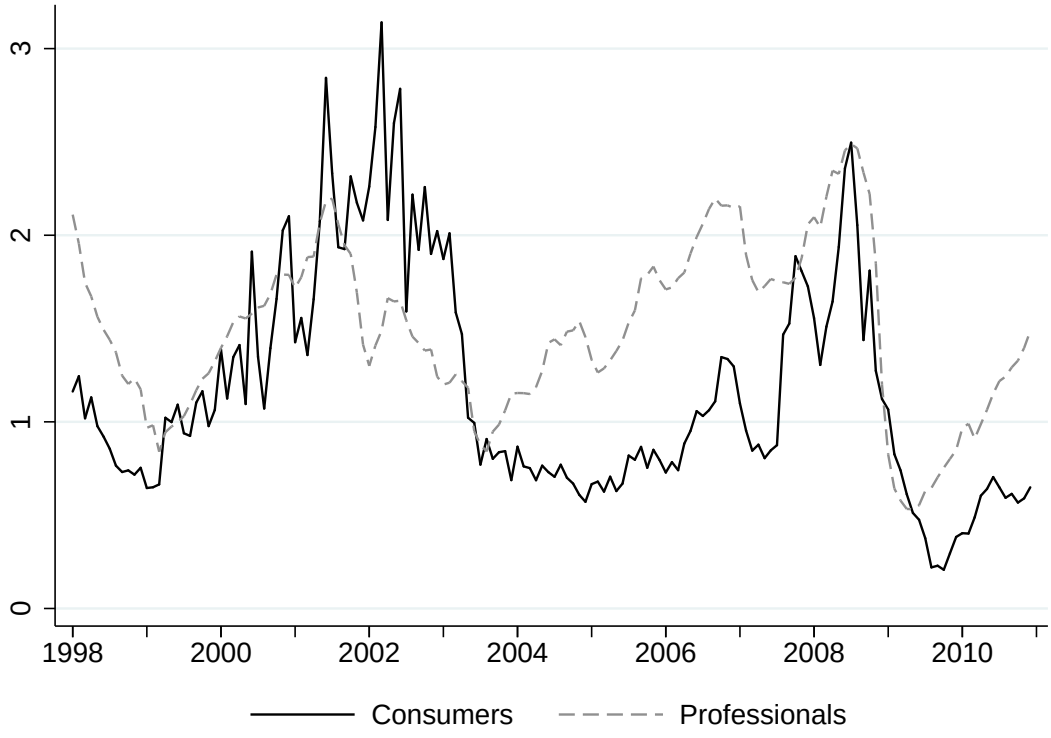
Inflation expectations from professional forecasters for Germany are constructed from Consensus Economics. In that survey, several professional economists are asked about the inflation prospects of the contemporary and upcoming year. Consensus Economics is a macroeconomic survey company. The survey of experts of private and public institutions in Germany asks for economists' inflation expectations for the rest of the current year and for the entire upcoming year. The consensus forecast, used in the paper as a measure of expert expectations, is the mean of these forecasts in Germany. As the time horizon used in this paper is always the 12-month expectation, the data have been transformed to obtain this fixed forecast horizon. We follow the approach commonly used for this type of data and transform the forecast as follows: for month m of a given year t, the expectation of inflation is defined as $(13-m)/12$ times the forecast for year t plus $(m-1)/12$ times the forecast for year $t+1$.

We see that consumers and professionals have quite similar inflation expectations. The difference is not statistically significant. Moreover, consumers' inflation expectations are more volatile. With respect to the ability to forecast inflation, Mestre (2007) shows that professional forecasters from consensus economics outperform the forecasts made by consumers. The latter contains even a small bias. Overall, both seem reasonable, and con-

[7] The calculation is described in detail in the Monthly Bulletin of November 2007.

sumers' expectations do not fare badly compared to simple parametric alternatives for forecasting inflation. Both series are shown in Figure 1. We observe three phases. In the first period, until mid-2001, consumers and professionals assessed future inflation about equally. This picture changes in 2001, when consumer inflation expectations increased substantially while the expectations of economists began to ease. After a peak in mid-2002, consumer inflation expectations began to fall again. Beginning in 2004, the expectations of professional economists increased, while consumer expectations remained at a rather low level. In the course of 2007, the two series converged again.

Figure 1: Inflation expectations: Consumers vs. professionals

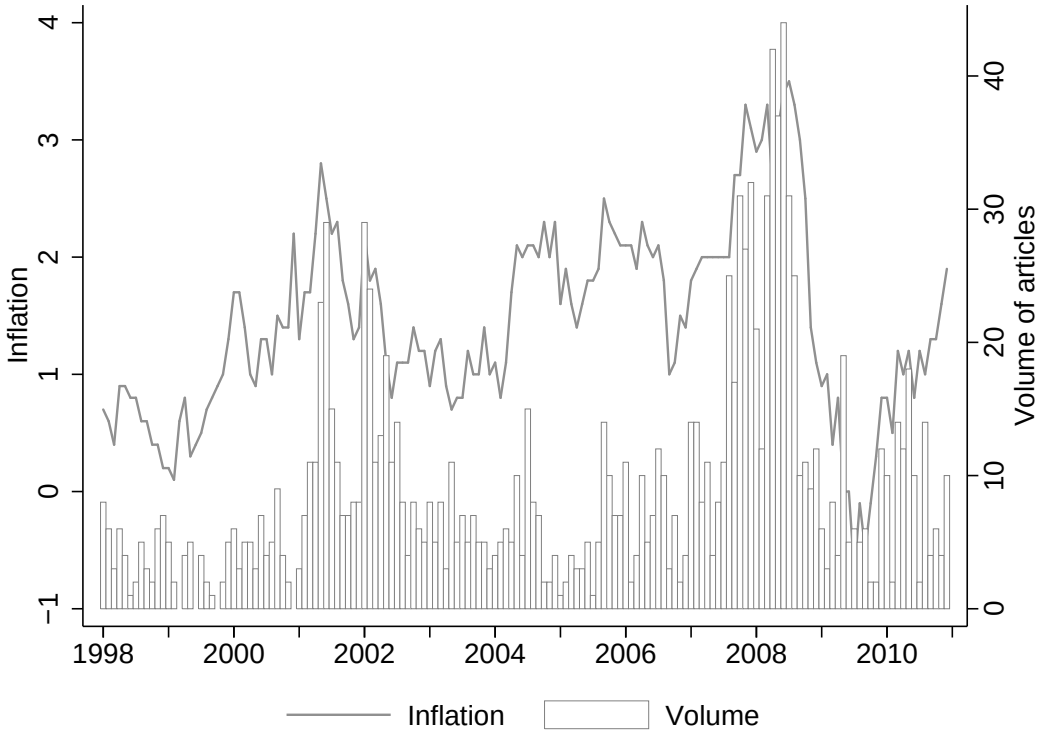


Solid line: Inflation expectations of German consumers; dashed line: Economists' inflation expectations for Germany from Consensus Economics.

An important issue is how media coverage is related to current inflation. Figure 2 pictures the variables *Volume* and HICP inflation. We can observe that in periods where

high inflation was present, coverage in the media increased. See, for instance, mid-2001, where, due to a bad harvest, the prices of vegetables substantially increased, HICP inflation picks up and also media coverage increases. Moreover, there are cases where media coverage was relatively high, although inflation was quite low. Examples for this phenomenon can be found in mid-2002 as well as in the beginning of 2003. Therefore, media coverage does not necessarily co-move with the level of inflation.

Figure 2: Media coverage and inflation



Solid line: HICP inflation (lhs); Bars: Amount of articles on inflation in the media.

The assessment of reports with respect to inflation is depicted in Figure 3. The reports of bad news were especially prevalent during the periods when inflation rose in 2001 and 2005. Interestingly, the reporting disappeared relatively quickly, even though inflation remained higher for several months. We can also observe substantial negative framing in the beginning of 2002, during the euro cash changeover. Positively toned news reports can

be observed especially during 1998, when inflation was very low.

As we are also interested in modelling the effect of the precision of the prior and the news signal we need to think of proxies for both. We decided to use the dispersion of consumers inflation expectations as a proxy for the quality of the a priori inflation expectations of the consumers. Analogously, we use the disagreement of professional forecasters to proxy for the information content of the public signal.[8]

Notably, disagreement is a distinct concept from uncertainty. Disagreement specifically is determined by the dispersion of information as well as the differences in the belief formation models across individuals (see Lahiri and Sheng, 2008; Lamla and Maag, 2012). However, both uncertainty and disagreement are interlinked. Lahiri and Sheng (2010) show that aggregate forecast uncertainty can be expressed as the disagreement among the forecasters plus the perceived variability of future aggregate shocks. Moreover, they exhibit a strong comovement. D'Amico and Orphanides (2008) find for instance that higher average expectations of inflation are associated with greater uncertainty as well as a higher degree of disagreement. Thus disagreement seems to be a good proxy for the reliability of the public signal as well as the for the precision of the prior of the consumers.

To quantify the dispersion of consumers' inflation expectations we use the index of qualitative variation which is defined by:

$$Q(X) \quad = \quad \frac{K}{K-1}\left(1-\sum_{i=1}^{K} p(x_i)^2\right). \tag{5}$$

Survey results are publicly available as aggregate shares over qualitative response categories. We quantify inflation forecast disagreement of households by computing an index of qualitative variation (IQV) based on the response shares in the 5 categories: where $K = 5$ is the number of categories in the EU consumer survey question on expected infla-

[8]We thereby assume that professionals are prone to sticky information and/or rational inattention and/or have heterogeneous predictor choices.

tion, and $p(x_i)$ the fraction of answers in category x_i. The scaling factor $\frac{K}{K-1}$ ensures that $0 \leq Q(X) \leq 1$. The index of qualitative variation has been applied by Lamla and Maag (2012).

For the professional forecaster inflation expectations which proxy the best possible inflation forecast we similarly use a disagreement measure. We follow Giordani and Söderlind (2003) and utilize the quasi standard deviation of the inflation expectations of professional forecasters. The quasi standard deviation is defined as half the distance of the 84th and 16th percentile of the point forecasts as a measure of disagreement. The quasi-standard deviation corresponds to the standard deviation if the underlying distribution is normal but is robust to outliers.

Both series are plotted in Figure 4. The disagreement of consumer expectations rises strongly with the introduction of the euro. Professionals also exhibit a rise in disagreement in the aftermath of the cash changeover, however, fall back to normal levels in the following years.

4 Econometric Framework

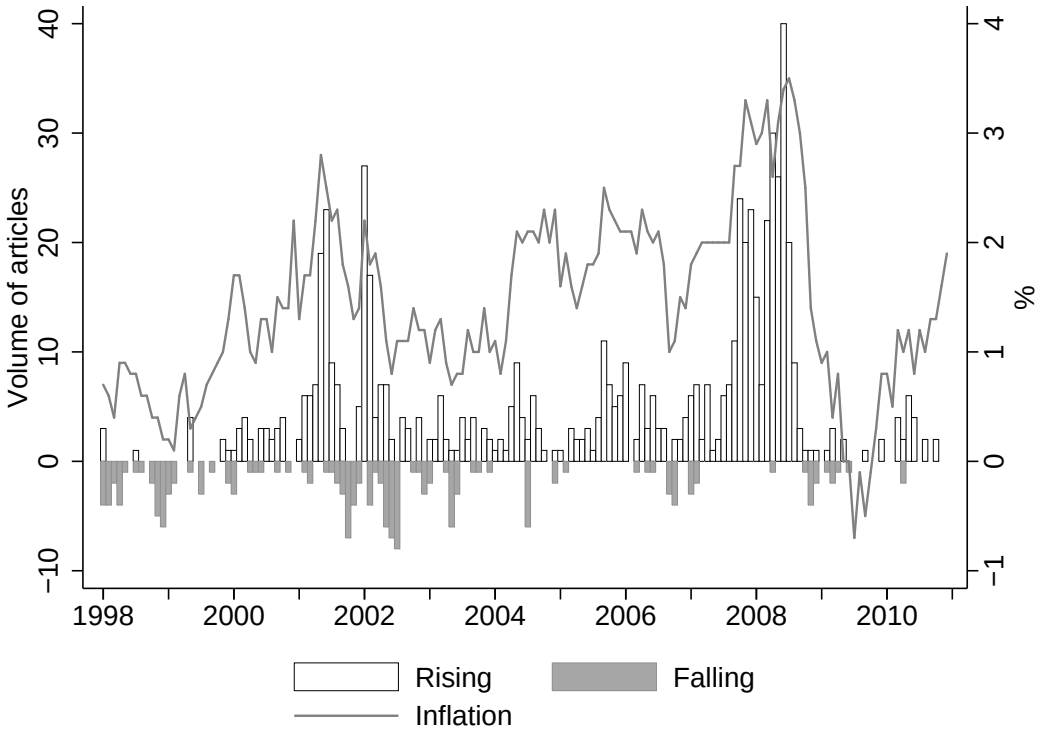
In this section we will introduce our empirical setup that is motivated by the model described in the preceding Section 2. Recall Equation (2) derived in Section 2:

$$E(\pi_{i,t+1}|\psi_{v,t}) = \rho_t \pi_{i,t} + (1-\rho_t)\bar{\psi}_t,$$

Assuming homogeneous consumers our statistical model then is defined as:

$$E(\pi_{t+1}|\psi_{v,t}) = \rho_t \pi_t + (1-\rho_t)\bar{\psi}_t + \varepsilon_t. \tag{6}$$

Figure 3: News tone and inflation

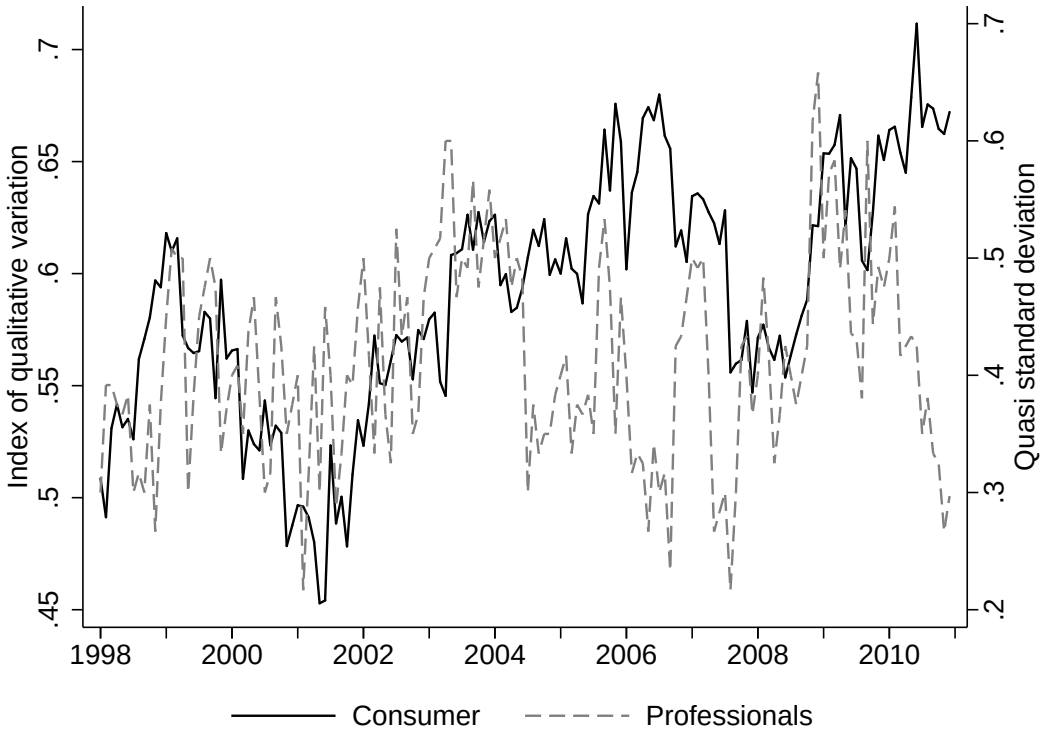


Solid line: HICP inflation (rhs); White bars: Volume of articles reporting rising inflation; Shaded bars: Volume of articles on falling inflation (inverted).

This equation model inflation expectations as a function of its own prior forecast and the average public signal. For the prior expectations π_i we employ the last months average of inflation expectations of the public while $\bar{\psi}_t$ will be replaced by the average forecast of the professional forecasters.

Additionally, we incorporate stochastic volatility to safeguard us against any possible events e.g. the cash changeover that may not only impact the conditional means of our variables but also their variances. It can be also motivated by the rational inattention setup of Maćkowiak and Wiederholt (2009) where the attention depends also on the variability of the idiosyncratic component. It can also be argued that when the variance of ε_t is actually time-varying but assumed to be fixed the estimated change in the parameter ρ_t

Figure 4: Disagreement on inflation expectations



Disagreement of consumer inflation expectations measured by the index of qualitative variation: solid line, left-hand scale; Disagreement of professional forecasters measured by the quasi standard deviation: dashed line, right-hand scale

is exaggerated.[9] In fact, Figure 1 indicates that the error term ε_t is of heteroscedastic nature. To model the time variation in the variance, we introduce stochastic volatility into Equation (6).[10] In particular, we assume that $\varepsilon_t = e^{h_t}\zeta_t$, with $\zeta_t \sim N(0,1)$. Thus, Equation (6) can be rewritten as:

$$E(\pi_{t+1}|\psi_{v,t}) = \rho_t\pi_t + (1-\rho_t)\bar{\psi}_t + e^{h_t}\zeta_t, \tag{7}$$

[9]See Sims (2001), Stock (2001), and Cogley and Sargent (2005) on a more detailed discussion of this issue.

[10]See, for instance, Jacquier et al. (1994) and Kim et al. (1998) for the estimation of univariate models with time-varying variances and Cogley and Sargent (2005) and Primiceri (2005) for the estimation of multivariate models with time-varying coefficients and stochastic volatilities.

where

$$h_t = h_{t-1} + \nu_t \tag{8}$$

is the stochastic volatility component with $\nu_t \sim N(0, \sigma_\nu^2)$.

Certainly, as ρ_t is time varying and is also determined by a very specific set of variables we need to harness a very flexible econometric approach that takes account for this. In our paper we therefore opt to use a state space framework. Moreover, we estimate the state space structure in a Bayesian fashion accounting additionally for a truncation in the parameter ρ_t and for a time-varying variance structure. Truncating ρ_t is necessary as ρ_t needs to lie between $[0, 1]$.

The dynamic of ρ_t follows the general expression:

$$\rho_t = \gamma_1 \rho_{t-1} + \sum_{i=2}^{K} \gamma_i x_{i,t} + \eta_t, \tag{9}$$

with $\eta_t \sim N(0, \sigma_\eta^2)$, $E(\nu_t, \eta_t) = 0$, and $\rho_t \in [0, 1]$. $\{x_{2,t}, \ldots, x_{K,t}\}$ are further explanatory variables such as disagreement of professional forecasters or the amount of news that will be added subsequently.

Moreover, we have to model the time varying parameter ρ_t. ρ_t as formulated in Equation (9) is a function of the variances of the signals as well as the volume of the public signal:

$$\rho_t = \frac{\frac{1}{V}\sigma_\psi^2}{\sigma_a^2 + \frac{1}{V}\sigma_\psi^2}.$$

For the volume variable V we use the sum of news reports on inflation in a given month. To model the precision of the public signal and the prior expectations of consumers we employ the introduced disagreement measures. σ_a^2 will be the dispersion of expectations of consumers calculated as the index of qualitative variation. The quality of the public signal provided by the average estimate of inflation expectations of professional forecasters σ_ψ^2 is

represented by the quasi standard deviation of the individual inflation forecasts.

The model can be cast into a state space framework, where Equation (7) is the observation equation and Equations (9) and (8) are the state equations.

4.1 Priors

Almost all priors were specified to be weakly informative and independent of each other. For the coefficients in Equation (9) and the initial states of ρ_t and h_t, we assumed normal distributions and for all variances we assumed inverted gamma distributions.

We begin with the normally distributed prior for the parameters in the state equation for ρ_t:

$$\gamma_{i,prior} \sim N(\gamma_i, G_{i,\gamma})$$

for $i = 1, \ldots, K$. For the mean of the parameters we assume $\gamma_1 = 1$, implying a persistent updating scheme, whereas for the other parameters we assume $\gamma_i = 0$. However, as we would like to put substantial weight on the sample information the prior for the parameters is specified to be relatively diffuse. Thus, $G_{i,\gamma} = 1000$ for $i = 1, \ldots, K$.

The prior on the variance of the disturbance in the equation for ρ_t follows an inverted gamma distribution and can be expressed as:

$$\sigma^2_{\eta,prior} \sim IG\left(\frac{\alpha_\eta}{2}, \frac{\delta_\eta}{2}\right),$$

where $\alpha_\eta = 10$ and $\delta_\eta = 0.1$ which represents a relatively weak prior.

The prior on the variance of the error term in the equation for h_t follows an inverted gamma distribution and is described as:

$$\sigma^2_{\nu,prior} \sim IG\left(\frac{\alpha_\nu}{2}, \frac{\delta_\nu}{2}\right),$$

where $\alpha_\nu = 60$ and $\delta_\nu = 1$. To check robustness, we experimented with different combinations of α_ν and δ_ν. It turns out that our main conclusions are robust against variations in α_ν and δ_ν.

Because $\rho_t \in [0, 1]$ the prior on the initial state of ρ_t follows a truncated normal distribution and can be expressed as:

$$\rho_0 \sim TN(\rho_0, G_\rho),$$

where TN is the truncated normal distribution, $\rho_0 = 0$, and $G_\rho = 10$. The prior on the initial state of h_t is specified as:

$$h_0 \sim N(h_0, G_h),$$

where $h_0 = log(0.5)$ and $G_h = 1$.

4.2 Estimation

To estimate the model we apply Bayesian methods. In the Bayesian approach prior distributions are multiplied with the likelihood function in order to obtain the posterior distributions. Taking a Bayesian perspective allows to characterize the uncertainty around our estimates in a natural way, i.e. the posterior characteristics are summarized using graphical representations or by simply calculating means and variances. Moreover, truncating ρ_t to the interval $[0, 1]$ given an appropriate indicator function is also straightforward in this context. Most importantly, the Gibbs sampling procedure used for estimation is very efficient for this class of models.

The Gibbs sampler, which is a Markovian updating scheme, is usually applied when draws from the joint probability distribution of the model are not available. Instead of using the joint distribution directly, the Gibbs sampling algorithm uses a full set of conditional distributions to simulate draws from the desired joint distribution. By simulating

iteratively from the conditional distribution the simulated draws from the conditional distributions converge (under mild conditions) to draws from the true joint distribution (e.g Geman and Geman (1984); Gelfand and Smith (1990)). See the Appendix for a more detailed discussion of the estimation procedure and the particular Gibbs sampling algorithm.

5 Results

The Gibbs sampling algorithm described in Section 4.2 was used to generate 100,000 draws from the posterior, with the first 80,000 discarded as burn-in. We saved each 10th draw to reduce autocorrelation across the draws resulting into a sample of 2000 draws. To ensure convergence we conducted several tests, some visual and others numerical. All convergence diagnostics were satisfactory.[11]

Figure 5 plots the estimated parameter ρ_t over time. The figure refers to a model following closely Equation (6) which corresponds to our Model 2 in Table 1. The light grey area corresponds to a one standard deviation and the dark grey area to a two standard deviation band. Several lessons can be learned from this figure.

First of all, we can see that there is substantial variation in the parameter ρ_t. This clearly justifies the effort to set up the very flexible econometric framework and implies that inflation expectations can be very persistent but also change very rapidly. For instance, during the years 2005–2007 inflation expectations of consumers were very persistent. To the contrary during the years 1999–2001 inflation expectations were adjusted very frequently. Note that estimating the observation equation via OLS assuming a constant $\rho_t = \bar{\rho}$ results in an average coefficient estimate for $\bar{\rho} = 0.84$ which is highly significant at

[11] We made use of traceplots, running mean plots, and autocorrelations at various lags and calculated Geweke's χ^2 test (e.g. Geweke, 1999). Additionally, we repeatedly started from different relatively dispersed starting values and experimented with different numbers of draws. The traceplots, running mean plots, and the detailed numerical convergence diagnostics are available on request. The statistical tests were calculated using the Matlab code provided by James P. LeSage.

the 1% level. This result implies that agents update their inflation expectations every six to seven months. Moreover, a test whether two separate coefficients on the prior forecast of the consumers and the public signal made by the professional forecaster sum up to one cannot be rejected (null hypothesis that both coefficients sum up to one) at any level of significance.[12] This indicates that the imposed restriction that $\rho_t \in [0, 1]$ is not very tight. Taking this OLS result as a benchmark to our time-varying representation also shows that the point estimate of 0.84 is not representative at all, as the updating frequency can vary between close to 1 and 0.6. Hence, the people adjust their inflation expectations between 2-3 months and 33 months.

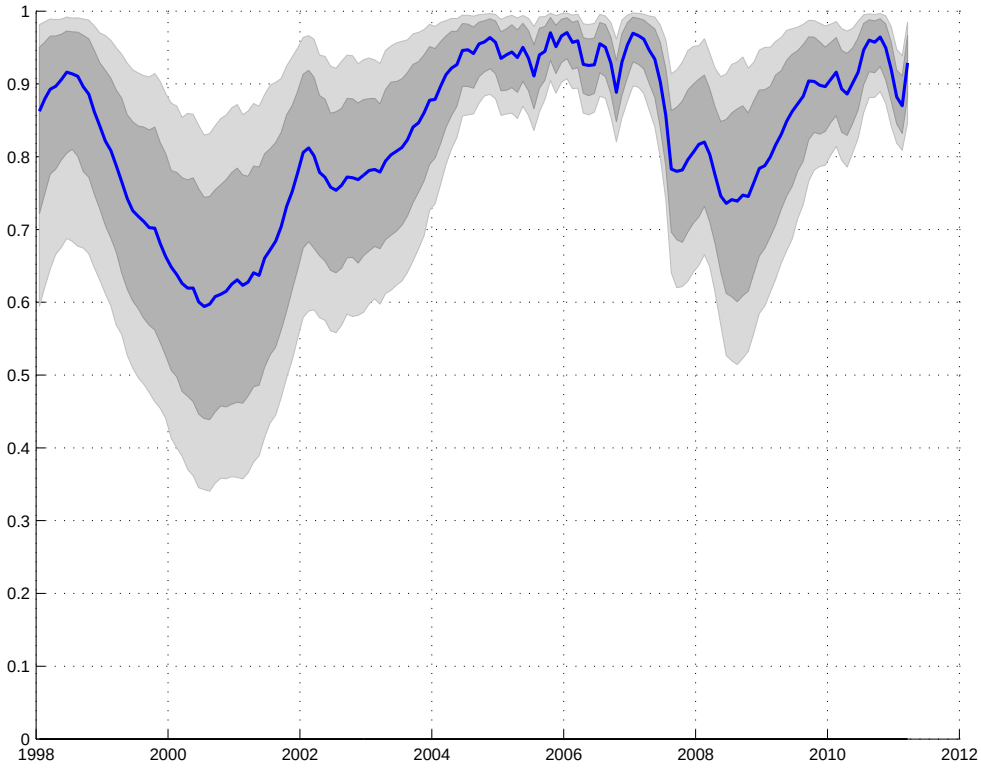
Figure 6 shows the evolution of the stochastic volatility. The evolution of the stochastic volatility reveals presence of time-variation in the variance. The euro cash change changeover seems to be an event where the variation of the model significantly changed. Moreover, there is a sharp increase due to the financial crisis. Furthermore, following Maćkowiak and Wiederholt (2009) we have argued that idiosyncratic components should be linked to the updating speed of expectations. Indeed the two peaks of the stochastic volatility match the two occasions where consumers have been very attentive as shown in Figure 5.[13] This implies that consumers are more attentive if there is a higher variation or uncertainty in the economy.

We are not only interested in describing the time path of the updating parameter ρ_t but have clear priors given by the Bayesian model which determinants should affect it. Table 1 contains the median as well as the standard errors of the coefficients of the explanatory variables put into the state equation. We start off with a very simplistic state equation where ρ_t depends on its own past value plus an error term which is denoted as Model 1. The median of γ_1 amounts to 0.89 and thus shows a high persistence of the updating

[12] $Prob > F = 0.8553$ and $F(1, 114) = 0.03$. This result is also in line with the findings of Carroll (2003) for the U.S.

[13] The correlation of both series amounts to -0.68.

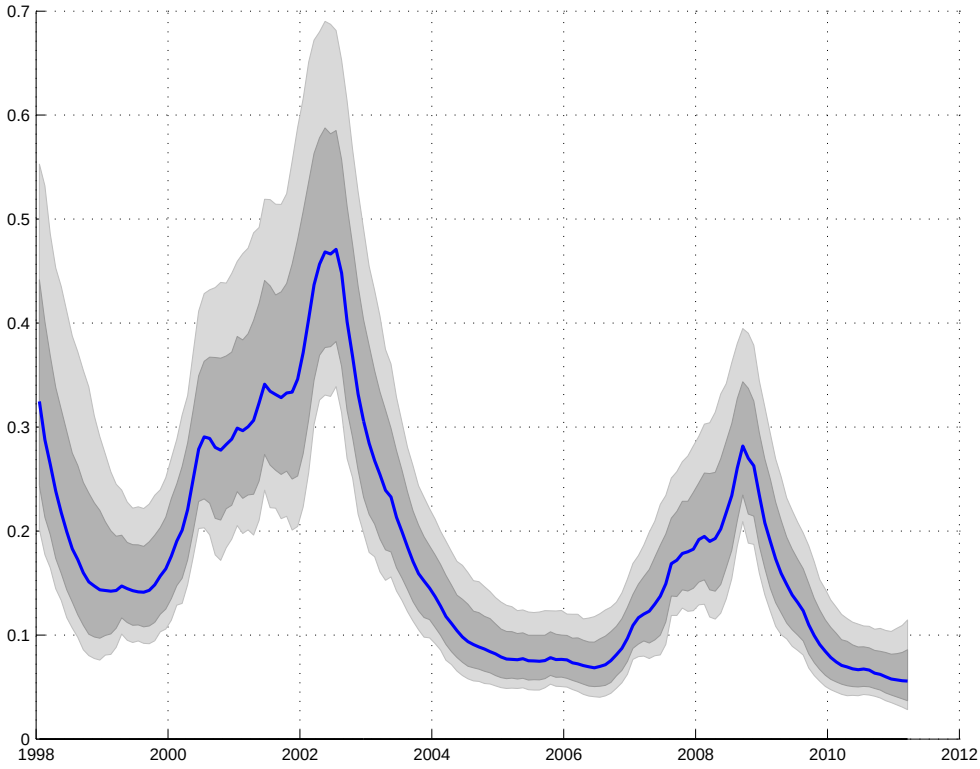
Figure 5: Plot of ρ_t



Solid line shows the median of posterior of ρ_t at each point in time; the dark shaded area indicates the 16th and 84th percentiles and the light shaded area the 5th and 95th percentiles of ρ_t.

parameter. This is not surprising as it is unlikely that all consumers will always switch from fully updating to the public signal and the next period being completely inattentive towards any new information. In Model 2 we add the dispersion variables and the amount of news. Notably we still leave the own lag of ρ_t in the equation. Figure 7 shows the posterior distributions of the corresponding coefficients of Model 2. As can be seen in the upper panel of Figure 7, the posterior probability mass is still centered around 0.9. Moreover, the relative dispersion between consumers and professionals has the expected sign and as can be seen from the middle panel in Figure 7 has sufficient probability mass above zero. A higher disagreement of the professional forecaster thus should lead to greater persistence

Figure 6: Stochastic Volatility



Solid line shows the median of the posterior of h_t at each point in time; the dark shaded area indicates the 16th and 84th percentiles and the light shaded area the 5th and 95th percentiles of h_t.

in the expectations of the consumers as they put less weight on the more imprecise public signal. Concerning the coefficient of the volume variable the probability mass is mostly centered around zero but has at least some probability for the right sign as can be inferred from the lower panel in Figure 7. More news should lead the consumers to be more in-line with the inflation expectations of the professionals. As previous papers have shown that there might be some asymmetry in the response to news, e.g., that especially bad news lead have an effect we now split up the volume variable into good news, bad news and neutrally toned news.[14] As can be seen from Model 3 again the own lag as well as the variable accounting for the dispersion in the news have ample probability mass above zero.

[14]See, e.g., Soroka (2006) or Lamla and Lein (2008).

Among the three new news variables good news are not relevant but bad news as well as neutrally toned news have some support by the data. It seems that especially the neutrally toned news trigger updating of consumers.

To sum up, we find that the updating speed of expectations varies substantially. There are times when people are very attentive and periods when people are less responsive. If they are very attentive they adjust their expectations every 2-3 months. If they are less attentive inflation expectations are modified after 33 months. Moreover, we show that adjustment of expectations can be triggered by some variables. Especially the quality of the signals is of great importance for the formation of inflation expectations of consumers. Moreover, we find only moderate support for the volume effect. In line with the model more news increases the weight put on the public signal stemming from the professional forecasters. However, consumers will follow this public signal if its information content is better than its own forecast quality. Finally, we also point to a link between the updating frequency and the stochastic volatility. The higher the stochastic volatility the more attention people assort to inflation expectations.

6 Conclusion

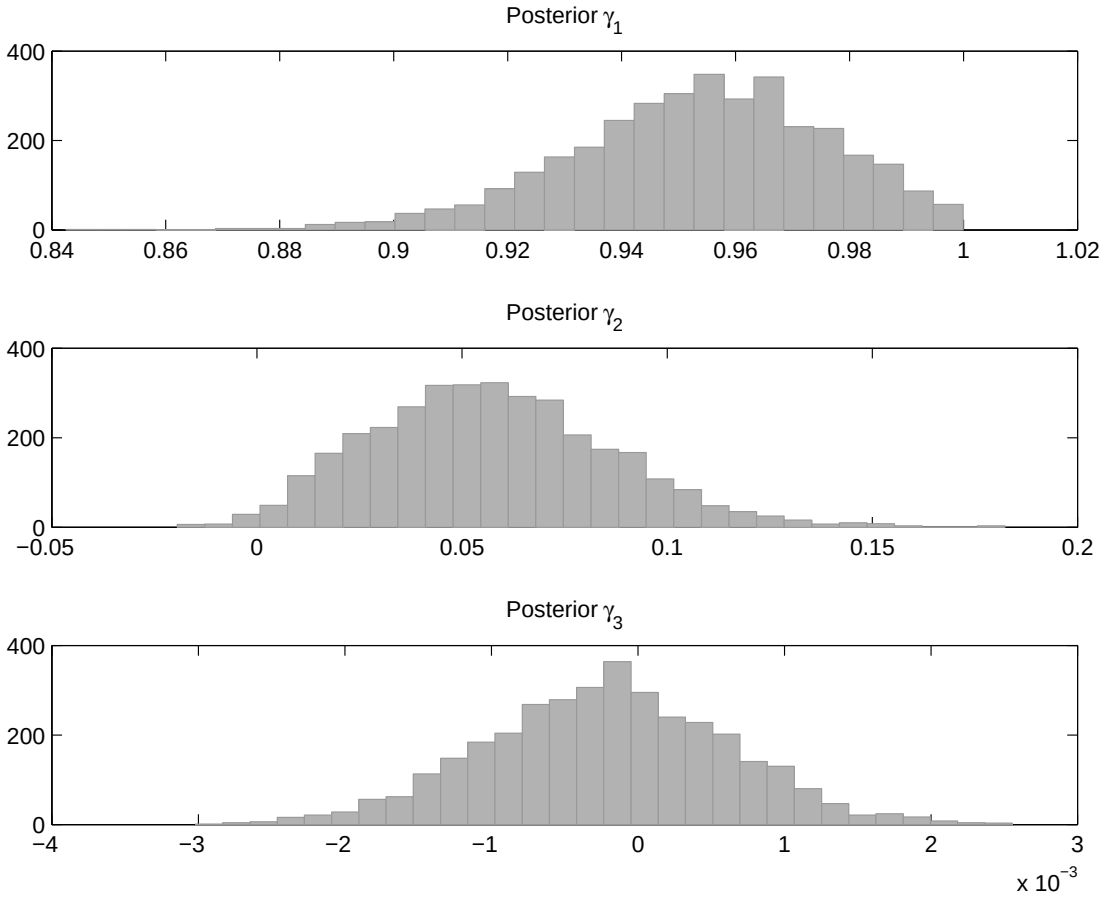
In this paper we shed light on the updating process of inflation expectations inspired by the idea that people face informational frictions. Based on a signal extraction framework we model the updating behavior of consumers. In that model we consider both the amount of news as well as their respective quality. While on the one hand more news increase the propensity to update on new information, signals of poor quality, on the other hand, reduce the weight people place on the incoming signal. We incorporate the implications of the model empirically by employing a very flexible econometric approach. The estimation approach allows us to track the attentiveness of consumers at each moment of time. Our

Table 1: Estimation results

Variable/Model	Posterior Mean	Standard Deviation
Modell 1		
γ_1 ρ_{t-1}	0.96	0.02
Modell 2		
γ_1 ρ_{t-1}	0.95	0.02
γ_2 σ_ψ^2/σ_a^2	0.08	0.05
γ_3 Volume	-0.0003	0.001
Modell 3		
γ_1 ρ_{t-1}	0.94	0.02
γ_2 σ_ψ^2/σ_a^2	0.10	0.04
γ_4 Good	0.002	0.003
γ_5 Bad	0.002	0.001
γ_6 Neutral	-0.004	0.002

estimation results indicate that the formation of inflation expectations is a very dynamic, state-dependent process. While we identify periods where people are very reluctant in changing their inflation expectations, we also observe episodes where people adopt to new information very rapidly. Furthermore, changes in the speed of adjustment of inflation expectations are linked to the quality of signals people receive. Moreover, there is some evidence that the adoption of new information is facilitated by more news. Finally, uncertainty as captured by stochastic volatility is positively related to the attentiveness of consumers.

Figure 7: Histograms of parameter estimates



Histograms show the posterior distributions of the parameters of Model 2 (γ_1, γ_2 and γ_3) for the remaining 2000 draws.

References

Ball, L., N. Gregory Mankiw, and R. Reis (2005): "Monetary policy for inattentive economies," *Journal of Monetary Economics*, 52, 703–725. 2

Berk, J. M. (1999): "Measuring Inflation Expectations: A Survey Data Approach," *Applied Economics*, 31, 1467–1480. 9

Bundesbank (2007): Monatsbericht november, Deutsche Bundesbank.

Carlson, J. A. and M. Parkin (1975): "Inflation Expectations," *Economica*, 42, 123–138. 9

Carroll, C. D. (2003): "Macroeconomic Expectations of Households and Professional Forecasters," *The Quarterly Journal of Economics*, 118, 269–298. 2, 3, 6

Carter, C. K. and R. Kohn (1994): "On Gibbs Sampling for State Space Models," *Biometrika*, 81, 541–553. 32, 36

Cogley, T. and T. J. Sargent (2005): "Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S." *Review of Economic Dynamics*, 8, 262–302. 15

Coibion, O. (2010): "Testing the Sticky Information Phillips Curve," *The Review of Economics and Statistics*, 92, 87–101. 3

Coibion, O. and Y. Gorodnichenko (2008): "What Can Survey Forecasts Tell Us About Informational Rigidities?" NBER Working Papers 14586, National Bureau of Economic Research, Inc. 3

D'Amico, S. and A. Orphanides (2008): "Uncertainty and disagreement in economic forecasting," Finance and Economics Discussion Series 2008-56, Board of Governors of the Federal Reserve System (U.S.). 12

DellaVigna, S. and E. Kaplan (2007): "The Fox News Effect: Media Bias and Voting," *The Quarterly Journal of Economics*, 122, 1187–1234. 4

Döpke, J., J. Dovern, U. Fritsche, and J. Slacalek (2008): "The Dynamics of European Inflation Expectations," *The B.E. Journal of Macroeconomics: Topics in Macroeconomics*, 8, 1540–1561. 3

Dräger, L. (2011): "Endogenous Persistence with Recursive Inattentiveness," Macroeconomics and Finance Series 201103, Hamburg University, Department Wirtschaft und Politik. 4

Dueker, M. (2006): "Kalman filtering with truncated normal state variables for Bayesian estimation of macroeconomic models," *Economics Letters*, 93, 58–62. 32

Frühwirth-Schnatter, S. (1994): "Data Augmentation and Dynamic Linear Models," *Journal of Time Series Analysis*, 15, 183–202. 32

Fuller, W. A. (1996): *Introduction to Statistical Time Series*, NJ: John Wiley & Sons, Inc. 35

Gelfand, A. E. and A. F. M. Smith (1990): "Sampling-Based Approaches to Calculating Marginal Densities," *Journal of the American Statistical Association*, 85, 398–409. 19

Geman, D. and S. Geman (1984): "Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-6, 721–741. 19, 31

Geweke, J. (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development,and Communication," *Econometric Reviews*, 18, 1–73. 19

Giordani, P. and P. Söderlind (2003): "Inflation Forecast Uncertainty," *European Economic Review*, 47, 1037–1059. 13

Hamilton, J. (2004): *All the News Thats Fit to Sell: How the Market Transforms Information into News*, Princeton, N.J.: Princeton University Press. 8

Holsti, O. R. (1969): *Content Analysis for the Social Sciences and Humanities*, Reading, MA: Addison-Wesley. 7

Jacquier, E., N. G. Polson, and P. E. Rossi (1994): "Bayesian Analysis of Stochastic Volatility Models," *Journal of Business & Economic Statistics*, 12, 371–89. 15

Kim, S., N. Shephard, and S. Chib (1998): "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," *Review of Economic Studies*, 65, 361–393. 15, 35, 36

Lahiri, K. and X. Sheng (2008): "Evolution of Forecast Disagreement in a Bayesian Learning Model," *Journal of Econometrics*, 144, 325–340. 12

——— (2010): "Measuring forecast uncertainty by disagreement: The missing link," *Journal of Applied Econometrics*, 25, 514–538. 12

Lamla, M. J. and S. M. Lein (2008): "The Role of Media for Consumers' Inflation Expectation Formation," KOF Working Papers 201, ETH Zurich. 4, 8, 22

Lamla, M. J. and T. Maag (2012): "The Role of Media for Inflation Forecast Disagreement of Households and Professionals," *Journal of Money, Credit and Banking*, forthcoming. 4, 12, 13

Maćkowiak, B. and M. Wiederholt (2009): "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, 99, 769–803. 2, 6, 14, 20

Mankiw, N. G. and R. Reis (2011): "Imperfect Information and Aggregate Supply," *Handbook of Monetary Economics*, 3A, 182–230. 2

Mankiw, N. G., R. Reis, and J. Wolfers (2003): "Disagreement about Inflation Expectations," Working Paper 9796, NBER. 3

Mestre, R. (2007): "Are survey-based inflation expectations in the Euro area informative?" ECB Working Paper 721, European Central Bank. 9

Paciello, L. and M. Wiederholt (2012): "Exogenous Information, Endogenous Information and Optimal Monetary Policy," mimeo. 2

Primiceri, G. E. (2005): "Time Varying Structural Vector Autoregressions and Monetary Policy," *Review of Economic Studies*, 72, 821–852. 15

Reis, R. (2006a): "Inattentive consumers," *Journal of Monetary Economics*, 53, 1761–1800. 2

——— (2006b): "Inattentive Producers," *Review of Economic Studies*, 73, 793–821. 2

——— (2012): "When Should Policymakers Make Announcements?" mimeo, Columbia University. 2

Sims, C. A. (2001): "Evolving Post-World War II U.S. Inflation Dynamics: Comment," *NBER Macroeconomics Annual*, 16, 373–379. 15

——— (2003): "Implications of Rational Inattention," *Journal of Monetary Economics*, 50, 665–690. 2

——— (2010): "Rational Inattention and Monetary Economics," Elsevier, vol. 3 of *Handbook of Monetary Economics*, 155 – 181. 2

Soroka, S. N. (2006): "Good News and Bad News: Asymmetric Responses to Economic Information," *The Journal of Politics*, 68, 372–385. 22

Stock, J. H. (2001): "Evolving Post-World War II U.S. Inflation Dynamics: Comment," *NBER Macroeconomics Annual*, 16, 379–387. 15

A Appendix

A.1 Overview over the Gibbs Sampler

The Gibbs sampler for our model proceeds as follows. First of all, suppose that $\Theta \equiv [\gamma_1, \gamma_2, \gamma_3, \sigma_\eta^2, \sigma_\nu^2]$ contains all parameters, ρ^T contains all ρ_ts, h^T all h_ts, where the two latent variables are summarized in the vector $L \equiv [\rho^T, h^T]$, to simplify exposition. Moreover, suppose that the joint distribution $p(\Theta, L, y^T)$, where y^T denotes the data used, can be divided into the conditional distributions $p(\Theta|L, y^T)$ and $p(L|\Theta, y^T)$.

The sampler can be initiated with an arbitrary value $L^{(0)}$.[15] A first draw $\Theta^{(1)}$ for Θ given the starting value $L^{(0)}$ is simulated from the conditional distribution $p(\Theta|L^{(0)})$. The drawn $\Theta^{(1)}$ can in turn be used to sample a first draw for L from the conditional distribution $p(L|\Theta^{(1)})$. More generally, $\Theta^{(w)}$ given the previous cycle's draw for L can be sampled from $p(\Theta|L^{(w-1)})$ and $L^{(w)}$ given $\Theta^{(w)}$ can be drawn from $p(L|\Theta^{(w)})$. This results into the Gibbs sequence $\{\Theta^{(w)}, L^{(w)}\}$, which converges, under mild conditions, to draws from the desired joint distribution $p(\Theta, L)$ at a geometric rate in w (Geman and Geman (1984)). Note that, $\{\Theta^{(w)}\}$ and $\{L^{(w)}\}$ are themselves Gibbs sequences, cycling over the parameter space and latent variables, respectively. The Gibbs sampling involves the following steps

1. Initialize h^T,s^T, σ_η^2, and $\Gamma \equiv [\gamma_1, \gamma_2, \gamma_3]'$.[16]

2. Sample ρ^T from $p(\rho^T|h^T, \Gamma, \sigma_\eta^2, y^T)$.

3. Sample h^T from $p(h^T|\rho^T, s^T, \sigma_\nu^2, y^T)$.

4. Sample s^T from $p(s^T|h^T, y^T)$.

5. Sample Γ from $p(\Gamma|\rho^T, \sigma_\eta^2, y^T)$.

[15] Note that superscripts in parenthesis denote draws from the conditional distributions.

[16] s^T is needed for the calculation of the stochastic volatility and will be explained in more detail in Section A.3.

6. Sample σ_η^2 from $p(\sigma_\eta^2|\rho^T, \Gamma, y^T)$.

7. Sample σ_ν^2 from $p(\sigma_\nu^2|h^T)$.

8. Go to step 2.

A.2 Sampling the truncated state variable ρ^T

Given the the stochastic volatility h^T, the hyperparameters Γ and σ_η^2, and the data y^T we draw the vector of truncated updating variables ρ^T by using a modified version of the forward filtering and backward sampling algorithm of Carter and Kohn (1994) and Frühwirth-Schnatter (1994). Because $\rho_t \in [0, 1]$, the usual Gaussian Kalman filter cannot be applied anymore. Hence, we employ the procedure as described in Dueker (2006), which allows to apply the Kalman filter to state space systems with truncated state variables. This is accomplished by simply readjusting the forecast error and forecast error variance of the Kalman filter equations and thus making the algorithm of Carter and Kohn (1994) applicable again.

Note that our state space system consists of Equation (7) as the observation equation:

$$\pi_{t+1}^* = \rho_t \pi_t^* + e^{h_t} \zeta_t, \text{[17]} \tag{10}$$

and Equation (10) as the state equation:

$$\rho_t = \gamma_1 \rho_{t-1} + \theta x_t + \eta_t,$$

where θ is a parameter vector and x_t a matrix containing the explanatory variables. Most importantly, because ρ_t is restricted to interval between zero and one the median and the

[17] This just is a rewritten version of Equation (7) with $\pi_{t+1}^* = E(\pi_{t+1}|\psi_{v,t}) - \bar{\psi}_t$ and $\pi_t^* = \pi_t - \bar{\psi}_t$

variance of η_t has to be modified. Conditional on the truncation the mean of η_t changes to $\sigma_\eta^2\kappa$, where

$$\kappa = \frac{\phi(\underline{\alpha}) - \phi(\overline{\alpha})}{\Phi(\overline{\alpha}) - \Phi(\underline{\alpha})}$$

with

$$\underline{\alpha} = \sigma_\eta^{-1}(-\gamma_1\rho_{t-1|t-1} - \theta x_t)$$

and

$$\overline{\alpha} = \sigma_\eta^{-1}(1 - \gamma_1\rho_{t-1|t-1} - \theta x_t).$$

Accordingly, the variance of η_t conditional on the truncation changes to $\sigma_\eta^2\mathcal{K}$, where

$$\mathcal{K} = 1 - \kappa^2 + \frac{\underline{\alpha}\phi(\underline{\alpha}) - \overline{\alpha}\phi(\overline{\alpha})}{\Phi(\overline{\alpha}) - \Phi(\underline{\alpha})}$$

Given the truncation, the prediction steps of the Kalman filter can be modified to:

$$\rho_{t|t-1} = \gamma_1\rho_{t-1|t-1} + \theta x_t + \sigma_\eta\kappa$$

$$P_{t|t-1} = \gamma_1 P_{t-1|t-1}\gamma_1 + \sigma_\eta^2\mathcal{K}.$$

To update the prediction steps we first have to derive the forecast error

$$f_{t|t-1} = \pi_{t+1}^* - \rho_{t|t-1}\pi_t^*,$$

its variance

$$w_t = \pi_t^* P_{t|t-1}\pi_t^* + \sigma_{\epsilon,t}^2$$

and the Kalman gain

$$K_t = P_{t|t-1}\pi_t^* w_t^{-1}.$$

The updating equations of the Kalman filtering procedure thus are:

$$\rho_{t|t} = \rho_{t|t-1} + K_t f_{t|t-1}$$

and

$$P_{t|t} = P_{t|t-1} + K_t \pi_t^* P_{t|t-1}.$$

As in Carter and Kohn (1994) the truncated version of the joint distribution of ρ^T can be decomposed into:

$$p(\rho^T|h^T, \Gamma, \sigma_\eta^2, y^T) = p(\rho_T|h^T, \Gamma, \sigma_\eta^2, y^T) \prod_{t=1}^{T-1} p(\rho_t|\rho_{t+1}, h^t, \Gamma, \sigma_\eta^2, y^t), \tag{11}$$

with $h^t = [h_1, h_2, \ldots, h_t]$ and $y^t = [y_1, y_2, \ldots, y_t]$. Because ρ_t follows a truncated normal distribution the joint distribution in (11) can also be written as:

$$p(\rho^T|h^T, \Gamma, \sigma_\eta^2, y^T) = TN(\rho_{T|T}, P_{T|T}) \prod_{t=1}^{T-1} TN(\rho_{t|t,\rho_{t+1}}, P_{t|t,\rho_{t+1}}), \tag{12}$$

where TN is the truncated normal distribution.

We begin by generating ρ_T from $TN(\rho_{T|T}, P_{T|T})$, where $\rho_{T|T}$ and $P_{T|T}$ are taken from the last step of the Kalman filter iteration. To generate draws for $\rho_{T-1}, \rho_{T-2}, \ldots, \rho_1$, we sample from $TN(\rho_{t|t,\rho_{t+1}}, P_{t|t,\rho_{t+1}})$, using a backwards moving updating scheme which incorporates at time t information about ρ_t contained in period $t+1$. More precisely, we move backwards and generate ρ_t for $t = T-1, \ldots, 1$ at each step while using the updated Kalman filter estimates and the generated ρ_{t+1} from the previous step. We continue with this procedure until we arrive at the first period. The updating equations for the backward sampling algorithm are described as:

$$\rho_{t|t,\rho_{t+1}} = \rho_{t|t} + P_{t|t}\gamma_1 P_{t+1|t}^{-1}(\rho_{t+1} - \rho_{t+1|t})$$

and

$$P_{t|t,\rho_{t+1}} = P_{t|t} - P_{t|t}\gamma_1 P_{t+1|t}^{-1}\gamma_1 P_{t|t}.$$

A.3 Sampling the stochastic volatility h^T

Given the time-varying updating parameter ρ^T, and the data y^T we observe y_t^*, which is defined as:

$$y_t^* = \pi_{t+1}^* - \rho_t \pi_t^* = e^{h_t}\zeta_t \tag{13}$$

This can be linearized by squaring and taking logarithms of Equation (13):

$$\log\ (y_t^*)^2 = 2h_t + \zeta_t^*,\ \ i = 1, ..., n,$$

where $\zeta_t^* = \log \zeta_t^2$. As $(y_t^*)^2$ can be very small, an offset constant is introduced to make the estimation procedure more robust. This results in the following approximating linear state-space form

$$y_t^{**} = 2h_t + \zeta_t^*, h_t = h_{t-1} + \nu_t, \tag{14}$$

where $y_t^{**} = \log[(y_t^*)^2 + \bar{c}]$. The offset constant $\bar{c}$ was introduced by (Fuller, 1996, pp. 494-7) and is set to 0.001. Although the representation is linear, it is not Gaussian, as the innovations in the measurement equation are distributed as $\log \chi(1)^2$. The Gaussian representation can be found by approximating each element of ζ_t^* by a mixture of normal densities as shown by Kim et al. (1998). In their paper Kim et al. (1998) match a number of moments of the $\log \chi(1)^2$ distribution using a mixture of seven normal densities with component probability q_j, and means m_j and variance v_j^2, $j = 1, .., 7$, as tabulated in Table 2. Hence, each element of ζ_t^* can be approximated as

$$f(\zeta_t^*) \approx \sum_{j=1}^{7} q_j f_N((\zeta_t^*|m_j - 1.2704, v_j^2)).$$

Table 2: Selection of Mixing Distributions

ω	$q_j = Pr(\omega = j)$	m_j	v_j^2
1	0.00730	-10.12999	5.79596
2	0.10556	-3.97281	2.61369
3	0.00002	-8.56686	5.17950
4	0.04395	2.77786	0.16735
5	0.34001	0.61942	0.64009
6	0.24566	1.79518	0.34023
7	0.25750	-1.08819	1.26261

Source: Kim et al. (1998).

An alternative way to express this is

$$\zeta_t^* | s_t = j \sim N(m_j - 1.2704, v_j^2), Pr(s_t = j) = q_j,$$

where $s^T = [s_1, \ldots, s_T]$ is a matrix of unobserved indicator states $s_t \in 1, ..., 7$, selecting at every period which member of the normal distribution mixture is used for the approximation of each element in ζ_t^*. Using the normal approximation to the $\log \chi(1)^2$ innovations transforms the system in (14) in a linear and Gaussian one, making the sampling algorithm of Carter and Kohn (1994) again applicable.

Conditional on y^{**T} and the new h^T, it is possible to sample the new indicator states s^T. This is done by independently drawing each s_t from the probability mass function defined by

$$Pr(s_t = j | y_t^{**}, h_t) \propto q_j f_N(y_t^{**} | 2h_t + m_j - 1.2704, v_j^2),$$

with $j = 1, ..., 7, \quad i = 1, ..., n$, and $t = 1, ..., T$.

A.4 Sampling the hyperparameters Γ, σ_η^2, and σ_ν^2

Given the time-varying updating parameters ρ^T and the stochastic volatility h^T, and the data y^T we obtain draws for the hyperparameters. After combining the conjugate prior distributions described in Section 4.1 and the likelihood function, the coefficients of the state equation Γ are drawn from a multivariate normal distribution, whereas the variance of error term of the state equation σ_η^2 and the variance of the stochastic volatility σ_ν^2 are each drawn from an inverted gamma distribution.